EconS 424 - Repeated Games II

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Harrington, Ch. 14 "Check your Understanding" 1

- Recall the Christie's and Sotheby's infinitely repeated game from class:

		Sotheby's		
		4%	*6%*	*8%*
Christie's	*4%*	0, 0	2, 1	4, -1
	6%	1, 2	4, 4	7, 1
	8%	-1, 4	1, 7	5, 5

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- Suppose that Christie's and Sotheby's want to collude to charge a commission rate of 8%, giving payoffs of 5 for both auction houses. Suppose that if deviation were detected, the punishment is a two period price war in which both auction houses set a commission rate of 4%.
- Derive the conditions for this strategy pair to be a subgame perfect Nash equilibrium.

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- First, note that the Nash equilibrium of the unrepeated game is $\{6\%, 6\%\}$:

		Sotheby's 4%	Sotheby's 6%	Sotheby's 8%
Christie's	4%	0, 0	2, <u>1</u>	4, -1
	6%	<u>1</u>, 2	<u>4</u>, <u>4</u>	<u>7</u>, 1
	8%	-1, 4	1, <u>7</u>	5, 5

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- Histories can be partitioned into three types.
- First, suppose the history is such that both auction houses are to set a rate of 8%. Then the equilibrium condition is

$$\frac{5}{1-\delta} \geq 7 + \delta \times 0 + \delta^2 \times 0 + \delta^3 \left(\frac{5}{1-\delta} \right)$$

where the 7 represents the instantaneous gain in utility from deviating towards 6%, when the other auction house still chooses the cooperative rate of 8%. In the following two periods the grim-trigger strategy prescribes that both players punish by setting a rate of 4% (price war), yielding a payoff of zero in both of these periods. Afterwards, cooperation is reestablished with a corresponding payoff of 5 thereafter.

- Now consider a history whereby there is to be a punishment starting in the current period. Then the equilibrium condition is

$$0 + \delta \times 0 + \delta^2 \left(\frac{5}{1-\delta} \right) \geq 1 + \delta \times 0 + \delta^2 \times 0 + \delta^3 \left(\frac{5}{1-\delta} \right)$$

In this case note that in the right hand side we consider the payoff that I obtain if I deviate towards 6% when my opponent still selects 4% (implementing the price war), which constitutes my most profitable deviation. If I were to do that, however, I would be punished with a price war for two periods, returning to the cooperation rate thereafter.

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- Finally, if the auction houses are in the second period of the punishment, then the equilibrium condition is

$$0 + \delta \left(\frac{5}{1-\delta} \right) \geq 1 + \delta \times 0 + \delta^2 \times 0 + \delta^3 \left(\frac{5}{1-\delta} \right)$$

A similar situation as in the previous slide applies.

- Recall the ABM Treaty infinitely repeated game with imperfect monitoring game from class:

		USSR		
		No ABMs	*Low ABMs*	*High ABMs*
	No ABMs	10, 10	6, 12	0, <u>18</u>
USA	*Low ABMs*	12, 6	8, 8	2, <u>14</u>
	High ABMs	<u>18</u>, 0	<u>14</u>, 2	<u>3</u>, <u>3</u>

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- Suppose a technological advance improves the monitoring technology, so that the probability of detecting ABMs are as follows:

Number of ABMs	Probability of Detecting ABMs
None	0
Low	.3
High	.75

- Using the strategy profile just described, derive the equilibrium conditions.
  - If you answer correctly, then you will find that the restriction factor is less stringent, indicating that better monitoring makes cooperation easier.

- No ABMs is preferred to Low ABMs when

$$\frac{10}{1-\delta} \geq 12+\delta\left[.3\times\left(\frac{3}{1-\delta}\right)+.7\times\left(\frac{10}{1-\delta}\right)\right]$$
$$\Longrightarrow \delta \geq \frac{2}{4.1} \approx .49$$

where $0.3 \times \frac{3}{1-\delta} = 0.3 \times \left[3 + 3\delta + 3\delta^2 + ...\right]$ indicates the expected and discounted stream of profits that the deviating country obtains if its deviation to Low is detected and therefore punished thereafter by reverting to the psNE of the unrepeated game, (High, High), with a corresponding payoff of 3.

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- Similarly, $0.7 \times \frac{10}{1-\delta} = 0.7 \times \left[10 + 10\delta + 10\delta^2 + ...\right]$ represents the expected and discounted stream of profits that the deviating country obtains if its deviation to Low is undetected, and then returns to the cooperative outcome (NoABMs, NoABMs) thereafter.

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- No ABMs is preferred to High ABMs when

$$\begin{aligned}\frac{10}{1-\delta} &\geq 18+\delta\left[.75\times\left(\frac{3}{1-\delta}\right)+.25\times\left(\frac{10}{1-\delta}\right)\right]\\ \Longrightarrow &\quad \delta \geq \frac{8}{13.25}\approx .60\end{aligned}$$

  This strategy pair is a subgame perfect Nash equilibrium when the discount factor is at least 0.6, since $\delta \geq 0.6$ is more restrictive than $\delta \geq 0.49$. With the weaker monitoring technology discussed in class, the discount factor had to be at least 0.74.

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- A similar intuition as above is now applicable for the expected stream of payoffs that the deviating country obtains if its deviation to High is detected, $0.75 \times \frac{3}{1-\delta} = 0.75 \times \left[3 + 3\delta + 3\delta^2 + \ldots\right]$, or undetected, $0.25 \times \frac{10}{1-\delta} = 0.25 \times \left[10 + 10\delta + 10\delta^2 + \ldots\right]$. Therefore, cooperation can be sustained for a larger set of discount factors as the monitoring technology becomes more accurate (closer to perfect monitoring).

Oligopoly and Collusion when firms compete a la Cournot

- Assume that there are two firms competing in quantities (a la Cournot) with zero marginal costs and a demand function given by:

$$p(q_1, q_2) = 1 - q_1 - q_2$$

so the profit function for firm 1 is:

$$\begin{aligned} \pi_1(q_1, q_2) &= p \cdot q_1 = (1 - q_1 - q_2) \cdot q_1 \\ &= q_1 - q_1^2 - q_1 q_2 \end{aligned}$$

Oligopoly and Collusion when firms compete a la Cournot

- Find the equilibrium if the game is played just one time.

Oligopoly and Collusion when firms compete a la Cournot

- The FOCs with respect to q_1 are

$$1 - 2q_1 - q_2 \leq 0$$

assuming we have an interior solution (meaning that both q_1 and q_2 are positive), this FOC holds with equality. Solving for q_1, we have firm 1's best response function

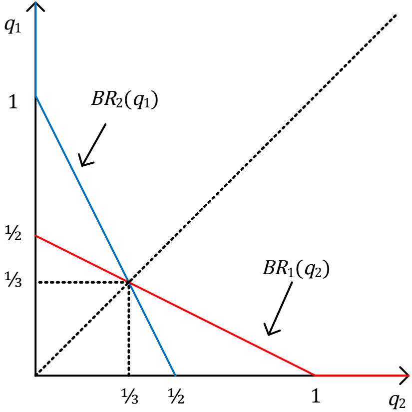
$$q_1 = \frac{1}{2} - \frac{1}{2}q_2$$

- Similarly by firm 2 (since firms 1 and 2 are symmetric):

$$q_2 = \frac{1}{2} - \frac{1}{2}q_1$$

Oligopoly and Collusion when firms compete a la Cournot

- Hence,



Oligopoly and Collusion when firms compete a la Cournot

- The Nash equilibrium is given by:

$$\begin{aligned} q_1 &= \frac{1}{2} - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}q_1\right) \\ &\Longrightarrow \quad q_1^{Cournot} = q_2^{Cournot} = \frac{1}{3} \end{aligned}$$

- So the total production in the market is $Q^{Cournot} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ and the market price is

$$p^{Cournot} = 1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}$$

- In terms of profits, each firm i will get:

$$\begin{aligned} \pi_i &= p \cdot q_i \\ \pi_i^{Cournot} &= \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \end{aligned}$$

Oligopoly and Collusion when firms compete a la Cournot

- Find the equilibrium in a cartel (that is if the two firms collude)

Oligopoly and Collusion when firms compete a la Cournot

- In a cartel, they maximize their joint profits, as a monopoly.

$$\begin{aligned} \Pi(Q) &= p \cdot Q = (1 - Q) \cdot Q \\ &= Q - Q^2 \end{aligned}$$

- The FOC with respect to Q is:

$$1 - 2Q = 0$$

Oligopoly and Collusion when firms compete a la Cournot

- Solving for Q

$$Q^{Cartel} = \frac{1}{2}$$

- So each firm will product half of Q^{Cartel}:

$$q_1^{Cartel} = q_2^{Cartel} = \frac{Q^{Cartel}}{2} = \frac{1}{4}$$

- Thus, the price is

$$p^{Cartel} = 1 - \frac{1}{2} = \frac{1}{2}$$

Oligopoly and Collusion when firms compete a la Cournot

- So the profits for each firm i in the cartel are

$$\pi_i^{Cartel} = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

and as we can see

$$\pi_i^{Cartel} = \frac{1}{8} > \pi_i^{Cournot} = \frac{1}{9}$$

- Can the two firms achieve cooperation with the cartel quantity?

Oligopoly and Collusion when firms compete a la Cournot

- Using the following Grim-Trigger strategy, in time period t:
  - If $t = 1$

$$q_i = q_i^{Cartel} = \frac{1}{4}, \text{ thus } \pi_i = \frac{1}{8}$$

  - If $t > 1$

$$q_i = \begin{cases} q_i^{Cartel} = \frac{1}{4} \text{ as long as } q_i = q_j = \frac{1}{4} \text{ in every previous period} \\ q_i^{Cournot} = \frac{1}{3} \text{ forever otherwise} \end{cases}$$

    with corresponding profit levels

$$\pi_i = \begin{cases} \pi_i^{Cartel} = \frac{1}{8} \text{ as long as } q_i = q_j = \frac{1}{4} \text{ in every previous period} \\ \pi_i^{Cournot} = \frac{1}{9} \text{ forever otherwise} \end{cases}$$

Oligopoly and Collusion when firms compete a la Cournot

- Thus, by cooperating:

$$\frac{1}{8} + \delta\frac{1}{8} + \delta^2\frac{1}{8} + ... = \frac{1}{8} \cdot \frac{1}{1-\delta}$$

- Let us now evaluate the firm's profits from *deviating*: (But towards what?)
- The optimal deviation is:

$$\max_{q_i} \left(1 - q_j^{Cartel} - q_i\right) q_i$$

where firm j sticks to the collusive agreement (producing $q_j^{Cartel} = \frac{1}{4}$) and firm i deviates. The profit maximization problem for firm i is now

$$\max_{q_i} \left(1 - \frac{1}{4} - q_i\right) q_i$$

- We are now ready to find out which value of q_i minimizes firm i's profits given that the other firm (firm j) repects the collusive agreement (j cooperates). i.e.,

$$\max_{q_i} \left(q_i - q_i \frac{1}{4} - q_i^2\right)$$

Oligopoly and Collusion when firms compete a la Cournot

- The FOC with respect to q_i is

$$1 - \frac{1}{4} - 2q_i \leq 0$$

and like before, we assume an interior solution and solve for q_i,

$$q_i^{Dev} = \frac{3}{8}$$

- With profits as

$$\begin{aligned} \pi_i^{Dev} &= \left(1 - \frac{1}{4} - q_i^{Dev}\right) q_i^{Dev} \\ &= \left(1 - \frac{1}{4} - \frac{3}{8}\right) \frac{3}{8} = \frac{9}{64} \end{aligned}$$

Oligopoly and Collusion when firms compete a la Cournot

- So the cooperation will be observed if and only if:

$$\overbrace{\pi_i^{Cartel} \cdot \frac{1}{1-\delta}}^{\text{Payoff from cooperating}} \geq \overbrace{\pi_i^{Dev}}^{\text{Payoff from deviating}} + \overbrace{\pi_i^{Cournot} \frac{\delta}{1-\delta}}^{\text{Punishment thereafter}}$$

$$\frac{1}{8} \cdot \frac{1}{1-\delta} \geq \frac{9}{64} + \frac{1}{9} \cdot \frac{\delta}{1-\delta}$$

- Solving for δ, this condition holds when

$$\delta \geq \frac{9}{17}$$

so collusion is still possible among firms if they do not discount the future too much.