EconS 424 - Bargaining Games and an Introduction to Repeated Games

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Watson, Ch. 19 # 8

- In experimental tests of the ultimatum bargaining game, subjects who propose the split rarely offer a tiny share of the surplus to the other party. Furthermore, sometimes subjects reject positive offers. These findings seem to contradict our standard analysis of the ultimatum bargaining game.
- Many scholars conclude that the payoffs specified in the basic model do not represent the *actual* preferences of the people who participate in the experiments. In reality, people care about more than their own monetary rewards.

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- For example, people also act on feelings of spite and the ideal of fairness. Suppose that, in the ultimatum game, the responder's payoff is $y + a(y - z)$, where y is the responder's monetary reward, z is the proposer's monetary take, and a is a positive constant.
  - That is, the responder cares about how much money he gets *and* he cares about relative monetary amounts (the difference between the money he gets and the money the other player gets).
- Assume that the proposer's payoff is as in the basic model.
- Represent this game in the extensive form, writing the payoffs in terms of m, the monetary offer of the proposer, and the parameter a.

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- First, player 1 (the proposer) offers a division of the pie (of size 1), m, to player 2 (the responder), who either accepts or rejects (this time structure coincides with the ultimatum bargaining game we analyzed in class).
- However, payoffs are not the same as in the standard ultimatum bargaining game.
- In particular, the payoff of player 1 is just the remaining share of the pie that he does not offer to player 2, $1-m$, and the payoff of player 2 is

$$\underbrace{y}_{\text{Responder's share}} + a(y - \underbrace{z}_{\text{Proposer's share}}) = m + a[m-(1-m)] = m + a(2m-1)$$

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- Intuitively, the responder not only cares about his payoff but also about the payoff difference (inequality) between his payoff and that of the proposer.
- Therefore, parameter a denotes how much the responder cares about payoff inequality. These types of preferences have been confirmed in several experiments.

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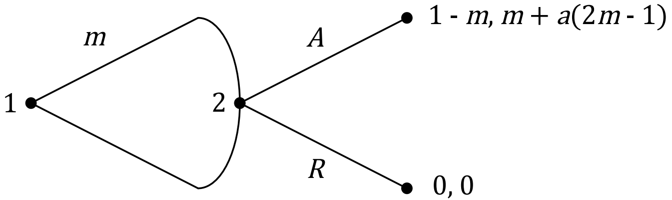
- We depict this modified ultimatum bargaining game in the following figure.



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- Find and report the subgame perfect equilibrium. Note how equilibrium behavior depends on a.

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- Player 2 accepts any offer m from player 1 such that $m + a(2m - 1) \geq 0$. Solving for m, this implies that player 2 will accept any offer such that

$$m \geq \frac{a}{1 + 2a}$$

and since Player 1 wants to maximize his own payoff, he will make the minimum offer that guarantees that player 2 will accept, i.e., $m = \frac{a}{1+2a}$.

- This implies that equilibrium payoffs are

$$\left(\underbrace{1 - \frac{a}{1+2a}}_{\substack{\text{Proposer's} \\ \text{Payoff}}}, \underbrace{\frac{a}{1+2a} + a\left[\frac{2a}{1+2a} - 1\right]}_{\substack{\text{Responder's} \\ \text{Payoff}}} \right) = \left(\frac{1+a}{1+2a}, 0 \right)$$

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- The proposer's equilibrium share and payoff is decreasing in a since its derivative with respect to a is

$$\frac{1(1+2a)-2(1+a)}{(1+2a)^2} = -\frac{1}{(1+2a)^2} < 0$$

whereas the responder's equilibrium share increases in a since its derivative with respect to a is

$$\frac{1(1+2a)-2(a)}{(1+2a)^2} = \frac{1}{(1+2a)^2} > 0$$

however, the equilibrium payoff for player 2 does not change with respect to a (It will always be 0).

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- What is the equilibrium monetary split as a becomes large? Explain why this is the case.

- Taking the limit of our split as a approaches infinity,

$$\lim_{a \to \infty} \frac{a}{1+2a} = \frac{1}{2}$$

which intuitively makes sense, as the more effect that the inequality has on the responder, the closer the payoffs will have to be in order for him to accept. At the extreme, the payoffs will have to be identical and the inequality eliminated completely for the responder to accept.

  - Interestingly, $a = \infty$ is actually the only point where the responder will have a non-zero payoff. Everywhere else, the proposer will guarantee that the responder's payoff is zero. The case where the responder will not accept anything other than an equal split of the pie is the only exception to this.

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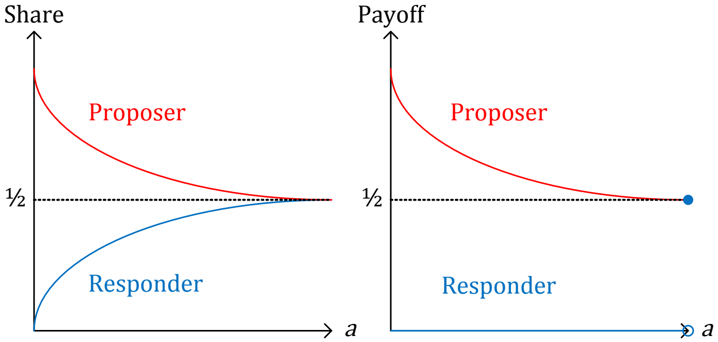
- In the following figure, we represent how the proposer's equilibrium share and payoff (in red) decreases in a, and how the share of the responder (in blue) increases in a.



Watson, Ch. 23 # 1

- Consider the Bertrand oligopoly model, where n firms simultaneously and independently select their prices $p_1, p_2, ..., p_n$, in a market. (These prices are greater than or equal to 0.) Consumers observe these prices and only purchase from the firm (or firms) with the lowest price $\hat{p}$, according to the demand curve $Q = 110 - \hat{p}$. ($\hat{p} = \min\{p_1, p_2, ..., p_n\}$.) That is, the firm with the lowest price gets all of the sales.
- If the lowest price is offered by more than one firm, then these firms equally share the quantity demanded. Assume that firms must supply the quantities demanded of them and that production takes place at a constant cost of 10 per unit. (That is, the cost function for each firm is $c(q) = 10q$.)

Watson, Ch. 23 # 1

- Suppose that this game is infinitely repeated. Define δ as the discount factor for the firms. Imagine that the firms wish to sustain a collusive arrangement in which they all select the monopoly price $p^M = 60$ in each period. What strategies might support this behavior in equilibrium?
  - Don't worry about solving for the conditions on the parameters (yet). Just explain what the strategies are. Remember, this requires specifying how the firms punish each other. Use the Nash equilibrium price as punishment.)

Watson, Ch. 23 # 1

- Consider all players selecting $p_i = \hat{p} = 60$ which is the monopoly price. They keep this price until the end unless someone defects. Otherwise, everyone chooses the perfectly competitive (Bertrand equilibrium) price $p_i = \hat{p} = 10$ thereafter as a punishment.
  - Because $\hat{p} = \min\{p_1, p_2, ..., p_n\}$ and when $\hat{p} = 10$, profit is zero ($p = MC$).

Watson, Ch. 23 # 1

- Derive a condition on n and δ that guarantees that collusion can be sustained.

- Let us first find the total output that firms should jointly produce in order to maximize joint profits:

$$\pi = pQ - c(Q) = \overbrace{(110 - Q)}^{\text{Demand function solved for } p} Q - 10Q$$

taking FOCs with respect to Q,

$$100 - 2Q = 10 \implies Q = 50$$

and the price is

$$p = 110 - Q = 110 - 50 = 60$$

  - Which confirms our monopoly values given in the previous part.

- Hence, every colluding firm produces

$$q^C = \frac{Q}{n} = \frac{50}{n}$$

and its individual profits are

$$\pi^C = pq^C - c(q^C) = 60\left(\frac{50}{n}\right) - 10\left(\frac{50}{n}\right) = \frac{2500}{n}$$

Watson, Ch. 23 # 1

- We know from past exercises that the profit under the Nash equilibrium of the stage game is zero (Since $p = MC$), i.e., $\pi^B = 0$.
- If player i chooses to defect, his best deviation is to set $p_i = 60 - \varepsilon$ (remember that ε is any arbitrarily small positive number).
  - Because the player wants to set price lower than others' prices to get an advantage in the market.
  - i.e., the firm with the lowest price gets all of the sales. If $\varepsilon = 0.000...001$, then the price will be close to 60 and his profit will be close to $\pi^D = 2500$ (since now he gets the whole market).

Watson, Ch. 23 # 1

- To support collusion, it must be that every firm's stream of profits under collusion exceeds its profits from deviating from the collusive agreement. That is,

$$\begin{aligned} \pi^C + \delta\pi^C + \delta^2\pi^C + ... &\geq \pi^D + \delta\pi^B + \delta^2\pi^B + ... \\ \frac{2500}{n} + \delta\left(\frac{2500}{n}\right) + \delta^2\left(\frac{2500}{n}\right) + ... &\geq 2500 + 0 + 0 + ... \end{aligned}$$

  or rearranging,

$$\frac{2500}{n}(1 + \delta + \delta^2 + ...) \geq 2500$$

- Where $\frac{2500}{n}$ represents every firm's profit from colluding at a particular time period, which is discounted along time. In the right-hand side of the inequality, 2500 is the firm's instantaneous profit from defecting (setting a price $p_i = 60 - \varepsilon$ and capturing as a consequence all of the market).
- Such defecting is, however, punished by all other firms in the form of Bertrand equilibrium prices ($p_i = 10$ for all firms $i = 1, 2, ..., n$), thereby reducing profits to zero thereafter.

- Rearranging the above inequality, we obtain:

$$\frac{2500}{n}\left(\frac{1}{1-\delta}\right) \geq 2500$$

and solving this inequality for δ, we have

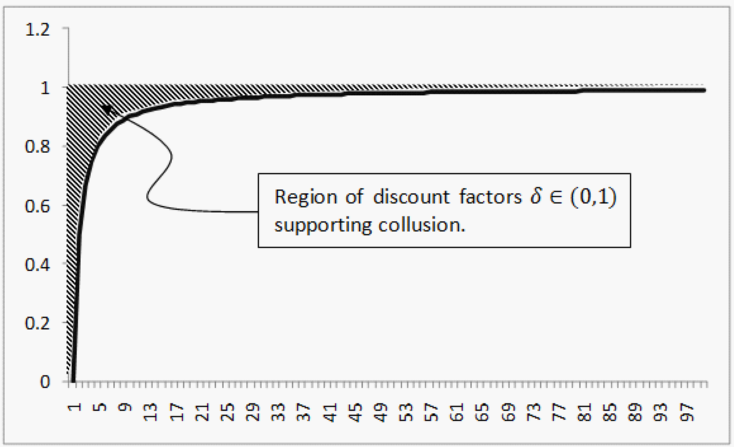
$$\delta \geq \frac{n-1}{n}$$

Watson, Ch. 23 # 1

- What does your answer in the previous part imply about the optimal size or cartels?

Watson, Ch. 23 # 1

- Taking our answer from the previous section. We plot the minimum value for δ (vertical axis) in order to sustain collusion as a function of n (horizontal axis).



Watson, Ch. 23 # 1

- Collusion therefore becomes more difficult to sustain as the number of firms participating in the collusive agreement increases, i.e., collusion can only be supported for a more restrictive set of discount factors.

Watson, Ch. 16 # 6

- Imagine a market setting with three firms. Firms 2 and 3 are already operating as monopolists in two different industries (they are not competitors). Firm 1 must decide whether to enter firm 2's industry and thus compete with firm 2, or enter firm 3's industry and thus compete with firm 3.
- Production in firm 2's industry occurs at zero cost, whereas the cost of production in firm 3's industry is 2 per unit.
- Demand in firm 2's industry is given by $p = 9 - Q$, whereas demand in firm 3's industry is given by $p' = 14 - Q'$, where p and Q denote the price and total quantity in firm 2's industry and p' and Q' denote the price and total quantity in firm 3's industry.

Watson, Ch. 16 # 6

- The game runs as follows: First, firm 1 chooses between E^2 and E^3. (E^2 means "enter firm 2's industry" and E^3 means "enter firm 3's industry.") This choice is observed by firms 2 and 3.
- Then, if firm 1 chooses E^2, firms 1 and 2 compete as Cournot duopolists, where they select quantities q_1 and q_2 simultaneously. In this case, firm 3 automatically gets the monopoly profit of 36 (corresponding to $q_3 = 6$) in its own industry.
- On the other hand, if firm 1 chooses E^3, then firms 1 and 3 compete as Cournot duopolists, where they select quantities q_1' and q_3' simultaneously; and in this case, firm 2 automatically gets its monopoly profit of $\frac{81}{4}$ (corresponding to $q_2 = \frac{9}{2}$).
- Calculate and report the subgame perfect Nash equilibrium of this game. In the equilibrium, does firm 1 enter firm 2's industry or firm 3's industry?

Watson, Ch. 16 # 6

- If firm 1 enters into firm 2's industry, firm 1 and 2 compete a la Cournot. In this context firm 1's profit maximization problem is

$$\max_{q_1} pq_1 = (9 - Q)q_1 = (9 - q_1 - q_2)q_1$$

Taking FOCs with respect to q_1 gives

$$9 - 2q_1 - q_2 = 0$$

and solving for q_1 gives our best response function

$$q_1 = \frac{9 - q_2}{2}$$

Watson, Ch. 16 # 6

- By symmetry, we know that firm 2's best response function will be

$$q_2 = \frac{9 - q_1}{2}$$

and substituting firm 2's best response function into firm 1's gives

$$q_1 = \frac{9 - \left(\frac{9-q_1}{2}\right)}{2} \implies q_1 = q_2 = 3$$

- Then $q_1 = 3$ and $q_2 = 3$, yielding an aggregate output of $Q = 6$. Replacing $Q = 6$ into the demand function $p = 9 - Q = 9 - 6 = 3$. We can then calculate firm 1's profits as

$$\pi_1 = pq_1 = 3(3) = 9$$

- If instead firm 1 enters into firm 3's industry, firm 1 and 3 compete a la Cournot. In this context, firm 1's profit maximization problem becomes

$$\max_{q_1'} p' q_1' - c(q_1') = (14 - Q')q_1' - 2q_1' = (14 - q_1' - q_3')q_1' - 2q_1'$$

Taking FOCs with respect to q_1 gives

$$14 - 2q_1' - q_3' - 2 = 0$$

and solving for q_1', we have firm 1's best response function

$$q_1' = \frac{12 - q_3'}{2}$$

- Again, by symmetry, we know that firm 3's best response function will be
$$q_3' = \frac{12 - q_1'}{2}$$
and substituting firm 3's best response function into firm 1's gives
$$q_1' = \frac{12 - \left(\frac{12 - q_1'}{2}\right)}{2} \implies q_1' = q_3' = 4$$

- Then $q_1' = 4$ and $q_3' = 4$, yielding an aggregate output of $Q' = 8$. Replacing $Q' = 8$ into the demand function $p' = 14 - Q' = 14 - 8 = 6$. We can then calculate firm 1's profits as

$$\pi_1' = p' q_1' - c(q_1') = 6(4) - 2(4) = 16$$

- As we can observe, $\pi_1' = 16 > \pi_1 = 9$, thus firm 1 enters firm 3's industry. The SPNE of this game is

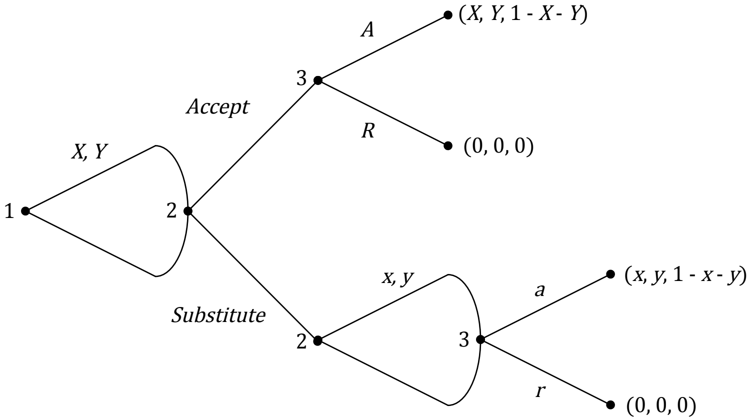
$$\left(E^3/3/4, 3/\frac{9}{2}, 6/4 \right)$$

Watson, Ch. 19 # 7

- Consider a three-player bargaining game, where the players are negotiating over a surplus of one unit of utility. The game begins with player 1 proposing a three-way split of the surplus.
- Then, player 2 must decide whether to accept the proposal or to substitute his own alternative proposal.
- Finally, player 3 must decide whether to accept or reject the current proposal (whether it is player 1's or player 2's). If he accepts, then the players obtain the specified shares of the surplus. If player 3 rejects, then the players each get 0.
- Draw the extensive form of this perfect-information game and determine the subgame perfect equilibria.

Watson, Ch. 19 # 7

- The extensive-form representation of this game is drawn below. See the next slide for an explanation of the notation.



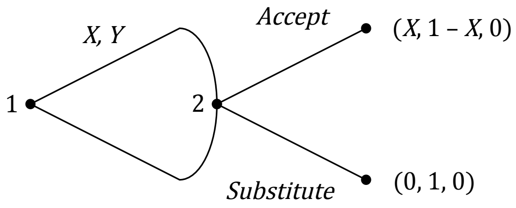
Watson, Ch. 19 # 7

- Where the capital letters represent the shares offered by player 1. X represents the share of the surplus for player 1, Y represents the share of the surplus for player 2, and $1 - X - Y$ represents the share of the surplus for player 3.
- Likewise, the lowercase letters represent the shares substituted by player 2. x represents the share of the surplus for player 1, y represents the share of the surplus for player 2, and $1 - x - y$ represents the share of the surplus for player 3.

Watson, Ch. 19 # 7

- Looking at the lower subgame (initiated after player 2 chooses to substitute player 1's offer), when player 2 makes an offer $1 - x - y$ to player 3, player 3 accepts it if and only if $1 - x - y \geq 0$. Hence, player 2 offers $y = 1$ (which maximizes his own payoff), with resulting payoffs $(0, 1, 0)$.
- Looking at the upper subgame (initiated after player 2 chooses to accept player 1's offer), when player 2 accepts player 1's proposal, player 3 accepts it if and only if $1 - X - Y \geq 0$. Hence, player 1 offers $X + Y = 1$ (which is the minimal offer that guarantees acceptance) with resulting payoffs $(X, 1 - X, 0)$.

- We can then draw a reduced version of this game



Watson, Ch. 19 # 7

- Hence, player 2 chooses to substitute for any $1 - X < 1$, or $X > 0$.
- If $X > 0$, player 2 substitutes, and player 1 (anticipating the substitution) chooses any combination of X and Y. Since his payoff in that event will be zero for all X (See first component of the triple).
- If $X = 0$, then player 2 accepts, and player 1 (anticipating the acceptance) chooses $X = 0$ and $Y = 1$.

Watson, Ch. 19 # 7

- SPNE (Part 1):
  - If $X > 0$:
  - Player 1 chooses any combination of (X, Y).
  - Player 2 substitutes and makes an offer $x = 0$, $y = 1$.
  - Player 3 accepts any offer $(1 - X - Y) \geq 0$ from player 1 or $(1 - x - y) \geq 0$ from player 2.

Watson, Ch. 19 # 7

- SPNE (Part 2):
  - If $X = 0$:
  - Player 1 chooses $X = 0$ and $Y = 1$.
  - Player 2 accepts.
  - Player 3 accepts any offer $(1 - X - Y) \geq 0$ from player 1 or $(1 - x - y) \geq 0$ from player 2.

Appendix - Geometric Series

- The sum of the first n terms of a geometric series is:

$$a + a\delta + a\delta^2 + a\delta^3 + ... + a\delta^{n-1} = \sum_{k=0}^{n-1} a\delta^k = a\frac{1-\delta^n}{1-\delta}$$

- To see why, let:

$$s = a + a\delta + a\delta^2 + a\delta^3 + ... + a\delta^{n-1}$$

and

$$\delta s = a\delta + a\delta^2 + a\delta^3 + a\delta^4 + ... + a\delta^n$$

Appendix - Geometric Series

- Thus,

$$\begin{aligned} s - \delta s &= a - a\delta^n \\ s(1-\delta) &= a - a\delta^n \\ &\implies s = \frac{a - a\delta^n}{1-\delta} = a\frac{1-\delta^n}{1-\delta} \end{aligned}$$

- This series is convergent as $n \to \infty$ if and only if $\delta < 1$, then:

$$s = a \cdot \frac{1}{1-\delta}$$