Rationalizing Time Inconsistent Behavior: The Case of Late Payments*

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Abstract

Consumers often sign contracts in which they consume a good over a period of time, paying for it with a fee due at a later period. Most contracts of this form impose a penalty if the fee is paid late. Despite anecdotal evidence that customers intend to pay on time, many pay late; which we refer to as "preference reversal." In this paper, we show that late payment requires present biasedness, and that shocks expand the range of parameter values under which consumers pay late. Moreover, we show how sellers can increase profits by setting penalties to induce late payments from consumers, who fall prey to their preference reversals over time.

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1 Introduction

Consumers often sign contracts in which they consume a good over a period of time, paying for it with a fee due at a later period. Examples of such goods include cable TV, internet, and cell phone. Although contracts normally specify that customers are expected to pay the bill on time, and most impose a penalty if the fee is paid late, many customers pay late, despite anecdotal evidence that customers intend to pay on time, and can afford to. In the U.S., for instance, 28.4 million households pay at least one bill late per month (Wall Street Journal 3/15/2007), and penalties for late payment increased from $7 billion in 2000 to $22 billion in 2004 (Wall Street Journal April 2004). In the same line, according to the Citi Simplicity Survey (2013) 59% of Americans have paid a bill late in their lifetime (including credit card, utility, cable, etc.), and 88% of those have done so in the past 12 months. The most common reason that consumers use to justify their late payments is forgetfulness (61%), and being busy with work and family obligations (39%). Lack of available funds (42%), while significant, is not the main reason a bill is paid late.

From these reports, it appears people who could pay on time, and sign a contract expecting to do so, often end up paying their bills late. Hence, while some of this consumer behavior can be attributed to financial reasons, it may be that often individuals are simply so busy with family and work that they forget to pay their bills. In this paper, we present two different explanations for such behavior. We also identify the conditions under which a seller can use such consumer behavior by strategically designing fees and penalties to exploit late payments and increase profits. Our paper seeks to answer two questions:

1. Why do consumers sign contracts anticipating paying them on time, but when payment is due, do not pay, thus exhibiting dynamically inconsistent behavior and incurring additional penalties? and,

2. Under what conditions can sellers increase profit by strategically designing fees and penalties that induce consumers to pay their bills late?

While considering the first question, an immediate answer is that consumers exhibit present bias. One explanation for present bias is quasi-hyperbolic discounting (Laibson, 1997, O'Donoghue and Rabin, 1999). When consumers care more about the present than the future, it can lead them to pay bills late. However, this does not provide a complete answer to the question. In particular, as the due date for a bill approaches, there is a small time difference between the current and the future (when consumers are penalized). For this small time period, we show that present bias can only explain late bill payments under restrictive parameter conditions.[1] Hence, as a further explanation we hypothesize that some consumers pay their bill late due to shocks, such as forgetting or family obligations, which, according to the above Citi Survey, account for 61% and 39%, respectively of why payments are late.

[1] Another possibility is procrastination. While procrastination can result from hyperbolic discounting (Murooka and Schwarz, 2016), Herweg and Muller (2011) establish that procrastination can be viewed as rational behavior when the cost of paying today outweighs the costs of paying the penalty.

There is evidence that consumers, over time, often forget contract details, including due dates and late penalties. Decreasing attention abilities have been well documented in the psychological literature since Ebbinghaus (1885) and Craik and Lockhart (1972) both in short and long periods of time (for a survey of this literature see Schacter, 1999). More recently, Ericson (2011) experimentally shows that individuals suffer from overconfidence in their own memory, that is, they overestimate their ability to remember future events. As a result of this, they are unwilling to incur costs (such as setting a reminder) required to remember due dates,[2] leading consumers to pay bills late and incur late fees.[3]

To answer the second question, we consider how consumer behavior can be exploited by firms. In a market where the probability of an exogenous shock is high, the seller can increase her profit by setting stringent penalties for late payment. However, if the probability of a shock is low, the seller may keep penalties so low that present bias alone drives all individuals to pay late. In essence, the seller's choice is between a low penalty that extracts a small surplus from all individuals, and a high penalty that extracts a late payment penalty from only a few customers. Our findings suggest that, under certain parameter conditions, sellers optimally charge a high penalty, thus fully exploiting those individuals who suffer a shock. This analysis is consistent with Chetty (2015) who shows how firms can extract welfare from present biased consumers, but goes beyond because our analysis does not rely on present biasedness alone and allows for shocks. If there is no present bias, buyers would still pay late, but only when suffering a shock, thus hindering the seller's ability to induce late payment.

Firms' optimizing penalties can be substantial, consistent with what is observed in the market.[4] Hence, we extend our analysis by evaluating how legal limits on the penalty can improve the utility of consumers who suffer a shock, without inducing those who do not suffer such a shock to stop paying on time. In this effort we support Armstrong and Vickers (2012). They discuss the importance of consumer protection on contingent charges. This plays a similar role as penalties in our model, although Armstrong, Vickers and Zhou (2009) point out that excessive consumer protection can have undesirable welfare effects.

Section 2 describes the model, and section 3 presents the consumer's problem for each of our behavioral settings. In section 4 we examine the seller's problem for each type of consumer he may

[2] In the experiment, subjects had to choose between receiving a large payment, conditional on them remembering to claim their payment with a six-month delay, or a smaller payment that would automatically be sent to them after six months. Ericson (2011) finds that, while three quarters of the subjects choose the large payment and said they would remember to claim their payment after six months, only half of them claimed it, thus reflecting overconfidence in their own memory.

[3] There is good empirical evidence that reminders help people meet their commitments when the reminders came at no set-up cost to the consumers. For instance, Karlan et al. (2016) examine data from three banks in Peru, Bolivia and the Philippines, and show that, among people who have recently opened a savings account, reminders increase the probability of meeting their commitments. Similarly, Calzolari and Nardotto (2012) conducted a field experiment on a sample of individuals joining a gymnasium, and found that a weekly email reminder increases attendance by up to 25%.

[4] In the U.S., for example, Time Warner Cable (TV services) and Verizon (phone) charge a penalty of up to 1.5% of the monthly fee in case of late payment. Penalties are larger in other countries; Reliance India Mobile charges a late penalty equivalent to 2.5% of the monthly fee in case of late payment.

face. Section 5 discusses our main conclusions and policy implications.

2 Model

Consider an individual who discounts future payoffs by a factor $\delta \in (0,1]$, and exhibits a present bias parameter $\beta \in [0,1]$. Present bias is absent if $\beta = 1$; as in Laibson (1997). A monopolist offers a contract at time $t = 0$. The consumer can choose to either sign it or not. If he signs the contract, he gets one unit of the good in every period t thereafter. This yields him a utility of u_t in period t. He pays for this service at a later time $t = n$. This setting embodies as special cases contracts in which payment is due immediately after signing (i.e. $n = 1$) as well as those allowing the customer to enjoy the good for multiple periods by having to pay at a "due date" period n (e.g., $n = 30$ for monthly). Examples for such goods are cable, internet and cell phone services. The billing cycles are repeated z times, where $z \geq 1$. For instance, if $n = 30$ and $z = 12$, the bill is repeated 12 months, for a total of $zn = 360$ days. Therefore, period zn is the last period of the billing cycle and, more generaly, xn is one of the periods when the bill is due, where $x \in \{1, 2, ...z\}$.

Paying on time. In every period t, the stream of payoffs that the consumer obtains from paying the bill due at the end of the current billing cycle xn is

$$\begin{aligned} & u_t + \beta \sum_{i=1}^{zn-t} \delta^i u_{i+t} - \beta F \left(\delta^{xn-t} + \delta^{(x+1)n-t} + ... + \delta^{zn-t} \right) \\ = \quad & u_t + \beta \sum_{i=1}^{zn-t} \delta^i u_{i+t} - \beta F \sum_{i=x}^{z} \delta^{in-t}. \end{aligned} \tag{2.1}$$

The first two terms in expression (2.1) reflect the discounted utility that the individual enjoys from the good, while the third captures the discounted value of paying the fee F at the end of every billing cycle xn, where $x \in \{1, 2, ...z\}$.

Not paying on time. For generality, we assume that, if the individual is late in his payment at period $t = xn$, he pays k periods late, i.e., he pays at period $t = xn + k$, where $k \geq 1$.[5] If he pays one period late, $k = 1$, the fee F increases to $(1 + a)F$, where penalty a satisfies $a > 0$.[6] Generally, if he is $k \geq 1$ periods late, he pays a total fee of $(1 + ka)\,F$, thus indicating that penalties increase in time. Both F and a are choice variables for the seller, which we characterize in Section 4. Therefore, if the consumer at period t plans to not pay the bill when it is due at period $t = xn$, but instead pay k periods late, his stream of discounted payoffs becomes

$$u_t + \beta \sum_{i=1}^{zn-t} \delta^i u_{i+t} - \beta F \left(0 + 0 + ... + 0 + \delta^{xn+k-t}(1 + ka) + \delta^{(x+1)n-t} + ... + \delta^{zn-t} \right)$$

[5] For instance, if the bill cycle is $n = 30$ days, it is due in 3 months, $x = 3$, and the individual pays it $k = 4$ days late, $xn + k = 94$ days.

[6] In the U.S., for instance, Time Warner Cable (TV services) and Verizon (phone) charge a penalty of up to 1.5% of the monthly fee in case of late payment, i.e., $a = 0.015$. These penalties are larger in other countries. Reliance India Mobile, for example, charges a late penalty equivalent to 2.5% of the monthly fee in case of late payment.

$$= u_t + \beta \sum_{i=1}^{zn-t} \delta^i u_{i+t} - \beta F \delta^{xn+k-t}(1+ka) - \beta F \sum_{i=x+1}^{z} \delta^{in-t} \tag{2.2}$$

Similarly as in expression (2.1), the first two terms indicate the discounted utility from the good, the third term reflects the discounted cost of paying fee F after it was due at xn, i.e., k periods late, while the last term captures the discounted cost of the fees from all future billing cycles.

Service suspension. We allow for the service to be suspended if the bill is not paid S periods after being due, i.e., if the customer pays k periods late where $k \geq S$. In this case, the service is suspended, the firm takes action (e.g., bringing the case to court) and the consumer incurs a penalty C, yielding a stream of payoffs

$$\beta \left(\sum_{i=2}^{zn-(S-1)} u_{(S-1)+i}\delta^i - F \sum_{i=1}^{z-1} \delta^{in-(S-1)} \right) - C$$

In order to guarantee that the individual pays at least before the suspension period S, we assume that the stream of discounted payoffs in expression (2.3) is negative, which entails that penalty C is sufficiently large, that is,

$$C > \beta \left(\sum_{i=2}^{zn-(S-1)} u_{(S-1)+i}\delta^i - F \sum_{i=1}^{z-1} \delta^{in-(S-1)} \right).$$

Therefore, customers can pay k periods late, where $k \leq S-1$ holds; will never pay at $k > S-1$ because the penalty is too high.

We assume that every customer has the financial ability to pay the amount in any period he chooses. However, he may suffer from present bias or shocks. The probability of such a shock occurring in any period is given by $0 \leq q \leq 1$. We assume that shocks are i.i.d. in every period, and their effects last only for one period. For presentation purposes, we first analyze the case without shocks, i.e., $q = 0$, and then extend our results to a setting with shocks, $q > 0$.

The time structure of the game is as follows:

1. At period $t = 0$, the seller presents a contract to the customer, which specifies a penalty and fee pair, (a, F);
2. The buyer, observing the contract (a, F), responds signing or not signing it at $t = 0$;
   (a) If he does not sign the contract, both seller and customer obtain a payoff of zero;
   (b) If he signs the contract, the buyer chooses, at the time that the bill when it is due, $t = xn$, whether or not to pay F.
      i. If he pays the bill, a new billing cycle starts, and the buyer does not incur any penalty a.
      ii. If he does not pay the bill, the buyer incurs a penalty a into the next period $t = xn+1$ (late period $k_L = 1$), when he again decides whether to pay the bill. If he pays the

bill, a new bill cycle starts; otherwise, the buyer incurs an additional penalty a into the next period (late period $k_L = 2$), when he again chooses whether to pay. A similar argument applies until period $S - 1$.

Applying backward induction, we start by solving the consumer's problem (whether he signs the contract and, if so, when does he pay) and then move on to the seller (design of the optimal contract). All proofs are relegated to the appendix.

3 Consumer's problem

Lemma 1 (No Shocks.) *When the buyer suffers no shocks, $q = 0$, for a given billing cycle x:*

1. *At any period $t < xn$, he expects to pay on time the bill that is due at the end of that billing cycle, at xn, rather than paying k periods later, if the penalty satisfies $a > \underline{a}(k)$ for every period k.*

2. *At any period $k_L = t - xn \geq 0$, the consumer pays the bill:*

    (a) *on time ($k_L = 0$) if the penalty satisfies $a > \overline{a}(k, k_L)$ for all subsequent periods, $k > k_L$; or if he is at the penultimate late period, $k_L = S - 1$.*

    (b) *at a late period k (which entails $k_L > 0$) if k satisfies $a \leq \overline{a}(k, k_L)$; where*

$$\underline{a}(k) \equiv \frac{1 - \delta^k}{k\delta^k} \quad \textit{and} \quad \overline{a}(k, k_L) \equiv \frac{1 - \delta^{k-k_L}}{\beta k \delta^{k-k_L} - k_L}.$$

Intuitively, when signing the contract the buyer plans to pay the bill on time (at period $t = xn$) if penalty a is high enough, i.e., $a > \underline{a}(k)$ for every late period k. Once the bill is due, however, at $k_L = 0$, the buyer finds it profitable to not pay it if there is one late period k (e.g., 3 periods after the bill was due) satisfying $a < \overline{a}(k)$. He then follows a similar cutoff rule in subsequent periods to determine whether to pay the bill at that late period $k_L > 1$ or not.

Example 1. Consider that $S = 3$, and the bill is due at the signature of the contract (period 0), the buyer only has three periods at which he can pay the bill: at the time it is due, at late period 1, or at late period 2. For simplicity, assume that $\delta = 0.95$ and $\beta = 0.93$.[7] In this context, cutoff $\underline{a}(k)$ becomes $\underline{a}(0) = 0.052$ and $\underline{a}(1) = 0.054$, implying that condition $a > \underline{a}(k)$ holds for every late period k as long as $a > \max\{\underline{a}(k)\} = 0.054$. Regarding cutoff $\overline{a}(k, k_L)$, when the buyer considers whether to pay the bill at the period it is due, $k_L = 0$, this cutoff becomes $\overline{a}(1, 0) = 0.056$ and $\overline{a}(2, 0) = 0.058$. However, when he is one period late, and examines whether to pay the bill at that

[7] Takeuchi (2011) discusses that if the time gap is very small, there is even a chance of $\beta > 1$ (future bias). While we do not consider the case for future bias, note that $\beta > 1$ implies that $\underline{a}(k) > \overline{a}(k, k_L)$, entailing that the consumer would not exhibit preference reversal under any parameter values. Jakiela et. al (2016) report that the estimated β's from several empirical studies ranging from 0.901 to 0.937.

period, i.e., $k_L = 1$, this cutoff becomes $\overline{a}(2,1) = 0.065$. Therefore, when the bill is due, the buyer will pay on time if $a > \overline{a}(k,0)$ holds for all subsequent periods, $k > 0$, which in this context entails $a > \overline{a}(1,0) = 0.056$ and $a > \overline{a}(2,0) = 0.058$, thus implying $a > 0.058$. However, if this condition does not hold, the consumer does not pay the bill on time. When he is at late period one, $k_L = 1$, he pays the bill if $a > \overline{a}(2,1) = 0.065$; but otherwise he delays his payment to period 2. ■

The lemma identifies that time inconsistent behavior arises when penalties are intermediate. This behavior arises because of present bias, $\beta < 1$, but would not arise if present bias was absent. In particular, if $\beta = 1$, when the consumer reaches the period when the bill is due, xn, $k_L = 0$, and cutoff $\overline{a}(k, k_L)$ collapses to $\frac{1-\delta^k}{k\delta^k}$, which coincides with cutoff $\underline{a}(k)$. Thus, the range on a for which time inconsistent behavior can emerge is nil. In contrast, lower values of β, entail a larger range of penalties a's for which time inconsistent behavior exists. Similarly, if the consumer does not discount future payoffs, $\delta = 1$, then cutoff $\underline{a}(k)$ collapses to zero, and any positive penalty induces the consumer to pay the bill on time.

We next explore how the above results are affected if we allow for shocks.

Proposition 1 (Shocks). *When the buyer suffers from shocks, $q > 0$, and a given billing cycle x:*

1. *At any period $t < xn$, he expects to pay on time the bill that is due at the end of that billing cycle, at xn, rather than paying k periods later, if the penalty satisfies $a > \underline{a}(k,q)$ for every period k.*

2. *At a period $k_L = t - xn \geq 0$, the consumer pays the bill:*

   (a) *on time if the penalty satisfies $a > \overline{a}(k,q,k_L)$ for all subsequent periods, $k > k_L$; or if he is at the penultimate late period, $k_L = S - 1$.*

   (b) *at a late period k if if k satisfies $a \leq \overline{a}(k,q,k_L)$; where*

$$\underline{a}(k,q) \equiv \frac{1-q}{A} + \frac{B}{A}, \quad and \quad \overline{a}(k,q,k_L) \equiv \frac{(1-q) + \beta D}{\beta C - (1-q)k_L}$$

and terms A-D are defined in the appendix.

When shocks occur, the range of penalties under which time inconsistent behavior is possible, expands.

Example 2. Consider again the same setting as in Example 1, but allowing for shocks to occur with probability $q = 0.2$. In this context, cutoff $\underline{a}(k)$ becomes $\underline{a}(0, 0.2) = 0.053$ and $\underline{a}(1, 0.2) = 0.054$, implying that condition $a > \underline{a}(k)$ holds for every late period k as long as $a > \max\{\underline{a}(k,q)\} = 0.054$. Regarding cutoff $\overline{a}(k,q,k_L)$, when the buyer considers whether to pay the bill at the period it is due, $k_L = 0$, this cutoff becomes $\overline{a}(1, 0.2, 0) = 0.120$ and $\overline{a}(2, 0.2, 0) = 0.092$. However, when he is one period late, and examines whether to pay the bill at that period, i.e., $k_L = 1$, this cutoff becomes $\overline{a}(2, 0.2, 1) = 0.151$. Therefore, when the bill is due, the buyer will pay on time if $a > \overline{a}(k, 0.2, 0)$ holds for all subsequent periods, $k > 0$, which in this context entails $a > \overline{a}(1, 0.2, 0) = 0.120$ and $a > \overline{a}(2, 0.2, 0) = 0.092$, thus implying $a > 0.120$. However, if this condition does not hold, the consumer does not pay the bill on time. When he is at late period one, $k_L = 1$, he pays the bill if $a > \overline{a}(2, 0.2, 1) = 0.151$; but otherwise he delays his payment to period 2. ■

Our results show that time inconsistent behavior can only arise if the individual exhibits present bias, $\beta < 1$; when $\beta = 1$, the range of penalties a supporting time inconsistent behavior, $a^{TI}(k, q) \equiv \overline{a}(k, q, k_L) - \underline{a}(k, q) = \frac{1-\beta}{\beta} \frac{1-q}{A}$, collapses to zero. When $\beta < 1$, for most combinations of parameter values (δ, β, S, q, and k), the difference $a^{TI}(k, q) \equiv \overline{a}(k, q, k_L) - \underline{a}(k, q)$ increases in the probability of a shock, q.

4 Seller's Problem

The seller's problem consists of: finding the optimal a, and F. He does this using backward induction. We first find the optimal penalty a, then the fee F. For simplicity, we assume that $a > \underline{a}(k, q)$ for all k, so the buyer, at the time of signing the contract, expects to pay the bill on time. Nonetheless, we allow for the penalty to be higher or lower than $\overline{a}(k, q, k_L)$ thus inducing the buyer to reconsider his initial decision and thus pay late.

4.1 Finding the optimal penalty

When setting the optimal penalty rate, a, the seller can anticipate the results of the consumer's problem. The seller can either: (1) charge low penalties that induce every buyer to pay late, both those who suffer shocks and those who do not, inducing all of them to pay in the last period $S-1$; or (2) set a high enough penalty a such that only those who suffer a shock pay late (these individuals pay late because of the shock, except in period $S-1$). In the case where shocks are absent, $q = 0$, the seller can only choose option (1). We now examine the optimal penalties in each scenario. We first examine how the seller identifies optimal penalties when shocks are absent, $q = 0$, and then extend the analysis to a setting with shocks, $q > 0$.

4.1.1 Optimal penalty under no shocks

If the seller seeks to induce everyone to pay late, she must keep the value of a low enough to induce late payment in at least one period, i.e., $k \geq 1$. Let k_P be the late period when the consumer chooses to pay the bill. In particular, payment at $k_P \in \{0, ..., S-2\}$ periods late can be induced if, at every period k_L at which the consumer evaluates his decision on whether to pay or not, penalty

a satisfies

$$\begin{aligned} a &\leq \overline{a}(k, k_L) \text{ for at least one period } k > k_L, \text{ and} \\ a &> \overline{a}(k, k_P) \text{ for all } k > k_P, \end{aligned} \tag{C1}$$

as defined in Lemma 1. However, payment at the penultimate period $k_P = S - 1$ is induced as long as

$$a \leq \overline{a}(k, k_L) \text{ for at least one period } k > k_L \tag{C2}$$

for every evaluation period k_L. Let $A(k_P)$ denote the feasible set of penalties triggering a late payment at exactly k_P, i.e., a's satisfying the above two conditions C1-C2. Since the seller can anticipate the buyer's problem (see Lemma 1), he can identify which penalty $a \in A(k_P)$ induces the buyer to pay the bill exactly k_P periods late, which implies that the seller's problem can be characterized as follows.

Proposition 2. *Under no shocks, i.e., $q = 0$, the seller sets penalty a to solve*

$$\pi(k_P, 0) \equiv \max_{a \in A(k_P)} F(a)(1 + k_P a) \tag{3}$$

where a_P is the argmax to (3) as a function of k_P. Then, the seller chooses the penalty a_P that yields the highest profit, that is, the a_P that solves

$$\max \{\pi(0, 0), \pi(1, 0), \pi(2, 0), ..., \pi(S - 1, 0)\}.$$

The seller's problem can be understood as a three-step program: first, among all penalties a inducing a specific payment period k_P, i.e., those satisfying $a \in A(k_P)$, the seller selects the penalty that maximizes his profits; second, he evaluates the maximal profits from inducing such a payment period k_P; and, third, he identifies the penalty associated to the most profitable payment period. For compactness, let a^* be the argmax of the above problem. Example 3 illustrates the seller's choice of a following the above three-step approach.

Example 3. If the seller seeks to induce payment at $k_P = 0$ (no late payment), the buyer located at $k_L = 0$ must find that the penalty is sufficiently high to prevent him from delaying his payment, i.e., $a > \overline{a}(k, 0)$ for every late period k. In our example, this condition entails $a > 0.056$ and $a > 0.058$, thus implying that $a > 0.058$ is a sufficient condition.[8] Therefore, the profits from inducing on-time payment are $F(a)$.

Similarly, if the seller seeks to induce payment at $k_P = 1$, the penalty must satisfy: (1) $a \leq \overline{a}(k, 0)$ for at least one period k, which induces the individual to not pay the bill when it is due;

[8] Since we assumed that the seller chooses a such that the buyer anticipates to pay on time at the time of signing the contract, we need that $a > \underline{a}(k)$ for all k, or $a > \max\{\underline{a}(k)\}$. In this example, $a > \max\{\underline{a}(k)\} = \underline{a}(2) = 0.054$, which holds since $a > 0.058$.

and (2) $a > \overline{a}(k,1)$ for all subsequent periods $k > 1$. In our setting, cutoff $\overline{a}(k,0)$ becomes $\overline{a}(1,0) = 0.056$, and $\overline{a}(2,0) = 0.058$, implying that condition (1) holds as long as $a < 0.058$. Condition (2), however, cannot hold in this context since cutoff $\overline{a}(k,1)$ becomes $\overline{a}(2,1) = 0.065$, which is incompatible with (1). In words, the seller cannot set a to induce payment one period late, i.e., at $k_P = 1$.

Finally, when the seller seeks to induce payment at $k_P = 2$, the penalty needs to satisfy: (1) $a \leq \overline{a}(k,0)$ for at least one period k, which induces the individual to not pay the bill when it is due; and (2) $a < \overline{a}(k,1)$ for at least one of the subsequent periods $k > 1$, which induces him to not pay at $k = 1$ either. Since the consumer did not pay in the first two periods, and $S = 3$, he pays at the penultimate period $S-1 = 2$. In this example, condition (1) entails $a \leq \overline{a}(2,0) = 0.058$,[9] condition (2) yields $a < \overline{a}(2,1) = 0.065$. Among the set of penalties simultaneously satisfying conditions (1) and (2), the highest penalty is $a = 0.058$, which induces payment two periods late, and yields profits $F(a)(1 + 2 * 0.058)$.

4.1.2 Optimal penalty under shocks

Following a similar approach as in the above subsection, when shocks are present $q > 0$, the seller's problem can be defined as follows. Let $A(k_P, q)$ denotes the set of feasible penalties that induces payment at period k_P, following an analogous definition as $A(k_P)$ in the previous subsection.

Proposition 3. *Under shocks, $q > 0$, the seller sets penalty a to solve*

$$\pi(k_P, q) \equiv \max_{a \in A(k_P,q)} F(a) \left[(1-q)(1+k_P a) + (1-q) \left(\sum_{i=k_P+1}^{S-2} (1+ia) q^{i-k_P} \right) + [1 + a(S-1)]\, q^{S-1} \right] \tag{4}$$

where $a_P(q)$ is the argmax to (4) as a function of k_P. Then, the seller chooses the penalty a_P that yields the highest profit, that is, the a_P that solves

$$\max \left\{ \pi(0,q), \pi(1,q), \pi(2,q), ..., \pi(S-1,q) \right\}.$$

While the seller follows a three-step program to identify the optimal penalty, his profit function is now different from that under no shocks. In particular, the first term in expression (4) is analogous to the seller's profits at (3), i.e., when the buyer finds it profitable to pay at period k_P, he pays if not suffering a shock. The second term in (4), however, represents the possibility that, at period k_P, the buyer suffers a shock, delaying his payment into a future period $k > k_P$ at which he does not suffer a shock with probability $(1-q)$. Finally, the third term in (4) captures the profits in the event that the buyer suffers shocks in all periods until $S-1$ (which is the last period when he has the ability to pay). For compactness, let $a^*(q)$ be the argmax of the above problem. Example 2 illustrates the seller's choice of a in this context.

[9] Note that $\overline{a}(1,0) = 0.056$ and $\overline{a}(2,0) = 0.058$, meaning that $a \leq \overline{a}(2,0)$ holds for both $k = 1$ and $k = 2$.

Example 4. Consider the same parameter values as in Example 3, and assume a probability of shocks $q = 0.20$. In this context, when $k_L = 0$, cutoff $\overline{a}(k, 0.2, k_L)$ becomes $\overline{a}(1, 0.2, 0) = 0.120$ and $\overline{a}(2, 0.2, 0) = 0.092$; and when $k_L = 1$, it becomes $\overline{a}(2, 0.15, 1) = 0.151$. If the seller seeks to induce no late payment, $k_P = 0$, the buyer located at $k_L = 0$ must find that the penalty a satisfies $a > \overline{a}(k, 0.2, 0)$ for every late period k. In our example, this condition entails $a > 0.120$ and $a > 0.092$, thus implying that $a > 0.120$ is a sufficient condition.[10] Unlike in the case with no shocks, however, the buyer will not pay on time if he suffers a shock, even if he intends to. Therefore, the seller faces expected profits from inducing on-time payment, $\pi(0, 0.2)$.[11]

If the seller seeks to induce payment at $k_P = 1$, the penalty must satisfy: (1) $a \leq \overline{a}(k, q, 0)$ for at least one period k, inducing the individual to not pay the bill on time; and (2) $a > \overline{a}(k, q, 1)$ for all subsequent periods $k > 1$. In our setting, cutoff $\overline{a}(k, q, 0)$ becomes $\overline{a}(1, 0.2, 0) = 0.120$, and $\overline{a}(2, 0.2, 0) = 0.092$. Therefore, condition (1) holds as long as $a \leq 0.120$. Condition (2), however, cannot hold in this context since cutoff $\overline{a}(k, 1)$ becomes $\overline{a}(2, 0.2, 1) = 0.151$, which is incompatible with (1). In words, the seller cannot set a to induce payment one period late, i.e., at $k_P = 1$.

Last, when the seller seeks to induce payment at $k_P = 2$, the penalty needs to satisfy: (1) $a \leq \overline{a}(k, q, 0)$ for at least one period k; and (2) $a < \overline{a}(k, q, 1)$ for at least one of the subsequent periods $k > 1$. In this example, condition (1) entails $a \leq \overline{a}(1, 0.2, 0) = 0.120$.[12] Condition (2) yields $a < \overline{a}(2, 0.2, 1) = 0.151$. Among the set of penalties simultaneously satisfying conditions (1) and (2), the highest penalty is $a = 0.120$, which induces payment two periods late. Expected profit in this case becomes $\pi(2, 0.2)$. ■

Our example had the consumer paying in the last possible period, $S - 1$. However, this may not be optimal under certain contexts. The next section identifies optimal fees, and then evaluate overall profits of each (a, F) pair. Last, we find settings under which profits are higher when buyers are induced to pay before the last period.

4.2 Finding the optimal fee

4.2.1 Optimal fee under no shocks

We now analyze the seller's optimal choice of fee F. The fee is defined by the consumer's participation constraint because the seller tries to charge the maximum F to the consumer while still ensuring his participation. The consumer's participation constraint is

$$u_0 + \beta \sum_{x=1}^{n} \delta^x u_x - \beta \delta^n F \geq 0$$

[10] This condition on a, $a > 0.120$, satisfies the initial assumption that $a > \max\{\underline{a}(k, q)\}$, i.e., the consumer plans to pay on time when signing the contract, since $a > \max\{\underline{a}(k, q)\} = \underline{a}(1, 0.2) = 0.054$.

[11] The expected profit is now a function of the specific penalty a that the seller chooses, if he seeks to induce on-time payment, i.e., those penalties satisfying the sufficient condition $a > 0.120$. We identify the optimal penalty in subsection 4.2.2.

[12] Note that $\overline{a}(1, 0.2, 0) = 0.120$ and $\overline{a}(2, 0.2, 0) = 0.092$, meaning that $a \leq \overline{a}(1, 0.2, 0)$ holds for both $k = 1$ and $k = 2$.

which must hold with equality; otherwise the seller could still raise F and increase profits. Assuming that u_t is constant across periods, and solving for F, we find

$$F^* = \frac{\bar{u}\left(1 + \beta \sum_{x=1}^{n} \delta^x\right)}{\beta \delta^n}$$

which is independent on the penalty a since the consumer does not expect to pay late at the signature of the contract, thus is not concerned with future penalties.

Example 5. Continuing Example 3, assume that $\bar{u} = 1$ and $n = 15$. Thus, we obtain a fee of $F^* = 24.332$. We can insert this fee at the profits found in Example 1. If the penalty induces the buyer to pay on time, $a > 0.058$, profits become $\pi(0,0) = F^* = 24.332$. If, instead, penalties induce him to pay two periods late, $a = 0.058$, profits are $\pi(2,0) = F^*(1+2a) = 27.154$. Therefore, the seller prefers to induce payment at $k_P = 2$, setting an optimal penalty of $a^* = 0.058$ and fee of F^*. ■

4.2.2 Optimal fee under shocks

Following a similar approach, when the consumer suffers shocks, his participation constraint becomes

$$u_0 + \beta \sum_{x=1}^{n} \delta^x u_x - \beta \delta^n \left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2 q^2 (1-q)F(1+2a) + ... + \delta^{S-1} q^{S-1} F(1+(S-1)a)\right] \geq 0$$

which must hold with equality; otherwise the seller could still raise F. After solving for F, we find

$$F(a) = \frac{\overline{u}\left(1 + \beta \sum_{x=1}^{n} \delta^x\right)}{\beta \delta^n \left[(1-q)\left(1 + \sum_{i=1}^{S-2} (1+ia) q^i\right) + (1 + a(S-1)) q^{S-1}\right]}$$

Intuitively, while the consumer expects the pay the bill on time when signing the contract, he anticipates that he may suffer shocks once the bill is due, forcing him to pay the bill at a later period when he will be subject to penalties. It is straightforward to note that, when shokcs are absent $q = 0$, $F(a)$ coincides with F^*.

Example 6. Continuing with Example 4, fee $F(a)$ becomes

$$F(a) = \frac{24.332}{1 + \frac{8}{100}a + \frac{8}{10}\left[1 + \frac{2}{10}(1+a)\right]}$$

thus being a function of penalty a. As shown in Example 4, the seller can induce the buyer to pay on time when setting a penalty that satisfies $a > 0.120$. However, penalties are often bound for legal and institutional reasons, restricting the seller to set penalties below a_{Legal}.[13] For instance, if $a_{Legal} = 0.15$, the optimal fee becomes $F(0.15) = 12.190$, yielding profits of 12.629. If,

[13] If there were no legal limit on the value of a, the seller could set it arbitrarily high, i.e., $a > 0.120$. As shown in Example 2, however, penalties on that range induce consumers to pay bills as soon as possible: either at the time the bill is due (if he does not suffer a shock), or at a later time period (if he suffers a shock).

instead, the seller induces payment two periods late, using penalty $a = 0.120$, then the fee becomes $F(0.120) = 12.234$, entailing profits of 15.171. Since profits are larger when the buyer is induced to pay late at $k_P = 2$, the seller sets the corresponding penalty-fee pair $(a^*, F^*) = (0.120, 12.234)$. ■

The seller faces a trade-off in the design of fees and penalties: on one hand, buyers only accept to sign the contract if fees are relatively low when they face the probability of a future shock; and, on the other hand, the seller can set a higher penalty. Intuitively, the latter happens because buyers' present bias not only leads to an underestimation of future costs, but also of the probability of suffering a shock. In our numerical simulations in Examples 5 and 6, the seller finds that the second effect dominates, thus inducing buyers to pay in the final period. When present bias is absent, $\beta = 1$, the second effect is nil, leading the buyer to pay on time under all parameter combinations.

4.3 Comparative statics

Table I summarizes our results from Example 6 in the first row (benchmark), and compares them against changes in only one parameter at a time.

	Payment period k_P	a^*	F^*	Profits
Benchmark (Example 6)	2	0.120	12.234	15.171
δ increases to 0.99	2	0.073	8.778	10.060
β decreases to 0.5	1	0.15 (a_{Legal})	13.19	15.565
β and δ increase to 0.995	0	0.15 (a_{Legal})	8.328	8.628

Table I. Equilibrium variables under different parameters.

When δ increases from 0.95 to 0.99, there is a substantial drop in fee F^* and profits. This is because, at the signing of the contract, the individual puts a larger weight on paying fee F^* in a future period, and agrees to pay less today. A similar argument applies to future penalties, which increase their current value in today's terms, ultimately decreasing profits.

When β decreases from 0.93 to 0.5, the converse argument applies since the individual suffers a stronger present bias. First, the seller can set a higher fee and penalty which are accepted by the buyer at the time of signing the contract, as he assigns a low weight to future payoffs. Second, the penalty induces the buyer to delay his payment to the first period $k = 1$ and, once he considers whether to pay the bill at $k = 1$, he finds it optimal to do so.

In the last row of the table, both β and δ simultaneously increase to 0.995. In a setting without shocks, the seller faces a tradeoff: either set a low penalty a that would induce the buyer to pay late, or set a high penalty (in the extreme, a_{Legal}) that leads him to pay on time. However, since this type of buyer puts a large weight on future payoffs, the seller would have to set the penalty very low to entice the buyer to pay late, ultimately leading the seller to find it optimal to charge a high penalty that induces on-time payment. Under shocks, the seller faces a similar trade-off, but in this case he can earn profits from the penalty, as the individual, despite planning to pay on time, he might suffer a shock that leads him to pay later.

5 Discussion

In this paper, we offer two different explanations for why consumers with financial means pay contractual fees late, even when suffering substantial penalties. When there are no shocks, consumers only pay late if there is present bias and the penalty is relatively low. However, if consumers face exogenous shocks, paying late can be the norm even when the penalty is high (for similar levels of present bias).

Seller's choice. The seller has the choice of setting a high penalty which induces on-time payment, extracting higher rents from those suffering shocks; or, instead, set a low penalty which leads all buyers to pay their bills late. When customers assign a high value to future payoffs , the seller can only induce late payment with extremely low penalties. In this context, it is more profitable to induce on-time payment, and set a high penalty (potentially at the legal limit) and rely on shocks to extract rents from customers suffering from them. If, instead, buyers assign a relatively low value to future payoffs, the seller can induce late payment with comparatively higher penalties. In this case, inducing late payment becomes more attractive for the seller.

Role of legal limits. Consider a scenario where the seller finds it profitable to induce late payment, and where $\overline{a}(k, q, k_L) > a_{Legal}$, thus leading the seller to charge a penalty $a = a_{Legal}$; as discussed in Table I. In this setting, if regulatory authorities were to marginally reduce the legal limit on penalties, a_{Legal}, the seller would extract less profit from late payments, making on-time payment more attractive. However, he would not be able to induce on-time payment as a_{Legal} is too low.[14] If, instead, policy makers marginally increase the legal limit on penalties, inducing late payment would become more attractive for the seller, who would not have incentives to alter the penalty-fee pair. Nonetheless, if the legal limit is increased above $\overline{a}(k, q, k_L)$, then the seller may have incentives to switch his penalty strategy to one that induces on-time payment. In summary, while capping late penalties can be politically popular, low legal limits may encourage sellers to induce late payments under larger parameter conditions, which may not be favorable for consumers.

6 Appendix

6.1 Appendix 1 - Further numerical examples

We next alter extend our numerical examples with shocks, Examples 4 and 6, allowing for one parameter value to change at a time.

[14] Recall that on-time payment can only be induced if penalty a satisfies $a > \overline{a}(k, q, k_L)$ for at least one period k.

Higher discount factor ($\delta = 0.99$). *Penalties.* Consider the same parameter values as in Example 6, but assume $\delta = 0.99$. In this context, when $k_L = 0$, cutoff $\overline{a}(k, 0.01, k_L)$ becomes $\overline{a}(1, 0.2, 0) = 0.073$ and $\overline{a}(2, 0.2, 0) = 0.045$; and when $k_L = 1$, it becomes $\overline{a}(2, 0.2, 1) = 0.094$. If the seller seeks to induce no late payment, $k_P = 0$, the buyer located at $k_L = 0$ must find that the penalty a satisfies $a > \overline{a}(k, 0.2, 0)$ for every late period k. In our example, this condition entails $a > 0.073$ and $a > 0.045$, thus implying that $a > 0.073$ is a sufficient condition.[15] Unlike in the case with no shocks, however, the buyer will not pay on time if he suffers a shock, even if he intends to. Therefore, the seller faces expected profits from inducing on-time payment, $\pi(0, 0.2)$.

If the seller seeks to induce payment at $k_P = 1$, the penalty must satisfy: (1) $a \leq \overline{a}(k, q, 0)$ for at least one period k, inducing the individual to not pay the bill on time; and (2) $a > \overline{a}(k, q, 1)$ for all subsequent periods $k > 1$. In our setting, cutoff $\overline{a}(k, q, 0)$ becomes $\overline{a}(1, 0.2, 0) = 0.073$, and $\overline{a}(2, 0.2, 0) = 0.045$. Therefore, condition (1) holds as long as $a \leq 0.073$. Condition (2), however, cannot hold in this context since cutoff $\overline{a}(k, 1)$ becomes $\overline{a}(2, 0.01, 1) = 0.094$, which is incompatible with (1). In words, the seller cannot set a to induce payment one period late, i.e., at $k_P = 1$.

Last, when the seller seeks to induce payment at $k_P = 2$, the penalty needs to satisfy: (1) $a \leq \overline{a}(k, q, 0)$ for at least one period k; and (2) $a < \overline{a}(k, q, 1)$ for at least one of the subsequent periods $k > 1$. In this example, condition (1) entails $a \leq \overline{a}(1, 0.2, 0) = 0.073$[16]. Condition (2) yields $a < \overline{a}(2, 0.2, 1) = 0.094$. Among the set of penalties simultaneously satisfying conditions (1) and (2), the highest penalty is $a = 0.073$, which induces payment two periods late. Expected profit in this case becomes $\pi(2, 0.2)$.

Optimal fee. Now $F(a)$ becomes

$$F(a) = \frac{17.359}{1 + \frac{8}{100}a + \frac{8}{10}\left[1 + \frac{2}{10}\left(1 + a\right)\right]}$$

thus being a function of penalty a. We use the same value of $a_{Legal} = 0.15$. Now the optimal fee for inducing on time payment becomes $F(0.15) = 8.697$, yielding profits of 9.010. If, instead, the seller induces payment two periods late, using penalty $a = 0.073$, then the fee becomes $F(0.073) = 8.778$, entailing profits of 10.060. Since profits are larger when the buyer is induced to pay two periods late, the seller sets the corresponding penalty-fee pair $(a^*, F^*) = (0.073, 8.778)$.

Higher present bias ($\beta = 0.5$). *Penalties.* Consider the parameter values in Example 2. but this time with $\beta = 0.5$. In this context, when $k_L = 0$, cutoff $\overline{a}(k, 0.2, k_L)$ becomes $\overline{a}(1, 0.2, 0) = 0.945$ and $\overline{a}(2, 0.2, 0) = 0.560$; and when $k_L = 1$, it becomes $\overline{a}(2, 0.2, 1) = -10.500$. If the seller seeks to induce no late payment, $k_P = 0$, the buyer located at $k_L = 0$ must find that the penalty a satisfies $a > \overline{a}(k, 0.2, 0)$ for every late period k. In our example, this condition entails $a > 0.945$ and $a > 0.560$, thus implying that $a > 0.945$ is a sufficient condition.[17] Unlike in the case with

[15] This condition on a, $a > 0.073$, satisfies the initial assumption that $a > \max\{\underline{a}(k, q)\}$, i.e., the consumer plans to pay on time when signing the contract, since $a > \max\{\underline{a}(k, q)\} = \underline{a}(1, 0.0.2) = 0.058$.

[16] Note that $\overline{a}(1, 0.2, 0) = 0.073$ and $\overline{a}(2, 0.2, 0) = 0.045$, meaning that $a \leq \overline{a}(1, 0.2, 0)$ holds for both $k = 1$ and $k = 2$.

[17] This condition on a, $a > 0.945$, satisfies the initial assumption that $a > \max\{\underline{a}(k, q)\}$, i.e., the consumer plans

no shocks, however, the buyer will not pay on time if he suffers a shock, even if he intends to. Therefore, the seller faces expected profits from inducing on-time payment, $\pi(0, 0.2)$.

Similarly, if the seller seeks to induce payment at $k_P = 1$, the penalty must satisfy: (1) $a \leq \overline{a}(k, q, 0)$ for at least one period k, inducing the individual to not pay the bill on time; and (2) $a > \overline{a}(k, q, 1)$ for all subsequent periods $k > 1$. In our setting, cutoff $\overline{a}(k, q, 0)$ becomes $\overline{a}(1, 0.2, 0) = 0.945$, and $\overline{a}(2, 0.2, 0) = 0.560$. Therefore, condition (1) holds as long as $a \leq 0.945$. Condition (2) holds, since cutoff $\overline{a}(k, 1)$ becomes $\overline{a}(2, 0.2, 1) = -10.500$, which is compatible with condition (1). The seller, among the set of feasible a's that induce payment at $k_P = 1$, the seller selects the highest, $a = 0.945$. Expected profits in this case become $\pi(1, 0.2)$.

Last, when the seller seeks to induce payment at $k_P = 2$, the penalty needs to satisfy: (1) $a \leq \overline{a}(k, q, 0)$ for at least one period k; and (2) $a < \overline{a}(k, q, 1)$ for at least one of the subsequent periods $k > 1$. In this example, condition (1) entails $a \leq \overline{a}(1, 0.2, 0) = 0.945$[18]. Condition (2), however, is not satisfied as $\overline{a}(2, 1) = -10.500$ and penalties are positive by definition. Therefore, there are no penalties inducing the buyer to pay at $k_P = 2$.

Optimal fee. Now $F(a)$ becomes

$$F(a) = \frac{26.328}{1 + \frac{8}{100}a + \frac{8}{10}\left[1 + \frac{2}{10}(1 + a)\right]}$$

thus being a function of penalty a. We use the same value of $a_{Legal} = 0.15$. Now the optimal fee for inducing on time payment becomes $F(0.15) = 13.190$. In this case, the buyer will pay one period late (since $a_{Legal} < 0.945$, which was the value of penalties upto which buyers pay one period late), entailing profits of 15.565. Thus, the seller cannot choose between on-time and late payments (as she is bounded by a_{Legal}) and the optimal $(a^*, F^*) = (0.15, 13.190)$.

Higher discount factor and present bias ($\delta = 0.995$, $\beta = 0.995$). *Penalties.* Consider the same parameter values as in the previous examples, but now , $\delta = 0.995$, $\beta = 0.995$. In this context, when $k_L = 0$, cutoff $\overline{a}(k, 0.2, k_L)$ becomes $\overline{a}(1, 0.2, 0) = 0.009$ and $\overline{a}(2, 0.2, 0) = 0.007$; and when $k_L = 1$, it becomes $\overline{a}(2, 0.15, 1) = 0.010$. If the seller seeks to induce no late payment, $k_P = 0$, the buyer located at $k_L = 0$ must find that the penalty a satisfies $a > \overline{a}(k, 0.2, 0)$ for every late period k. In our example, this condition entails $a > 0.009$ and $a > 0.007$, thus implying that $a > 0.009$ is a sufficient condition.[19] Unlike in the case with no shocks, however, the buyer will not pay on time if he suffers a shock, even if he intends to. Therefore, the seller faces expected profits from inducing on-time payment, $\pi(0, 0.2)$.

If the seller seeks to induce payment at $k_P = 1$, the penalty must satisfy: (1) $a \leq \overline{a}(k, q, 0)$ for at least one period k, inducing the individual to not pay the bill on time; and (2) $a > \overline{a}(k, q, 1)$ for all subsequent periods $k > 1$. In our setting, cutoff $\overline{a}(k, q, 0)$ becomes $\overline{a}(1, 0.2, 0) = 0.009$, and

to pay on time when signing the contract, since $a > \max\{\underline{a}(k, q)\} = \underline{a}(1, 0.2) = 0.054$.

[18]Note that $\overline{a}(1, 0.2, 0) = 0.945$ and $\overline{a}(2, 0.2, 0) = 0.560$, meaning that $a \leq \overline{a}(1, 0.2, 0)$ holds for both $k = 1$ and $k = 2$.

[19]This condition on a, $a > 0.009$, satisfies the initial assumption that $a > \max\{\underline{a}(k, q)\}$, i.e., the consumer plans to pay on time when signing the contract, since $a > \max\{\underline{a}(k, q)\} = \underline{a}(1, 0.2) = 0.005$.

$\overline{a}(2, 0.2, 0) = 0.045$. Therefore, condition (1) holds as long as $a \leq 0.007$. Condition (2), however, cannot hold in this context since cutoff $\overline{a}(k, 1)$ becomes $\overline{a}(2, 0.2, 1) = 0.010$, which is incompatible with (1). In words, the seller cannot set a to induce payment one period late, i.e., at $k_P = 1$.

Last, when the seller seeks to induce payment at $k_P = 2$, the penalty needs to satisfy: (1) $a \leq \overline{a}(k, q, 0)$ for at least one period k; and (2) $a < \overline{a}(k, q, 1)$ for at least one of the subsequent periods $k > 1$. In this example, condition (1) entails $a \leq \overline{a}(1, 0.2, 0) = 0.073$[20]. Condition (2) yields $a < \overline{a}(2, 0.2, 1) = 0.010$. Among the set of penalties simultaneously satisfying conditions (1) and (2), the highest penalty is $a = 0.120$, which induces payment two periods late. Expected profit in this case becomes $\pi(2, 0.2)$.

Optimal fee. Now $F(a)$ becomes

$$F(a) = \frac{16.623}{1 + \frac{8}{100}a + \frac{8}{10}\left[1 + \frac{2}{10}(1 + a)\right]}$$

thus being a function of penalty a. We use the same value of $a_{Legal} = 0.15$. Now the optimal fee for inducing on time payment becomes $F(0.15) = 8.328$, yielding profits of 8.628. If, instead, the seller induces payment two periods late, using penalty $a = 0.009$, then the fee becomes $F(0.009) = 8.472$, entailing profits of 8.624. Since profits are larger when the buyer is induced to pay on time, the seller sets the corresponding penalty-fee pair $(a^*, F^*) = (0.15, 8.328)$.

6.2 *Proof of Lemma 1*

The individual prefers to pay at period t than not pay (and pay after k periods of being late), if and only if expression (2.1) in the Model section is larger than expression (2.2), that is,

$$u_t + \beta \sum_{i=1}^{zn-t} \delta^i u_{i+t} - \beta F \sum_{i=x}^{z} \delta^{in-t} > u_t + \beta \sum_{i=1}^{zn-t} \delta^x u_{i+t} - \beta F \delta^{n+k-t}(1 + ka) - \beta F \sum_{i=x+1}^{z} \delta^{in-t}$$

which reduces to $a > \underline{a}(k) \equiv \frac{1-\delta^k}{k\delta^k}$.

Once the consumer reaches a paying period xn, for any $x = \{1, 2, ..., z\}$, he pays his bill due at xn rather than paying it k periods later, at period $xn + k$, if and only if

$$u_{xn} - F + \beta \sum_{i=1}^{n(z-x)} \delta^i u_{i+xn} - \beta F \sum_{i=1}^{z-x} \delta^{in} \leq u_{xn} + \beta \sum_{i=1}^{n(z-x)} \delta^i u_{i+xn} - \beta F \delta^k(1 + ka) - \beta F \sum_{i=1}^{z-x} \delta^{in}$$

which simplifies to $a \leq \overline{a}(k) \equiv \frac{1-\delta^k}{\beta k \delta^k}$ where the number of delay periods, k, satisfies $k < S$ by definition. Then, the consumer expects to pay on time at the beginning of every paying cycle if $a > \underline{a}(k)$, but chooses to pay at period $xn + k$ rather than at the due date xn if $a \leq \overline{a}(k)$.

The above cutoff is when the evaluation takes place at late period $k_L = 0$. Thus, we define this

[20] Note that $\overline{a}(1, 0.2, 0) = 0.009$ and $\overline{a}(2, 0.2, 0) = 0.007$, meaning that $a \leq \overline{a}(1, 0.2, 0)$ holds for both $k = 1$ and $k = 2$.

set of $a < \overline{a}(k, k_L)$ where $k_L = 0$. If the buyer decides to pay late, he re-evaluates his cutoffs at $k_L = 1$ as well as each subsequent period. When re-evaluated at every subsequent k_L,

$$a \leq \overline{a}(k) \equiv \frac{1 - \delta^{k-kL}}{\beta k \delta^{k-k_L} - k_L}$$

The buyer will pay at the current late period k_L if $a \leq \overline{a}(k) \equiv \frac{1-\delta^{k-kL}}{\beta k \delta^{k-k_L} - k_L}$ for at least one subsequent k. Otherwise, the buyer will pay at a later period. It is important to note that at period $k_L = S-1$, the buyer pays the fee no matter what. ■

6.3 *Proof of Proposition 1*

In a setting with shocks, the stream of payoffs that, at every period t, the consumer obtains from paying the bill due at the end of that billing cycle is

$$u_t + \beta \sum_{i=1}^{zn-t} \delta^i u_{i+t}$$

$$-\beta\delta^{xn-t}\left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2 q^2(1-q)F(1+2a) + ... + \delta^{S-1}q^{S-1}F(1+(S-1)a)\right]$$

$$-\beta\delta^{(x+1)n-t}\left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2 q^2(1-q)F(1+2a) + ... + \delta^{S-1}q^{S-1}F(1+(S-1)a)\right] - ...$$

$$-\beta\delta^{zn-t}\left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2 q^2(1-q)F(1+2a) + ... + \delta^{S-1}q^{S-1}F(1+(S-1)a)\right]$$

$$u_t + \beta \sum_{i=1}^{zn-t} \delta^i u_{i+t}$$

$$-\beta \sum_{i=x}^{z} \delta^{in-t}\left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2 q^2(1-q)F(1+2a) + ... + \delta^{S-1}q^{S-1}F(1+(S-1)a)\right] \quad (3)$$

In words, the individual pays fee F at period n if he does not suffer a shock at that period, which occurs with probability $1-q$. If, instead, he suffers a shock, with probability q, then he pays the fee (augmented by the penalty) one period later, $F(1+a)$, if he does not suffer a shock on that subsequent period, which happens with probability $(1-q)$. A similar argument applies if he suffers two subsequent shocks, with probabilty q^2, and thus pays $F(1+2a)$ three two periods after the bill was due, which occurs with probability $q^2(1-q)$. Finally, if the individual suffers shocks until the period in which the service is suspended, $S-1$, we assume that the bill is paid.

If, instead, at period t, the consumer plans to not pay the bill that is due at period xn despite not suffering a shock (but pay it k periods after it was due), his stream of discounted payoffs becomes

$$u_t + \beta \sum_{i=1}^{zn-t} \delta^i u_{i+t}$$

$$-\beta\delta^{xn-t}[(1-q)(0+0+...+\delta^k F(1+ka)) + \delta q(1-q)F(1+a) + \delta^2 q^2(1-q)F(1+2a) +$$

$$\delta^{S-1}q^{S-1}F(1+(S-1)a)] - \beta\delta^{(x+1)n-t}\left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2 q^2(1-q)F(1+2a) + ... + \delta^{S-1}q^{S-1}F(1+(S-1)a)\right] - ...$$

$$-\beta\delta^{zn-t}\left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2 q^2(1-q)F(1+2a) + ... + \delta^{S-1}q^{S-1}F(1+(S-1)a)\right] \quad (4)$$

Comparing (3) and (4), we conclude that, at any period t, the consumer plans to pay the next bill, due at xn, if and only if

$$a \geq \underline{a}(k,q) \equiv \frac{(1-q)\left[1+\sum_{i=1}^{S-2}\delta^i q^i - \sum_{i=k}^{S-2}\delta^i q^{i-k}\right] + \delta^{S-1}q^{S-1}(1-q^{-k})}{(1-q)\left[\sum_{i=k}^{S-2} i\delta^i q^{i-k} - \sum_{i=1}^{S-2}\ i\delta^i q^i\right] + (S-1)\delta^{S-1}q^{S-1}(q^{-k}-1)}$$

where cutoff $\underline{a}(k,q)$ can be more compactly expressed as $\underline{a}(k,q) = \frac{1-q}{A} + \frac{B}{A}$, where

$$A \equiv (1-q)\left[\sum_{i=k}^{S-2} i\delta^i q^{i-k} - \sum_{i=1}^{S-2}\ i\delta^i q^i\right] + (S-1)\delta^{S-1}q^{S-1}(q^{-k}-1)$$

$$B \equiv (1-q)\left[\sum_{i=1}^{S-2}\delta^i q^i - \sum_{i=k}^{S-2}\delta^i q^{i-k}\right] + \delta^{S-1}q^{S-1}(1-q^{-k})$$

Note that when $q=0$ this cutoff reduces to $\underline{a}(k,0) \equiv \frac{1-\delta^k}{k\delta^k}$, which coincides with the cutoff found in the case with no shocks, $\underline{a}(k)$.

Once the bill is due at period xn, his discounted stream of payoffs from paying the bill on time becomes

$$u_{xn} - F(1-q) + \beta\sum_{i=1}^{zn-t}\delta^i u_{i+t}$$
$$-\beta[(1-q)F\delta(1+a) + ...(1-q)F\delta^{(S-2)}q^{(S-2)}(1+(S-2)a) + F\delta^{(S-1)}q^{(S-1)}(1+(S-1)a)$$
$$-\beta\sum_{i=1}^{z-x}\delta^{in}\left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2q^2(1-q)F(1+2a) + ... + \delta^{S-1}q^{S-1}F(1+(S-1)a)\right] \tag{5}$$

If, instead, he chooses to pay k periods late, his discounted stream of payoffs is

$$u_{xn}+\beta\sum_{i=1}^{zn-t}\delta^i u_{i+t}$$
$$-\beta\left[(0+0+...+(1-q)\delta^kF(1+ka)) + \delta^{k+1}q(1-q)F(1+(k+1)a) + ... + \delta^{S-1}q^{S-k}F(1+(S-1)a)\right]$$
$$-\beta\sum_{i=1}^{z-x}\delta^{in}\left[(1-q)F + \delta q(1-q)F(1+a) + \delta^2q^2(1-q)F(1+2a) + ... + \delta^{S-1}q^{S-1}F(1+(S-1)a)\right] \tag{6}$$

Comparing (5) and (6), we conclude that, at any period xn when bills are due, the consumer does not pay if and only if $a < \overline{a}(k,q)$, where cutoff $\overline{a}(k,q)$ is

$$\frac{(1-q)\left[1+\beta\sum_{i=1}^{S-2}\delta^i q^i - \beta\sum_{i=k}^{S-2}\delta^i q^{i-k}\right] + \beta\delta^{S-1}q^{S-1}(1-q^{-k})}{\beta\left[(1-q)\left(\sum_{i=k}^{S-2} i\delta^i q^{i-k} - \sum_{i=1}^{S-2} i\delta^i q^i\right) + (S-1)\delta^{S-1}q^{S-1}(q^{-k}-1)\right]}$$

In addition, cutoff $\overline{a}(k,q)$ can be more compactly expressed as

$$\overline{a}(k,q)=\frac{1-q}{\beta A}+\frac{B}{A}.$$

Note that when $q=0$ this cutoff reduces to $\overline{a}(k,0)\equiv\frac{1-\delta^k}{\beta k\delta^k}$, which coincides with that in the case with no shocks, $\overline{a}(k)$. Generally, consider that the consumer is at period xn, or at a later period after xn, k_L, where $k_L=t-xn\geq 0$ represents how many periods mediated since the bill was due at xn. In the case that $k_L=0$, he is at period xn; whereas when $k_L>0$, he is strictly beyond period xn. In that setting, he does not pay the bill at the current period, if there is a period k_L such that $a<\overline{a}(k,q,k_L)$, where cutoff $\overline{a}(k,q,k_L)$ is

$$\overline{a}(k,q,k_L)=\frac{(1-q)+\beta D}{\beta C-(1-q)k_L},$$

where

$$\begin{aligned}C \equiv\ & (1-q)\left(\sum_{i=k-k_L}^{S-2-k_L}(i+k_L)\delta^i q^{i-(k-k_L)}-\sum_{i=1}^{S-2-k_L}(i+k_L)\delta^i q^i\right)\\ & +(S-1)\delta^{S-1}q^{S-1}(q^{-(k-k_L)}-1)\end{aligned}$$

$$D\equiv(1-q)\left[\sum_{i=1}^{S-2-k_L}\delta^i q^i-\sum_{i=k-k_L}^{S-2-k_L}\delta^i q^{i-(k-k_c)}\right]+\delta^{S-1}q^{S-1}(1-q^{-(k-k_L)})$$

Note that, when the consumer evaluates his decision at period xn, $k_L=0$, thus reducing the above cutoff $\overline{a}(k,q,0)=\overline{a}(k,q)$. In addition, cutoff $\underline{a}(k,q)$ did not include late period k_L since the consumer considered, at the time of signing the contract, whether to pay the bill when it is due at xn or not. ■

References

[1] Armstrong, Mark, and John Vickers (2012) "Consumer protection and contingent charges." Journal of Economic Literature: 477-493.

[2] Armstrong, Mark, John Vickers, and Jidong Zhou (2009) "Consumer protection and the incentive to become informed." Journal of the European Economic Association: 399-410.

[3] Balakrishnan, Uttara, Johannes Haushofer, and Pamela Jakiela (2016) "How Soon Is Now? Evidence of Present Bias from Convex Time Budget Experiments." IZA Discussion Paper No. 9653.

[4] Calzolari, Giacomo, and Mattia Nardotto (2011) "Nudging with information: A randomized field experiment." Available at SSRN 1924901 .

[5] Caillaud, Bernard, and Bruno Jullien (2000) "Modelling time-inconsistent preferences." European Economic Review 44, no. 4: 1116-1124.

[6] Chetty, Raj (2015) "Behavioral economics and public policy: A pragmatic perspective." The American Economic Review: 1-33.

[7] Citi Simplicity Survey (2013) April 19-21, http://www.citigroup.com/citi/news/2013/130514a.htm.

[8] Craik, Fergus IM, and Robert S. Lockhart (1972) "Levels of processing: A framework for memory research." Journal of Verbal Learning and Verbal Behavior 11, no. 6: 671-684.

[9] Ebbinghaus, Hermann (1885) "Memory: A Contribution to Experimental Psychology." Leipzig, Germany: Duncker and Humblot.

[10] Ericson, Keith M. Marzilli (2011) "Forgetting we forget: Overconfidence and memory." Journal of the European Economic Association 9, no. 1: 43-60.

[11] Herweg, Fabian, and Daniel Müller (2011) "Performance of procrastinators: on the value of deadlines." Theory and Decision: 329-366.

[12] Hoch, Stephen J., and George F. Loewenstein (1991) "Time-inconsistent preferences and consumer self-control." Journal of Consumer Research: 492-507.

[13] Karlan, Dean, Margaret McConnell, Sendhil Mullainathan, and Jonathan Zinman (2016) "Getting to the top of mind: How reminders increase saving." Management Science 62, no. 12: 3391-3411.

[14] Laibson, David (1997) "Golden eggs and hyperbolic discounting." The Quarterly Journal of Economics: 443-477.

[15] Murooka, Takeshi, and Marco A. Schwarz (2016) "The timing of choice-enhancing policies." Working paper, LMU Munich.

[16] O'Donoghue, Ted, and Matthew Rabin (1999) "Doing it now or later." American Economic Review: 103-124.

[17] Schacter, Daniel L. (1999) "The seven sins of memory: Insights from psychology and cognitive neuroscience." American Psychologist 54, no. 3: 182.

[18] Spiegler, Ran (2011) *Bounded rationality and industrial organization.* Oxford University Press.

[19] Takeuchi, Kan (2011) "Non-parametric test of time consistency: Present bias and future bias." Games and Economic Behavior 71, no. 2: 456-478.

[20] The 2009 Consumer Financial Literacy Survey, March 13-16, 2009. The National Foundation for Credit Counseling (NFCC). https://www.nfcc.org/wp-content/uploads/2014/04/2009FinancialLiteracySurveyFINAL.pdf.

[21] The 2010 Consumer Financial Literacy Survey, March 4-8, 2010. The National Foundation for Credit Counseling (NFCC). https://www.nfcc.org/wp-content/uploads/2014/04/2010ConsumerFinancial LiteracySurveyFinalReport.pdf.

[22] The 2011 Consumer Financial Literacy Survey, March 4-7, 2011. The National Foundation for Credit Counseling (NFCC). https://www.nfcc.org/wp-content/uploads/2014/04/NFCC_2011Financial-Literacy Survey_FINALREPORT_033011.pdf.

[23] The 2012 Consumer Financial Literacy Survey, March 16-17, 2012. The National Foundation for Credit Counseling (NFCC). https://www.nfcc.org/NewsRoom/FinancialLiteracy/files2012/FLS2012FINAL REPORT0402late.pdf.

[24] The 2013 Consumer Financial Literacy Survey, March 4-6, 2013. The National Foundation for Credit Counseling(NFCC). https://www.nfcc.org/NewsRoom/FinancialLiteracy/files2013/NFCC_NBPCA_ 2013%20FinancialLiteracy_survey_datasheet_key%20findings_032913.pdf.

[25] The 2014 Consumer Financial Literacy Survey, March 4-6, 2014. The National Foundation for Credit Counseling(NFCC). https://www.nfcc.org/wp-content/uploads/2013/06/NFCC_2014-Financial LiteracySurvey_FINAL.pdf.

[26] The 2015 Consumer Financial Literacy Survey, March 11-13, 2015. The National Foundation for Credit Counseling (NFCC). https://www.nfcc.org/wp-content/uploads/2015/04/NFCC_2015_Financial_ Literacy_Survey_FINAL.pdf.

[27] The 2016 Consumer Financial Literacy Survey, March 22-April 6, 2016. The National Foundation for Credit Counseling (NFCC). https://nfcc.org/wp-content/uploads/2016/04/NFCC_BECU_2016-FLS_datasheet-with-key-findings_041516.pdf.

[28] Thaler, Richard H. (2015) *Misbehaving: The Making of Behavioral Economics*. WW Norton & Company.