

# Campaign Limits and Policy Convergence with Asymmetric Agents\*

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## Abstract

We extend previous work on the role of politically motivated donors who contribute to candidates in an election with single dimension policy preferences. In a two-stage game where donors observe candidate policy positions and then allocate funding accordingly, we find that reducing the cost of donations incentivizes candidates to position closer to one another, reducing policy divergence. Furthermore, we find that as donations become more effective at influencing voter decisions, candidates respond less to voter preferences and more to those of donors. In addition, we analyze the presence of asymmetries in this model using numerical analysis techniques. We also extend our model by allowing for public funding from governments. By implementing stringent campaign contributions limits, candidate positions align with voter preferences at the cost of increased policy divergence. In contrast, unlimited campaign contributions lead to candidate positions moving away from voters to donors' preferences, but increases policy convergence.

KEYWORDS: Asymmetries; Policy Preferences; Policy Divergence; Donors; Political Contributions; Campaign Limits.

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# 1 Introduction

The landscape of American campaign finance has changed drastically over the past decade. With the Supreme Court of the United States of America’s landmark 2010 decision in *Citizen’s United v. FEC*, the ability to contribute to political organizations by corporations, unions, and other similar organizations was declared protected as a form of free speech. Following that ruling, in 2014, the Supreme Court in *McCutcheon v. FEC* ruled that donations by private individuals to political candidates cannot be limited. These two rulings coincided with a drastic increase in campaign donations, and spending, by American politicians.<sup>1</sup> In this paper, we examine whether setting no limits on political contributions to candidates can bring their policies closer to the median voter or, instead, move them further away, hindering political representation. Specifically, we demonstrate that setting no limit on donations, as currently in the U.S., induces candidates to converge towards the average of the donors’ ideal policies in a race to receive as many contributions as possible. While more policy convergence arises in this setting, it occurs towards the donors’ preferred policies, which may often differ from those of voters. We also analyze the opposite scenario, where regulations set stringent limits on donations (as in Canada, where each individual may donate at most \$1,550, and political parties are limited to what they can spend). In this context, our results show that donors have no incentives to contribute to either candidate, who behave considering their own ideal policies and those of voters, but ignoring those of the donors. As a consequence, more policy divergence emerges than when donations are not severely limited.

We consider a two-stage game where, in the first period, every candidate simultaneously announces his policy position; and in the second period, donors observe candidates’ positions, and respond contributing to either (or both) candidates. Given political positions and donations, every candidate has a probability of winning the election which depends on the underlying distribution of voter preferences, and on the amount of campaign funding of each candidate (e.g., voters’ decision is affected by advertising, house visits, etc.). For generality, we assume that, when choosing his policy position, every candidate considers: (1) his probability of winning the election, as in Downs (1953); and (2) the difference between the policy that is elected and his ideal, as in Wittman (1983). Our setting thus allows for a linear combination of these two polar incentives, which also lets us examine the special cases of Downs or Wittman’s models.

Candidates face three separate forces when choosing their policy positions. First, as in the original Downs model (without contributions), candidates select their positions by considering their probability of winning the election alone (i.e., locating as close as possible to the median voter). Adding expected policy payoffs (i.e., motive (2) from above, but still without donations) would reduce candidates’ incentives to locate at the median voter, as they must also consider the utility they obtain from the implemented policy. Finally, if contributions are allowed, candidates must also take into account both donors’ ideal policies in order to maximize their share of donations.

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<sup>1</sup>Campaign spending levels in the US Presidential election increased by 45% from 2004 to 2008, and again by 40% from 2008 to 2012. This, comparing to a 32% decrease in donations in the UK parliamentary election from 2005 to 2010 and a subsequent increase of 12% from 2010 to 2015.

In the equilibrium of the second stage, we show that donors contribute at most to one candidate. Intuitively, if a donor contributing to one candidate donates to the other, he would be reducing the former’s probability of winning, which is contrary to his original objective. In the first stage, candidates anticipate the effect that a change in their political positions has on donations during the second stage before choosing their policies, thus considering the three forces mentioned above. Furthermore, we demonstrate the existence of a “donation stealing” incentive, whereby every candidate, by locating closer to his opponent’s donor, reduces his rival’s donations more significantly than the decrease he experiences in his own contributions. This plays a role when donations become less costly, where more policy convergence can be sustained in equilibrium as candidates try to deny donations to their rivals.

For presentation purposes, we first analyze a symmetric setting, where voters, donors, and candidates are symmetrically distributed around the midpoint of the policy spectrum. In that context, we show that adding donations has no effect on the Downsian results, regardless of their cost and effectiveness. In contrast, allowing for political contributions to the Wittman model, gives rise to the donation stealing effect, ultimately increasing policy convergence. In addition, we show that, as donations become more effective at determining the outcome of the election, candidates move away from the median voter toward the (average of) donors’ ideal policies.

In the asymmetric version of the model, we first demonstrate that equilibrium results are qualitatively unaffected when candidates assign a sufficiently high weight on the utility from the implemented policy. However, when such weight is low, candidates have the ability to gain a donation advantage relative to their opponent by positioning closer to one of the donors and further away from the median voter. If a candidate chooses such a position, his opponent mimics his strategy, leading the former to move back to a position close to the median voter; ultimately yielding cyclical behavior, and no stable policy profile emerging in equilibrium. In the special case where donors have similar ideal policies, we find that the candidate whose ideal policy is closest to those of the donors receives all donations, while his opponent chooses a position favorable to the voter distribution.

To build a more complete picture of international elections, we model public funding as an extension to our model. When one candidate receives a funding advantage prior to announcing his position (as is common in many publicly funded elections where funding is allocated based on votes in the previous election), we find that he positions closer to his ideal policy, while his opponent also positions closer to him in order to balance the advantage in public funding.

We also model the effect of campaign contribution limits, as seen in the US until 2014 and currently seen in Canada (for a list of countries setting contribution or spending limits, see Appendix 4). We find that when donation constraints are set low, candidates behave as if donations were absent, leading to maximal policy divergence, but their incentives align with voter preferences. Intermediate constraints yield situations where no equilibrium in pure strategies exists, and unbinding high constraints (or no limits) yield the least amount of policy divergence, but allow equilibrium candidate positions to be distorted towards the ideal policies of the donors rather than the voters.

Our results thus contribute to the discussion about the costs and benefits of limiting campaign contributions. On one hand, a social planner can implement stringent donations, which aligns candidate behavior with voter preferences at the cost of increased policy divergence. On the other hand, a social planner can leave donations unlimited, which allows candidate behavior to skew in favor of donor preferences, but increases policy convergence. Depending on donor preferences relative to those of voters and how effective donations are at influencing the election, either stance of the social planner could be optimal.

**Related Literature.** Our model extends previous work done by Ball (1999a), which considers a spatial setting where policy positions are ordered along a Hotelling line, which borrows from the works of Hotelling (1929) and d’Aspremont. et al (1979). The seminal work by Downs (1957) establishes that, when every candidate only seeks to maximize his own probability of winning an election, he positions at the median voter, thus achieving perfect policy convergence.<sup>2</sup> Wittman (1983) builds upon the work of Downs but assumes that candidates care about which policy is implemented (and have different desired policies), rather than their probability of winning the election. In this setting, policy divergence can be sustained in equilibrium, where each candidate positions closer to his own ideal policy and away from the median voter. Our model encompasses both approaches as special cases, adds the role of political contributions, and the effect of limiting campaign funding.<sup>3</sup>

In the campaign finance literature, the seminal work of Austen-Smith (1987) establishes that candidates announce their policy positions anticipating how their potential donors react to such an announcement.<sup>4</sup> Ball (1999a) builds upon this initial model and shows that candidates not only choose their policy positions based on how they expect their own potential donors to react, but they also take into consideration the effect that their position has on the amount of donations that their rival receives. Our work extends Ball’s by allowing for asymmetries, such as when both candidates or donors prefer a policy position that is above or below that of the median voter. In Ball’s original model, symmetry was required in order to guarantee a closed form solution, as described in Ball (1999b). By using numerical analysis, we examine asymmetries and analyze the cases where donors, candidates, or voters favor policies that are not perfectly balanced against one another. In addition, Ball’s work focused solely on the case in which candidates obtain utility from implemented policies (i.e., based on Wittman’s model). We consider candidates that are interested in winning the election (as in Downs), in the implemented policy (as in Wittman), or in both.

Several studies estimate ideal policy positions of elected officials. Poole and Rosenthal (1984)

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<sup>2</sup>For an example of an election with three candidates, see Evrenk and Kha (2011).

<sup>3</sup>Candidates’ announcements of policy positions might to be credible to donors or voters. Work by Alessina (1987) and Aragones. et al. (2007), however, suggest that when candidates face repeated elections, voters (and by extension, donors) recall when implemented and announced policy positions differ, and form their beliefs on a candidate’s true position accordingly. As a result, candidates announce the policy that they intend to implement and can be considered credible. We assume that candidates cannot shirk on their announced positions, but our setting can be extended to models where candidates build their reputation.

<sup>4</sup>Work by Hinich and Munger (1994) explores campaign contributions as a hinderance to a rival rather than a benefit to a preferred candidate.

measure policy divergence in the U.S. Senate from 1959 to 1980 and find that policy divergence increased over that time period. Hare and Poole (2014) follow up on this initial study, updating the data to the Obama administration and find that, while the Democrat party became slightly more liberal since 1980, the Republican party faced a large conservative shift. Our paper models these shifts in ideal policy positions and their resulting change in equilibrium policies and donations.<sup>5</sup>

In real life, special interest groups can offer donations at any stage of the electoral competition. While several studies consider donors offering contribution menus at the beginning of the game, Grossman and Helpman (2001), our setting allows contributions after candidates choose policy positions. As a consequence, our model closely fits recent elections such as the 2012 U.S. Presidential election.<sup>6</sup> When donors act first, contributions must specify a rather involved contract defining donations as a function of both candidates' positions, which is typically illegal and thus not enforceable. Our setting, however, allows donors to observe candidates' positions and respond to them without the need for a contract.

Our model's time structure is therefore similar to that in Herrera et al. (2008), which considers two parties choosing binding policy positions during the first stage, and their campaign effort (funds) in the second stage. Our setting, however, assumes that funding comes from donors. Anticipating the donors' ideal policies, we show that candidates can alter their announced policies in the first stage to capture a larger contribution than their rival. We also allow for public funding to play a role in candidates' policies, as in Ortuno and Schultz (2005). Unlike our model, they consider that candidates do not receive contributions from donors. Instead, they assume that candidates campaign is financed through two sources: public funding, based on the candidate's vote share; and non-public sources (a lump sum subsidy) which does not provide candidates with incentives to alter their policy positions to receive more funds. In contrast, our model allows for strategic effects to arise between candidates and donors.

Section 2 describes the model, while section 3 presents equilibrium results in the first and second stage. Section 4 provides a numerical simulation to illustrate our findings, and section 5 extends our model to asymmetric candidates, donors, and voter distributions; allows for public funding; and analyzes the effect of donation constraints in equilibrium policy positions. Section 6 concludes and discusses our results.

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<sup>5</sup>Empirical work has been done to estimate the effect of campaign contribution limits. Jacobson (1978) and Coate (2004) focus on how contribution limits affected the margin of victory between candidates (namely, an incumbent against a challenger). Our model, however, examines the effect of contribution limits on equilibrium positioning, rather than the margin of victory.

<sup>6</sup>In 2012, Mitt Romney faced a long primary challenge, which required him to spend 87% of the \$153 million raised up until June 2012, when he clinched the Republican nomination. In contrast, incumbent President Barack Obama spent 69% of the \$303 million that he had raised over the same period. To raise additional funds, Romney courted donors who had either supported his rivals in the primary election or stayed out completely. These donors were able to observe Romney's policy positions long before ever offering him their aid.

## 2 Model

Consider two candidates competing for office. Every candidate  $i \in \{A, B\}$  chooses a policy position  $x_i \in [0, 1]$ . Every voter has his own preferred policy position distributed along  $[0, 1]$ . Voters choose the candidate that gives them the higher utility level and are more likely to vote for a candidate whose policy position is closer to their own. Every candidate seeks to maximize a combination of his probability of winning the election (as in Downs (1957)), and his expected utility they receive from the elected policy (as in Wittman (1983)).

In addition to candidates and voters, we also include two donors, where each donor  $k$  is endowed with an exogenous sum of money,  $\bar{k}$ , and with a preferred position,  $\hat{d}_k$ , where  $k \in \{L, R\}$ . Donors face a marginal cost of contributions  $c$ , which represents the opportunity cost of money. As we describe in the following sections, every donor uses these donations to induce candidates to position closer to the donor's ideal policy. In turn, a campaign with more funding is able to better advertise their respective candidate, and will increase the probability that candidate wins the election.

The time structure of the game is the following: In stage 1, candidates simultaneously and independently announce their policy positions, which are observed by both donors and voters. In stage 2, every donor responds simultaneously and independently allocating donations to one, both, or no candidates. In the last stage, votes are a function of the candidates' chosen positions as in Downs' and Wittman's models; where this probability is affected by donors' contributions to each candidate. Since this probability function is exogenous, its effect is only considered when analyzing candidates' probability of winning (subsection 3.1), and donors' expected utility from the implemented policy (subsection 3.2).

## 3 Equilibrium results

### 3.1 Stage 2 - Donor's funding decisions

In stage 2, every donor  $k \in \{L, R\}$  seeks to maximize his expected utility based on the outcome of the election, which we can express with the following objective function:

$$\begin{aligned} \max_{k_i, k_j \geq 0} \quad & p_i(x_i, x_j, D_i, D_j) u_k(x_i; \hat{d}_k) + [1 - p_i(x_i, x_j, D_i, D_j)] u_k(x_j; \hat{d}_k) - (k_i + k_j) c \quad (1) \\ \text{s.t.} \quad & k_i + k_j \leq \bar{k} \end{aligned}$$

where  $k_i$  denotes donor  $k$ 's monetary contribution to candidate  $i$ 's campaign, where  $i \in \{A, B\}$ ,  $p_i(\cdot)$  represents the subjective probability that candidate  $i$  wins the election,  $x_i$  denotes candidate  $i$ 's policy position,  $D_i$  captures the aggregate donations received by candidate  $i$ , i.e.,  $D_i \equiv k_i + l_i$ ; and  $u_k(\cdot)$  represents the utility that donor  $k$  receives based on the outcome of the election. As in the standard Downsian (1967) and Wittman (1983) models, candidates do not have precise knowledge of voters' utility functions, which prevents candidates from knowing exactly how voters will vote. The next lemma analyzes equilibrium donations for every donor  $k$ . (All proofs are relegated to the

Appendix)

**Lemma 1.** *Donor  $k$ 's equilibrium donation to candidate  $i$ ,  $k_i$ , solves*

$$\frac{dp_i}{dD_i} \left[ u_k(x_i; \hat{d}_k) - u_k(x_j; \hat{d}_k) \right] \leq c \quad (2)$$

Intuitively, donor  $k$  increases his contribution  $k_i$  until his marginal benefit from further donations (left-hand side of equation (2)) coincides with his marginal cost (right-hand side of (2)). Note that the marginal benefit captures the additional probability that candidate  $i$  wins the election thanks to larger donation,  $\frac{dp_i}{dD_i} \geq 0$ , and the utility gain that donor  $k$  obtains when candidate  $i$  wins the election to candidate  $j$ ,  $u_k(x_i; \hat{d}_k) - u_k(x_j; \hat{d}_k)$ . Needless to say, if donor  $k$  prefers candidate  $j$  winning the election then  $u_k(x_i; \hat{d}_k) < u_k(x_j; \hat{d}_k)$ , and the left-hand side becomes unambiguously negative, ultimately yielding a corner solution where donor  $k$  does not contribute to candidate  $i$ 's campaign in equilibrium. This result suggests that ever donor  $k$  will only contribute to the candidate yielding the highest utility; as we prove in the next lemma.

**Lemma 2.** *In equilibrium, every donor  $k$  contributes to one candidate at most.*

Intuitively, if donor  $k$  contributes to candidate  $i$ , he does so to increase the probability that candidate  $i$  wins the election. On the contrary, any contribution that donor  $k$  makes to the other candidate  $j \neq i$  lowers the probability that candidate  $i$  wins the election. Thus, contributions to both candidates are counterproductive, and every donor  $k$  only donates to the candidate whose policy position yields him the highest utility level. As a remark, note that if both policy positions yield the same utility for donor  $k$ ,  $u_k(x_i; \hat{d}_k) = u_k(x_j; \hat{d}_k)$  then his marginal benefit of contributing to candidate  $i$  (left-hand side of (2)) becomes nil, inducing no donations to either candidate, i.e.,  $k_i^* = 0$  for all  $i$ .

Lemmas 1 and 2 allow us to characterize the solution of the second stage of the game into several cases, as detailed in Proposition 1. For compactness, let  $g_k(x_i, x_j) \equiv u_k(x_i; \hat{d}_k) - u_k(x_j; \hat{d}_k)$  be the utility gain that donor  $k$  obtains from candidate  $i$  relative to  $j \neq i$ . Solving for  $g_k(x_i, x_j)$  in expression (2) yields  $g_k(x_i, x_j) \leq \hat{p}_i$ , where  $\hat{p}_i \equiv \frac{c}{\frac{dp_i}{dD_i}}$ . Figure 1 depicts the utility gains of donor  $k$ ,  $g_k(x_i, x_j)$  in the horizontal axis; and includes  $\hat{p}_i$ ,  $\hat{p}_j$  and the origin where  $g_k(x_i, x_j) = 0$ , which gives rise to three different regions, A-C. For instance, region A satisfies  $g_k(x_i, x_j) \geq \hat{p}_i$ ; region B has  $\hat{p}_j < g_k(x_i, x_j) < \hat{p}_i$ ; and similarly for region C.

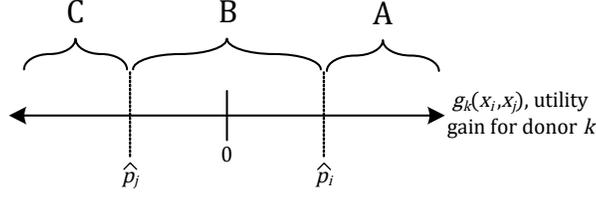


Figure 1. Donor behavior.

**Proposition 1.** *Equilibrium donations  $(k_i^*, k_j^*)$  satisfy:*

1. Region A.  $0 < k_i^* \leq \bar{k}$ ,  $k_j^* = 0$  if  $g_k(x_i, x_j) \geq \hat{p}_i$ .
2. Region B.  $k_i^* = k_j^* = 0$  if  $\hat{p}_j < g_k(x_i, x_j) < \hat{p}_i$ .
3. Region C.  $k_i^* = 0$ ,  $0 < k_j^* \leq \bar{k}$  if  $g_k(x_i, x_j) \leq \hat{p}_j$ .

In region A (C) of figure 1, donor  $k$ 's marginal benefit of contributions to candidate  $i$  ( $j$ ) is greater than or equal to his marginal cost. Intuitively, the policy position that candidate  $i$  ( $j$ ) presents to donor  $k$  is favorable relative to candidate  $j \neq i$ 's ( $i \neq j$ 's) and donor  $k$  makes a contribution to candidate  $i$ 's ( $j$ 's) campaign. When the marginal benefit of contributions is equal to the marginal cost, donor  $k$  contributes some positive amount,  $k_i^* > 0$  ( $k_j^* > 0$ ), whereas when the marginal benefit strictly exceeds the marginal cost, donor  $k$  contributes as much as possible to candidate  $i$  ( $j$ ), i.e.,  $k_i^* = \bar{k}$  ( $k_j^* = \bar{k}$ , respectively).

Lastly, in region B, the marginal benefit that donor  $k$  receives from contributions to either candidate  $i$  is strictly less than the marginal cost. This induces donor  $k$  to withhold all contributions from either candidate. Intuitively, candidate  $i$  and  $j \neq i$ 's policy positions do not differ enough for donor  $k$  to support either candidate monetarily.

With the equilibrium behavior of both donors defined as in proposition 1, we can sign the best response functions for both candidates as presented in corollaries 1 and 2.

**Corollary 1.** *When donors  $k$  and  $l \neq k$  each contribute to the same (opposite) candidate, their contributions are decreasing (increasing) in the other donor's contribution, i.e.,  $\frac{dk_i}{dl_i} = -1$  ( $\frac{dk_i}{dl_j} > 0$ ).*

When both donors support the same candidate, additional contributions from either candidate decreases the marginal benefit of further contributions. Intuitively, both donors are able to benefit from the contribution made to the supported candidate, and a similar situation as a public good arises, where donors free ride upon each others' contributions. In particular, every dollar donated to candidate  $i$  by donor  $l$  decreases donor  $k$ 's contribution by exactly one dollar. On the contrary,

when donors support opposite candidates, additional contributions from either candidate increases the marginal benefit of additional contributions to the other donor. Intuitively, contributions from one donor are detrimental to the other donor's outcome in the election, and the other donor has incentive to further contribute to his own candidate to protect his interests in the election.

**Corollary 2.** *Donor  $k$ 's contribution to candidate  $i$ ,  $k_i^*$ , weakly increases (decreases) in candidate  $i$ 's position,  $x_i$ , if  $x_i < \hat{d}_k$  ( $x_i > \hat{d}_k$ , respectively, where  $\hat{d}_k$  is donor  $k$ 's ideal policy position). Furthermore,  $k_i^*$  weakly decreases (increases) in candidate  $j$ 's position,  $x_j$  if  $x_j < \hat{d}_k$  ( $x_j > \hat{d}_k$ , respectively).*

When candidate  $i$  moves closer to  $k$ 's ideal policy position, the utility gain that donor  $k$  receives from candidate  $i$ 's position increases. If donor  $k$  is already contributing the candidate  $i$ , this change in position increases the contribution that  $k$  makes to candidate  $i$ 's campaign. On the other hand, if candidate  $j$  moves towards donor  $k$ 's ideal policy position, the utility that donor  $k$  receives from candidate  $j$ 's position increases, thus decreasing the marginal benefit of donor  $k$  contributing to candidate  $i$ . In the case of an interior solution, this leads donor  $k$  to contribute less to candidate  $i$ 's campaign.

### 3.2 Stage 1 - Policy decisions

In the first stage, every candidate  $i \in \{A, B\}$  seeks to maximize his expected payoff, given what he anticipates how donors will respond in the second stage. For simplicity, let  $p_i \equiv p_i(x_i, x_j, k_i^* + l_i^*, k_j^* + l_j^*)$  denote the probability that candidate  $i$  wins the election. Therefore, candidate  $i$  solves

$$\max_{x_i} \underbrace{\gamma[p_i(v_i(x_i) + w) + (1 - p_i)v_i(x_j)]}_{\text{Wittman}} + (1 - \gamma) \underbrace{p_i}_{\text{Downs}} \quad (3)$$

where  $v_i(x_i)$  represents the payoff that candidate  $i$  receives when his chosen policy is elected (i.e., he wins the election);  $v_i(x_j)$  denotes candidate  $i$ 's payoff from losing the election (but having candidate  $j$ 's policy position implemented);  $w$  represents the utility that candidate  $i$  receives from holding office; and  $\gamma$  reflects how candidate  $i$  weighs his expected payoff from policy implementation against his probability of winning the election. Hence, the first term in expression (3) coincides with the specification used in Wittman (1983), where candidates are expected payoff maximizers, while the second term in expression (3) corresponds with the original Downsian (1957) model where candidates maximize the probability to win the election. Thus, by setting  $\gamma = 1$ , our model becomes identical to Wittman's (1983), or if we set  $\gamma = 0$ , we obtain Downs' (1957) model. Function  $v_i(\cdot)$  is strictly concave, reaching a maximum at candidate  $i$ 's ideal policy position,  $\hat{x}_i$ , which we assume to be exogenously given.<sup>7</sup> The next lemma studies equilibrium conditions from candidate  $i$ 's problem in (3).

<sup>7</sup>Many studies consider that functions such as  $v_i(x_i) = -A(x_i - \hat{x}_i)^2$ , where  $A > 0$ , which is negative everywhere except at its max when  $x_i = \hat{x}_i$ , i.e., the implemented policy coincides with candidate  $i$ 's ideal, where  $v_i(\hat{x}_i) = 0$ .

**Lemma 3.** *Candidate  $i$ 's equilibrium position,  $x_i$ , solves*

$$\underbrace{\frac{dp_i}{dx_i} [\gamma(v_i(x_i) + w - v_i(x_j)) + (1 - \gamma)]}_{\text{Direct effect}} + \underbrace{p_i \gamma \frac{dv_i(x_i)}{dx_i}}_{\text{Indirect effect}} = 0 \quad (4)$$

where term  $\frac{dp_i}{dx_i}$  can be expressed as

$$\frac{dp_i}{dx_i} = \frac{\partial p_i}{\partial x_i} + \frac{\partial p_i}{\partial D_i} \left[ \frac{dk_i^*}{dx_i} + \frac{dl_i^*}{dx_i} \right] + \frac{\partial p_i}{\partial D_j} \left[ \frac{dk_j^*}{dx_i} + \frac{dl_j^*}{dx_i} \right] \quad (5)$$

Expression (5) measures how an increase in position  $x_i$  affects the probability that candidate  $i$  wins the election. In the first term on the right-hand side, an increase in  $x_i$  corresponds with candidate  $i$  moving closer to (away from) the median voter, thus making his policy position more (less) attractive to voters and increasing (decreasing) the probability that he wins the election. The second term depicts how candidate  $i$ 's policy position affects the contributions he receives from each donor. We know that  $\frac{\partial p_i}{\partial D_i} > 0$  since an increase in donations to candidate  $i$  increases his chances of winning the election. The signs of  $\frac{dk_i^*}{dx_i}$  and  $\frac{dl_i^*}{dx_i}$  depend on candidate  $i$ 's policy position relative to the ideal policies of donors  $k$  and  $l$ , respectively. Like the median voter, if candidate  $i$  is positioned below (above) either donor's ideal policy position, an increase in  $x_i$  makes his position more (less) desirable to the donor, thus increasing (decreasing) the donor's contribution to this candidate. An analogous argument applies to the third term in expression (5), which captures how an increase in  $x_i$  affects donations to his rival, candidate  $j$ , where  $\frac{\partial p_i}{\partial D_j} < 0$ . Overall, an increase in  $x_i$  improves candidate  $i$ 's probability of winning the election (making expression (5) positive) if: (1) the additional votes he receives offset the potential loss in contributions (relative to candidate  $j$ 's donations); (2) if, instead, the loss in votes he experiences from increasing  $x_i$  is compensated by the more generous contributions he receives from donors; or a combination of both cases.

In expression (4), the first term depicts how an increase in candidate  $i$ 's policy position affects his expected policy payoff. An increase in  $x_i$  causes the probability of candidate  $i$  winning the election to either rise or fall, as outlined in the previous paragraph. This alters candidate  $i$ 's expected policy payoff by an amount equal to the net utility gain,  $v_i(x_i) + w - v_i(x_j)$ , i.e., candidate  $i$ 's policy is implemented and he holds office, rather than having his opponent win the election. Likewise, an increase in  $x_i$  also produces an indirect effect as it changes candidate  $i$ 's utility from his new policy position,  $\frac{dv_i(x_i)}{dx_i}$  as depicted in the second term of expression (4). The sign of  $\frac{dv_i(x_i)}{dx_i}$  depends on candidate  $i$ 's policy relative to his ideal. If  $x_i < \hat{x}_i$  ( $x_i > \hat{x}_i$ ), he moves closer (farther away) to his ideal policy, thus increasing (decreasing) his utility if he wins the election. Finally, the second term in expression (4), associated with  $1 - \gamma$ , shows how candidate  $i$ 's payoff changes as the probability of candidate  $i$  winning the election changes; the details of which were explained in the previous paragraph.

**Corollary 3.** *In the Downsian specification ( $\gamma = 0$ ), expression (4) simplifies to  $\frac{dp_i}{dx_i} = 0$ . In*

the Wittman's specification ( $\gamma = 1$ ), expression (4) reduces to

$$\frac{dp_i}{dx_i}(v_i(x_i) + w - v_i(x_j)) + p_i \frac{dv_i(x_i)}{dx_i} = 0$$

whereas expression (5) remains unchanged in both specifications.

In the Downsian specification,  $\frac{dp_i}{dx_i} = 0$  indicates that candidate  $i$  changes his policy position until his probability of winning no longer increases (i.e., he seeks to maximize the probability of winning), disregarding the specific policy he implements. In the Wittman specification, candidates put less weight on winning the election and instead focus entirely on maximizing their expected policy payoff. We next limit our focus in finding our solution, as expressed in Lemma 4.

**Lemma 4:** *If  $x_i \leq \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$  then it is strictly dominated by  $\min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\} + \varepsilon$  where  $\varepsilon > 0$  and  $m$  represents the median voter's ideal policy. If  $x_i \geq \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ , then it is strictly dominated by  $\max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\} - \varepsilon$ .*

Intuitively, Lemma 4 implies that both candidates will position themselves within bounds set by their own ideal policy preferences, the donors' ideal policy preferences, and the median voter's. The candidates must balance the effects of both their own policy positions and the donations that those policy positions yield in order to maximize their expected payoff. Consider, for example, the case where candidate  $i$  positions below the donor on the left,  $\hat{d}_k$ , as depicted Figure 2. If candidate  $i$  were to increase his position, he could not only increase his probability of winning the election by moving towards the median voter, but he could also gain more donations from both donors by moving closer to their own preferred positions,  $\hat{d}_k$  and  $\hat{d}_l$ . Furthermore, since he is also positioned below his ideal policy position,  $\hat{x}_i$ , increasing his policy position increases the utility he receives if he wins the election. Since there are only gains to be made by increasing his position, all positions where  $x_i < \hat{d}_k$  are strictly dominated by the position  $x_i = \hat{d}_k$ .

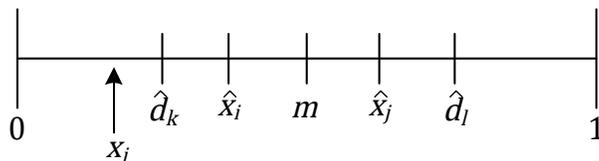


Figure 2. Candidate  $i$  positioning beyond donor  $k$ .

Next, consider the case where candidate  $i$  positions at donor  $k$ 's ideal position as depicted in Figure 3 below. In this case, a marginal increase in candidate  $i$ 's position increases his probability of winning the election by moving towards the median voter as well as the amount of donations he receives from donor  $l$  (through either direct donations or a reduction in donations to candidate  $j$ ). In addition, candidate  $i$  moves closer to his ideal policy position,  $\hat{x}_i$ , increasing the utility he

receives if he is elected. Thus, the position  $x_i = \hat{d}_k$  is also strictly dominated by the position  $x_i = \hat{d}_k + \varepsilon$  where  $\varepsilon > 0$  represents a marginal increase in candidate  $i$ 's position. However, if candidate  $i$  increased his position further, he would experience a tradeoff: on one hand, he gains more voters as he approaches the median voter, and contributions from donor  $l$ , as  $x_i$  approaches  $\hat{d}_l$ ; but, on the other hand, candidate  $i$  would lose donations from donor  $k$ , since candidate  $i$ 's position would now be further from donor  $k$ 's ideal position,  $\hat{d}_k$ . Hence, in equilibrium, candidate  $i$  balances these two effects when choosing  $x_i$ . A symmetric argument applies to candidate  $j$ 's position relative to donor  $l$ .

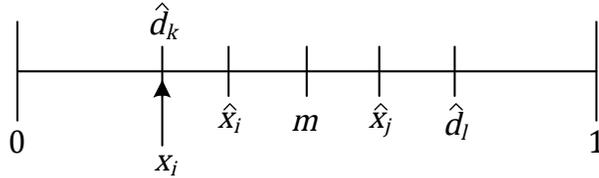


Figure 3. Candidate  $i$  positions at donor  $k$ 's ideal policy position.

We restrict our attention to undominated strategies  $\min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\} < x_i < \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ . Due to the presence of corner solutions outlined in the second stage, an analytical solution to Lemma 3 is not feasible. Furthermore, from work done in Ball (1999b), our model does not satisfy the sufficient assumptions to guarantee that a mixed strategy Nash equilibrium exists, as we cannot guarantee convergence of the fixed point theorems due to donations.<sup>8</sup> Nevertheless, we turn to numerical analysis to determine if an equilibrium exists and where that equilibrium is located.

Several of the below results are driven by every candidate's incentive to reduce his rival's donations, as shown in the following corollary. Let us define symmetric donors as those who, when having the same ideal policy  $x_k$ , experience the same utility from every policy  $x_i$ , i.e.,  $u_k(x_i; \hat{d}_k) = u_l(x_i; \hat{d}_k)$  for every  $x_i$ .

**Corollary 4.** *When donors are symmetric and support separate candidates, and if  $|x_i - \hat{d}_k| < |x_i - \hat{d}_l|$ , then  $\frac{dk_i}{dx_i} > \frac{dl_j}{dx_i}$ .*

In words, this corollary says that when candidate  $i$  positions himself closer to donor  $k$ 's ideal policy than donor  $l$ 's ideal policy, i.e.,  $|x_i - \hat{d}_k| < |x_i - \hat{d}_l|$ , an increase in policy position from candidate  $i$  causes donor  $l$  to reduce his contribution to candidate  $j$  by more than donor  $k$ 's reduction to candidate  $i$ .<sup>9</sup> Both donors reduce their donations to their respective candidates, but

<sup>8</sup>Specifically, our model does not satisfy assumption 4 of the Ball (1999) paper that requires  $p(x_i, x_j) = \Pi(x_i + x_j)$  where  $\Pi(x_i + x_j)$  is some continuous function.

<sup>9</sup>Since candidate  $i$  was relatively close to donor  $k$ 's ideal policy position, and donors' utility is concave by definition, candidate  $i$ 's increase in  $x_i$  causes only a small reduction in donor  $k$ 's expected utility, but gives donor  $l$  a much larger gain in his expected utility.

candidate  $j$ 's reductions decrease by a greater amount. Subsequent sections show that candidates have incentives to lower their rival's donations because of the above result.

## 4 Numerical Analysis

To simulate best response functions for candidate  $i \in \{A, B\}$ , our goal was to make the probability function as general as possible. Starting with the voters, we assume that their ideal policy positions are distributed according to the Beta distribution. This functional form was chosen for its general flexibility and because its support aligns with the range of our voter ideologies. Let  $B(x; \alpha, \beta)$  represent the cumulative distribution function for the Beta Distribution at point  $x$ , given shape parameters  $\alpha$  and  $\beta$ .<sup>10</sup> If  $x_i < x_j$  ( $x_i > x_j$ ), the contribution to the probability that candidate  $i$  wins the election based on their policy position relative to the voters' ideal policy positions is  $B\left(\frac{x_i+x_j}{2}; \alpha, \beta\right) (1 - B\left(\frac{x_i+x_j}{2}; \alpha, \beta\right))$  where  $\frac{x_i+x_j}{2}$  represents the midpoint between the candidate policy positions. Intuitively, point  $\frac{x_i+x_j}{2}$  is where the indifferent voter is located along the policy spectrum. If  $x_i < x_j$  ( $x_i > x_j$ ), all voters below  $\frac{x_i+x_j}{2}$  prefer the policy of candidate  $i$  ( $j$ ), while all those above prefer the policy of candidate  $j$  ( $i$ ).

With regard to donations, the contribution to the probability that candidate  $i$  wins the election from donations is assumed to follow a standard normal distribution. Let  $N(x)$  represent the cumulative distribution function of the standard normal distribution. The contribution to the probability that candidate  $i$  wins the election based on their received donations is  $N(D_i^\eta - D_j^\eta)$  where  $\eta \in (0, 1)$  guarantees that the probability of candidate  $i$  winning the election remains concave in  $D_i$  and convex in  $D_j$ .<sup>11</sup> Intuitively, a candidate that already receives more in donations than their rival will not benefit as much from additional donations.

Combining both of these contributions, the probability that candidate  $i$  wins the election is the linear combination of both contributions, i.e.,

$$p(x_i, x_j, D_i, D_j) = \begin{cases} (1 - \lambda)B\left(\frac{x_i+x_j}{2}; \alpha, \beta\right) + \lambda N(D_i^\eta - D_j^\eta) & \text{if } x_i < x_j \\ (1 - \lambda)\left[1 - B\left(\frac{x_i+x_j}{2}; \alpha, \beta\right)\right] + \lambda N(D_i^\eta - D_j^\eta) & \text{if } x_i > x_j \end{cases} \quad (6)$$

where  $\lambda \in (0, 1)$  denotes the weight of donations received relative to candidate policy position on the probability that candidate  $i$  wins the election.<sup>12</sup>

<sup>10</sup>Utilizing  $\alpha$  and  $\beta$ , we can simulate several different population distributions. When  $\alpha = \beta > 1$ , we obtain a symmetric population of voters whose mean is 0.5 and are more concentrated towards the center of the distribution. Alternatively, when  $\alpha = \beta < 1$ , we obtain a symmetric population of voters whose mean is 0.5 but are more concentrated on the tails of the distribution. With  $\alpha > \beta > 1$ , we obtain a distribution that has a mean above 0.5 and has a negative skew. Lastly, with  $\beta > \alpha > 1$ , we obtain a distribution that has a mean below 0.5 and has a negative skew.

<sup>11</sup>This holds as long as the difference between donations is not significantly larger than  $\eta$ , which holds throughout the simulation process.

<sup>12</sup>This is a significant departure from Ball's (1999a) original model, which did not consider the effectiveness of donations as a linear combination, but rather as a parameter (Ball refers to it as  $\gamma$ ). We choose this method as it guarantees that the probability that candidate  $i$  wins the election falls between 0 and 1, inclusive.

For utility functions, donors face the following utility functions,

$$u_k(x_i; \hat{d}_k) = -(x_i - \hat{d}_k)^2$$

while candidates face similar utility functions,

$$v_i(x_j) = -(x_i - \hat{x}_j)^2$$

These functions were chosen since they allow for a bliss point, where donors and candidates maximize their utility at exactly their most preferred policy position, and lose utility as they move away from that position.

The analysis was performed as follows:

1. We first divided the  $[0, 1]$  interval into 1,001 equally spaced points (e.g., 0, 0.001, 0.002, etc.).
2. We pick one value of  $x_j$  from the above range at a time. For each value of  $x_j$ , we consider all 1,001 values of  $x_i$ , calculating for each policy pair the donations made to each candidate  $i$  and  $j$ , and their corresponding expected payoffs.<sup>13</sup>
3. We identify the value of  $x_i$  that maximizes candidate  $i$ 's expected payoff. This identifies candidate  $i$ 's best response to the chosen value of  $x_j$ .
4. We then repeat the process for all values of  $x_j$ , to characterize candidate  $i$ 's best responses.
5. We then repeat steps (1)-(4) picking all values of  $x_i$  at a time, to identify candidate  $j$ 's best response.
6. We finally find where the two best responses intersect to identify the Nash equilibrium of the donation game.

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<sup>13</sup>For instance, if parameters are  $\gamma = 1$ ,  $\lambda = 0.5$ ,  $w = 0$ ,  $\alpha = \beta = 2$ ,  $\hat{d}_k = 0.2$ ,  $\hat{d}_l = 0.8$ ,  $\hat{x}_i = 0.3$ ,  $\hat{x}_j = 0.7$ ,  $\bar{k} = \bar{l} = 1$ ,  $\eta = 0.5$ , and  $c = 0.03$ ; if we start with  $x_j = 0.1$ , candidate  $i$ 's highest payoff occurs at  $x_i = 0.3$  where his payoff becomes  $-0.0053$ .

Figure 4 summarizes the results of this numerical simulation.

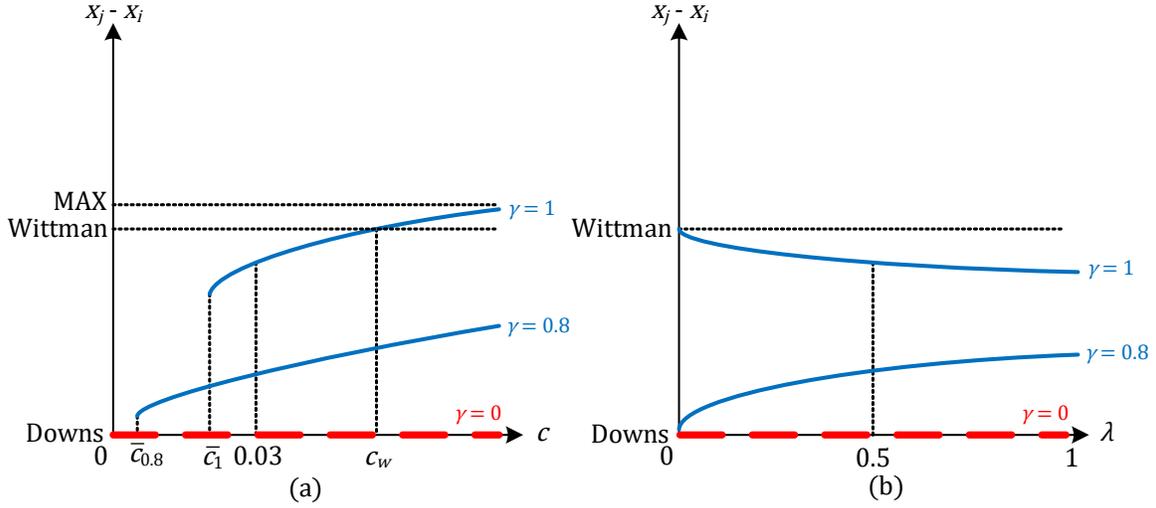


Figure 4. Policy divergence as a function of  $c$  and  $\lambda$  for different values of  $\gamma$ .

Figure 4a depicts policy divergence as a function of  $c$  when  $\lambda$  is held at 0.5. When  $\gamma = 0$ , as in the Downsian (1957) specification, we obtain zero policy divergence and both candidates position at the median voter’s ideal policy. This result holds regardless of the costs of donations,  $c$ , and their effectiveness,  $\lambda$ . Intuitively, since candidates reduce their rivals donations by moving closer to one another, positioning at the median voter maximizes the probability of winning when no donations are made. When  $\gamma = 1$ , as in the Wittman (1983) specification, policy divergence is a function of the marginal cost of donations,  $c$ , as well as their effectiveness,  $\lambda$ , as depicted in figure 4b. Beginning from the right side of figure 4a, when  $c$  approaches infinity, candidates receive no donations, and respond positioning at maximal policy divergence.<sup>14</sup> This result is more divergent than that in the original Wittman model due to the reduced voter sensitivity, as explained in Wittman (1983). As  $c$  decreases, candidates converge in order to deny donations to their rival. For intermediate values of  $c$ ,  $\bar{c} < c < c_w$ , our model produces less policy divergence than in the original Wittman model as candidates continue to position closer to one another. Lastly, for values of  $c < \bar{c}$ , no equilibrium emerges.

Figure 4b depicts policy divergence as a function of  $\lambda$ , where  $c$  is held at 0.03. When  $\gamma$  is sufficiently low ( $\gamma < 0.655$ ), policy convergence arises for low values of  $\lambda$ ; which includes the case of  $\gamma = 0$  where policy convergence occurs for all values of  $\lambda$ , as illustrated in the figure. In contrast, when  $\gamma$  is relatively high, positive policy divergence emerges as depicted in the figure. Graphically, this is represented in the vertical axis of figure 4b, where donations are ineffective ( $\lambda = 0$ ), and where policy divergence increases in  $\gamma$ . When donations become more effective ( $\lambda > 0$ ), this policy

<sup>14</sup>In this situation, candidate  $i$  positions between his own ideal policy,  $\hat{x}_i$  and the location of the median voter, but prefers positioning closer to  $\hat{x}_i$ .

divergence is attenuated, since candidates seek to position closer to their donors.

In Appendix 2, we show how equilibrium candidate policy position shifts as the parameters  $c$  and  $\lambda$  vary. Consistent with the results in Corollary 4, as the marginal cost of donations decreases, candidates position themselves closer to the median voter to deny donations to their opponent.

On the other hand, as the effectiveness of donations increases, we see candidates shift from targetting the median voters' ideal preferences to the donors' ideal preferences. When donors' ideal policies are not symmetric around the median voters, this leads to a scenario where candidates' chosen policies are skewed towards donors relative to those preferred by the voters. Intuitively, if voters have a high value of  $\lambda$ , they are easily influenced by campaign contributions (in the form of advertisements, etc.). In this setting, they can be convinced to vote for a candidate that receives large contributions whose policy is farther away from their own ideal policy rather than a candidate whose policy is closer to their own ideal, but receives fewer contributions. This leads candidates to align themselves with donor preferences in order to maximize his own contributions and minimize those received by his opponent.

## 5 Extensions

In this section, we analyze the robustness of our results to two extensions in our model: asymmetries (either in voter's distribution, candidates' ideal policies, or donors' ideal policies), and the effect of limiting the amount of contributions that donors can make to candidates.

### 5.1 Asymmetries

The previous section assumes a symmetric voter distribution (where  $\alpha = \beta = 2$  in the beta distribution). In addition, it considers that donors' and candidates' ideal policies are located on either side of and equidistant to the median voter. Introducing either form of asymmetry into our model (voter distribution, candidates' ideal policies, and donors' ideal policies), causes significant changes to models with low values of  $\gamma$ , but minimal changes to models with high values of  $\gamma$ .

For models with high values of  $\gamma$  (which contains the Wittman (1983) model where  $\gamma = 1$ ), asymmetries simply change the location of the equilibrium, but do not prevent equilibria from emerging. With an asymmetric voter distribution that results in the median voter having a higher (lower) ideal policy position, candidates respond by increasing (decreasing, respectively) their equilibrium location to be closer to the median voter. Likewise, for asymmetric ideal policy positions among candidates or donors that result in the midpoint of their ideal policies being above (below) the midpoint of the policy spectrum, both candidates respond by increasing (decreasing, respectively) their equilibrium location. Intuitively, candidates have strong incentives to move in the same direction as changes to the ideal policies of the voters, donors, and themselves.

When we allow for an asymmetric voter distribution, we observe situations in settings with low values of  $\gamma$  (including the Downsian (1957) model where  $\gamma = 0$ ) because such distributions give rise to profitable deviations from the previous policy convergence at the median voter. Since the median

voter is no longer located at the midpoint between the donors' ideal policies, deviations from his position induce large contributions from the donor located farther away from the median voter, while the donor located nearer to the median voter contributes little in comparison. When the marginal cost of donations,  $c$ , is low or their effectiveness,  $\lambda$ , is high, it becomes profitable for candidate  $i$  to deviate from the median voter and target the larger of the two donations available. In response to this deviation, candidate  $j$  positions himself  $\varepsilon$  closer to the median voter than candidate  $i$ , which leads to cyclical behavior between the two candidates and no equilibrium emerges. In summary, no stable policy positions arise in equilibrium when contributions are relatively cheap (low  $c$ ) and/or they are effective (high  $\lambda$ , indicating that voters' decisions are highly affected by large campaigns and advertising).

For asymmetries in donors' or candidates' ideal policies, we experience a similar problem in models with low values of  $\gamma$ . In the case of donors' ideal policies, again we find that the ability to obtain a large amount from the donor located farther away from the median voter induces cyclical behavior in the candidates when either  $c$  is low or  $\lambda$  is high. Regarding asymmetries in candidates' ideal policy positions, the effect is more subtle.<sup>15</sup> For candidates with extreme, yet similar ideal policy positions, a profitable deviation from the median voter exists which is closer to the candidates' own ideal policy. These models also produce no equilibrium.

**Extreme cases.** For presentation purposes, we next consider three extreme cases about candidate preferences, donor preferences, or the voter distribution.

*Candidate preferences.* If candidates have identical policy preferences, i.e.,  $\hat{x}_i = \hat{x}_j$ , we find that both candidates converge towards their (common) ideal policy.<sup>16</sup> Intuitively, since candidates are identical, they choose the same policy in equilibrium, leading donors to not contribute to either candidate. Therefore, both candidates maximize their expected utility by positioning at their common ideal policy. This includes the case where both candidates' ideals lie at extreme positions, such as  $\hat{x}_i = \hat{x}_j = 0$ .

*Donor preferences.* When donors have identical policy positions,  $\hat{d}_k = \hat{d}_l$ , we still experience policy divergence among the candidates, where the candidate whose ideal policy is closest to the donors' (common) ideal policy locates closer to the donors and receives all donations, while the other candidate positions closer to the median voter to remain competitive by targetting voter preferences.<sup>17</sup>

*Voter distribution.* For extreme voter distributions, we examine two cases. First, when the voter distribution has a single peak at the very end of the policy spectrum (by setting  $\alpha = 2$  and  $\beta = 0.5$ , or vice versa), both candidates position below (above, in the case of  $\alpha = 0.5$  and  $\beta = 2$ ) the median voter, as locating above (below, respectively) such a position would be strictly dominated,

<sup>15</sup>When  $\gamma = 0$ , we return to policy convergence, as candidate policy preferences do not impact the Downsian equilibrium.

<sup>16</sup>This is the only case of complete policy convergence in the Wittman specification we found.

<sup>17</sup>The candidate receiving donations has a higher probability of winning the election under these circumstances. The candidate who positions closer to the median voter (and receives no donations), however, still has a positive probability of winning the election.

as explained in Lemma 4. Second, we examine a voter distribution with dual peaks, such as one where voters are concentrated at the tails of the beta distribution (by setting  $\alpha = \beta = 0.5$ ). In this context, policy divergence increases as candidates seek to accommodate voters at the ends of the distribution. Intuitively, since so few voters are located at the center of the policy line, candidates do not compete as fiercely for them, instead favoring their own ideal policies and those of the donors.

## 5.2 Public Funding

Several countries provide publicly funded lump-sum contributions to political parties based on votes received in the previous election; as in Canada, Australia, and most European nations (For more details, see table in Appendix 4). This leads to potential donation advantages that are entirely exogenous. If a candidate knows he has an initial advantage over his rival in donations, he may choose to position closer to his own ideal policy position.

We can adjust our model to include public funding by adding two terms into our probability of winning the election function. Let  $F_i$  and  $F_j$  denote the amount of public funding candidates  $i$  and  $j$  receive from the government, which we incorporate into the probability that candidate  $i$  wins the election, equation 6, as follows,  $N(D_i^\eta + F_i^\eta - (D_j^\eta + F_j^\eta))$ .

For compactness, we relegate the numerical results to Appendix 5, and discuss here the main difference with respect to the model without public funding. Specifically, we find that both candidates position closer to the ideal of the candidate with the public funding advantage, where the latter capitalizes on his funding advantage by moving toward his own ideal more than his opponent. This leads to an increase in policy divergence that remains approximately constant in the cost of donations,  $c$ , relative to the scenario with no public funding; thus producing a similar result as in figure 7a. Intuitively, if candidate  $i$  has a public funding advantage over his opponent, he can use the donation advantage to offset a less appealing position to the voters, moving closer to his own ideal. Candidate  $j$  must respond to this by also moving toward candidate  $i$ 's ideal in order to mitigate his own disadvantage by choosing a position more favorable to voters.

Candidates represent better voter preferences when public funding is available only if the candidate with the public funding advantage has an ideal close to the median voter. Otherwise, candidate's policy positions and voter preferences are misaligned, yet public funding provides less incentives for candidates to position closer to the median voter. Since public funding is distributed according to votes in previous elections, this may lead to the candidate who won previous electoral contests having a substantial funding advantage. If his ideal policies changed (e.g., becoming more radicalized), he could position at a more extreme policy, driving his rival to similar positions, and yet win the election again.

## 5.3 Donation Constraints

Of interest to policy makers is the effect of constraints that limit donors' ability to contribute to their respective candidates. A binding donation constraint can significantly alter the equilibrium

of our model. Under certain circumstances, a donation constraint can prevent an equilibrium from emerging at all; as shown in this section. Under specifications with high  $\gamma$  (which includes the Downsian (1957) specification), a binding donation constraint has no effect on the equilibrium behavior of either candidate, since, as shown above, no donations are made in equilibrium to either candidate.

In contrast, in specifications with low  $\gamma$  (which captures the Wittman (1983) specification as a special case), when donors are constrained to a maximum donation, candidate behavior changes relative to the size of those constraints. For simplicity, we categorize the equilibrium behavior when donors are constrained into three cases, with cutoff values of the donation constraint,  $\bar{k}$  at  $k_1$  and  $k_2$ , where  $k_1 < k_2$ .<sup>18</sup>

*Case 1:  $\bar{k} > k_2$ .* For high donation constraints, the constraint does not bind and we obtain the same equilibrium as described in the previous section. Of interest is the fact that  $k_2$  is significantly higher than the unconstrained amount that donors contribute to their respective candidates in the unconstrained equilibrium, which we denote as  $k_U$ . In other words, if  $\bar{k}$  is set to the unconstrained donation level, candidates do not position at the unconstrained equilibrium, as explained in the next case.

*Case 2:  $\bar{k} \in [k_1, k_2]$ .* For intermediate constraints, we find that every candidate  $i$  deviates from the unconstrained equilibrium. Figure 5 below demonstrates the effect of a binding constraint on

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<sup>18</sup>For calculated values of  $k_1$  and  $k_2$ , refer to the selected simulation results table in Appendix 3.

the unconstrained equilibrium.

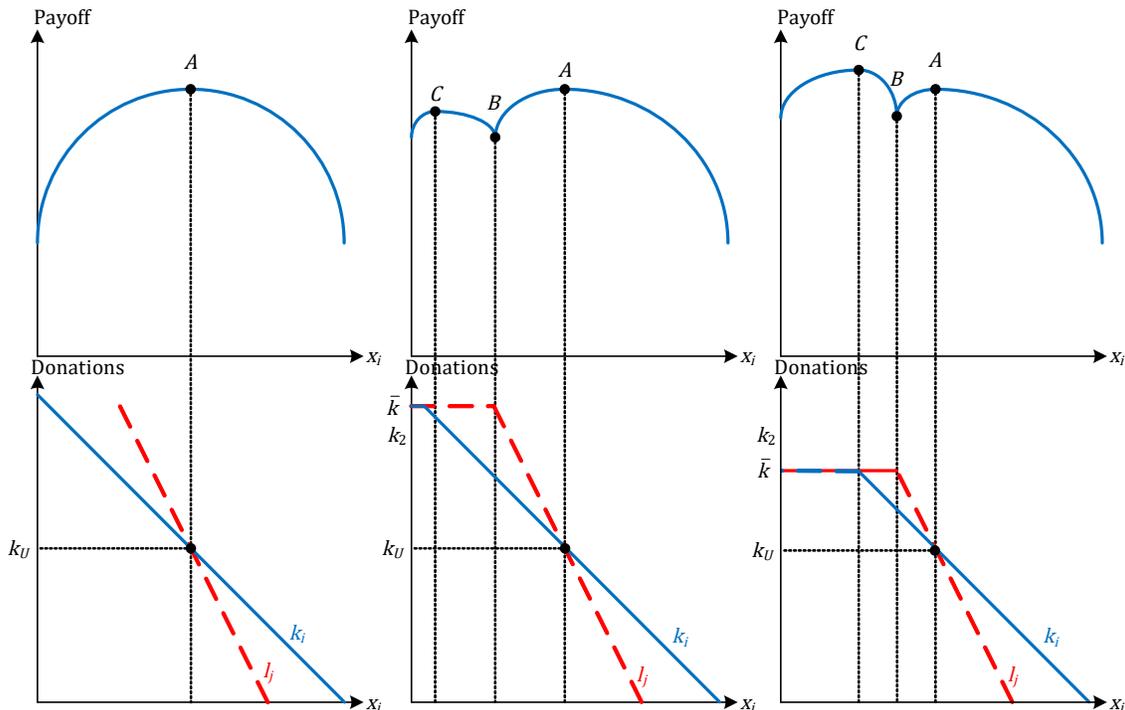


Figure 5. Candidate  $i$ 's best response at different donation constraints.

The left panel of figure 5 depicts candidate  $i$ 's payoff function and both candidates' donation levels when donations are unconstrained. In this case, candidate  $i$  maximizes his expected payoff by also positioning at the unconstrained equilibrium (located at point  $A$ ), and receives the same donation level as candidate  $j$ .

The center panel of figure 5 plots a constraint  $\bar{k} > k_2$ , as described in case 1. From Corollary 4, the donations that candidate  $j$  receives from donor  $l$  are more sensitive to candidate  $i$ 's position than the donations that candidate  $i$  receives from donor  $k$ . Thus, as candidate  $i$  decreases his position from the unconstrained equilibrium, candidate  $j$  receives more donations than candidate  $i$  up until point  $B$ , where donor  $l$  reaches the donation constraint. At this point, candidate  $i$  can continue to decrease his position further without candidate  $j$  receiving any additional donations. This causes candidate  $i$ 's payoff to increase since he is able to move closer to his own ideal position as well as receive more donations relative to candidate  $j$  (since he is constrained). In this situation,  $\bar{k}$  is sufficiently high to allow candidate  $i$  to maximize his expected policy by positioning himself at the unconstrained equilibrium (since point  $A$  yields a higher payoff than point  $C$ ).

The right panel of figure 5 represents the case which  $\bar{k}$  is above the unconstrained donation level  $k_U$ , but below  $k_2$ . In this situation, since  $\bar{k}$  is sufficiently low, candidate  $i$  reaches a higher utility at point  $C$  than he does at point  $A$ , and thus deviates from the unconstrained equilibrium.

The best response functions for both candidate  $i$  and  $j$  are depicted below in figure 6.

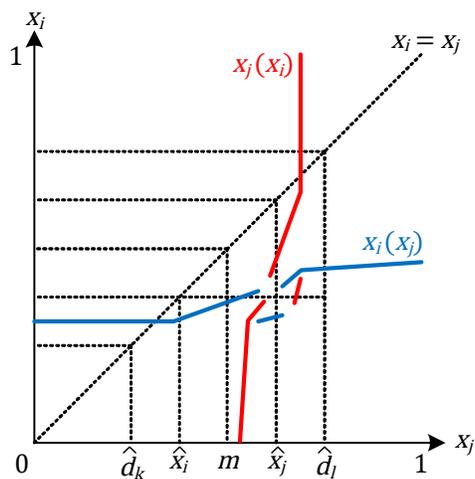


Figure 6. Best response functions with intermediate constraints.

As shown in figure 6, the profitable deviations from the unconstrained equilibria cause significant discontinuities to appear in the best response function, leading to no Nash equilibrium.

*Case 3:  $\bar{k} < k_1$ .* For low donation constraints, donors contribute  $\bar{k}$  to their respective candidates, but the candidates behave as if they receive no donations at all. When every candidate  $i$  is constrained, he cannot reduce the amount of donations his opponent receives by increasing his own policy position (as with low constraints, both candidates remain constrained). Thus, the incentive to position close to one another as described in Corollary 4 does not exist, and candidates position themselves at the location which maximizes their expected policy payoff had they not received any donations.<sup>19</sup>

## 6 Discussion

*Cheaper donations.* Our results suggest that, policies lowering donors' cost of contributions to political campaigns (such as making a larger portion of them tax deductible) induces donors to increase their ability to contribute to either candidate. Anticipating the availability of more donations, we demonstrated that candidates converge in their policy positions, seeking to reduce each other's donations as much as possible; see Figure 4a. However, we also showed that, when donations become extremely cheap, candidates become so concerned about monetary contributions that a cyclical behavior arises, preventing a stable profile of political platforms from emerging. Hence, our findings indicate that societies that make contributions to political campaigns sufficiently cheap may

<sup>19</sup>Which, as described in the numerical analysis section, has more policy divergence than in the original Wittman (1983) model, but the candidates' do not experience the skewness towards donors' ideal policies due to  $\lambda > 0$ .

not experience more policy convergence among candidates, but instead unstable political platforms (e.g., candidates who change their position after his rival alters his own, without ever reaching an equilibrium).

*More effective donations.* When voters become more influenced by large political campaigns featuring TV ads (higher  $\lambda$ ), our results suggest that candidates shift their policy position, from targeting the median voter to targeting the midpoint of the donors. Essentially, money distorts the incentives in the Wittman’s model, where candidates care about voter’s preferences and their own ideal policy position, since now candidates must also care about the donors’ ideal policies. In addition, we show that as donations become more effective at winning votes, both candidates position themselves closer to the midpoint of donors’ ideal policies, yielding more policy convergence. Therefore, electorates highly influenceable by campaigns yield more policy convergence than otherwise. This holds, however, when candidates assign a sufficiently high weight to the policy that wins the election. Otherwise, policy convergence emerges around the median voter for all values of  $\lambda$ .<sup>20</sup>

*Cynical candidates.* Following Wittman’s results, our paper confirms that, as candidates assign a larger weight to the utility they obtain from the policy that wins the election (higher  $\gamma$ ), political platforms become more divergent, as candidates put less emphasis on maximizing their probability of winning the election and, as a result, seek to position themselves closer to their ideal policies. This occurs even in the absence of political contributions. When donations are present, this effect is attenuated, as candidates balance their own ideal policies with those of the donors.

*Incumbent advantage.* For public campaign funding systems that allocate based on the results of the previous election, an incumbent candidate starts with a funding advantage, leveraging it to position closer to his own ideal. If previous elections that are closely contested, this effect is minimal. In the case of a previous landslide electoral victory, however, the public funding advantage granted to the incumbent leads to a significant skewing of both candidate positions closer to the incumbent’s ideal.

*Limiting campaign budgets.* Last, our findings help shed light on the effect of setting limits on the amount of money donors can contribute to political campaigns. We show that these constraints yield different results, depending on the severity of the constraint. When campaign contributions are substantially constrained, our findings entail more policy divergence between candidates. However, the midpoint of their policies is now closer to the median voter, so policies can be interpreted as becoming more aligned with voter preferences. In contrast, when constraints are relatively lax, our results indicate that candidates’ positions become cyclical, with no stable policy pair emerging in equilibrium. Therefore, if the social planner seeks to minimize policy divergence among candidates, donors must remain unconstrained. On the contrary, if the social planner’s objective is to have candidates’ policies in line with those of the voter distribution, a low donation constraint is optimal. In summary, aligning candidate policies with those of the median voter comes at a cost of increased

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<sup>20</sup>Of note, there are intermediate values of  $\gamma$  that yield policy convergence only for low values of  $\lambda$ . For example, when  $\gamma = 0.5$  with our standard parameters, policy convergence occurs for  $\lambda < 0.74$ , but policies diverge slightly for values above this.

policy divergence.

*Further Research.* Our work leads to several venues for future research. Many democratic countries elect their government from more than two candidates, and adding additional candidates to the model would allow for more robust results. In addition, we consider a one dimensional policy spectrum, where candidates typically chose among many dimensions (different policies) when forming their campaign policy. Lastly, we consider the effectiveness of campaign contributions,  $\lambda$ , to be uniform among all voters and candidates. For asymmetric values of  $\lambda$ , we could find situations where one candidate favors donations much more than the other.

## 7 Appendix

### 7.1 Appendix 1 - Further details on the numerical simulation

**Downsian (1957) specification** ( $\gamma = 0$ ). Under the Downsian (1957) specification, candidates seek to solely maximize their probability of winning the election. By setting both  $\gamma = 0$  and  $\lambda = 0$ , we obtain the original model and result proposed by Downs in that each candidate maximizes his probability of winning the election by positioning himself at exactly the median voter. For values of  $\lambda > 0$ , campaign contributions also determine the probability of winning the election, and our results are presented in figure A1 below.

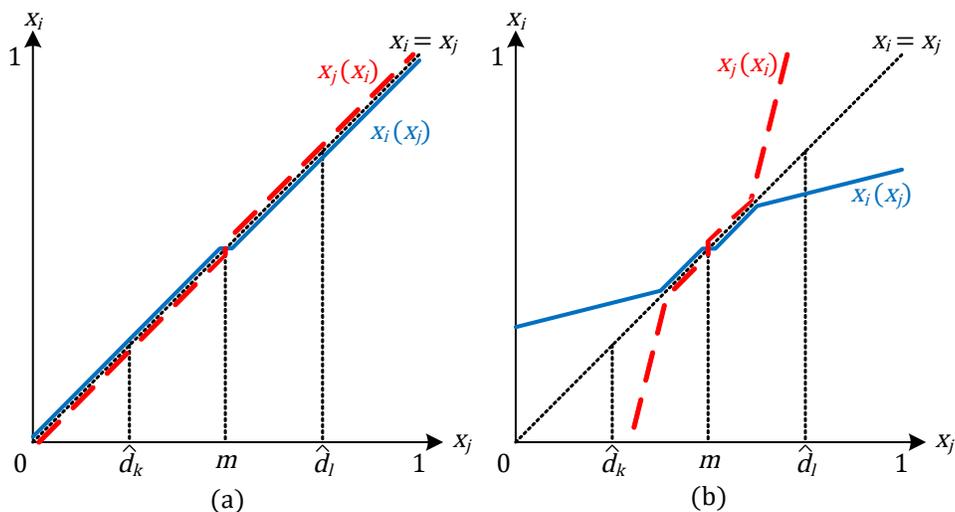


Figure A1. Downsian specification with  $\lambda = 0$  and  $\lambda > 0$ , respectively.

Figure A1(a) plots the results in the original Downsian (1957) model since  $\lambda = 0$ . The best response for either candidate  $i$  is to position himself  $\varepsilon > 0$  closer towards the median voter relative to his opponent. This behavior continues until both candidates converge at the median voter and have even odds of winning the election. In figure A1(b), we have the Downsian specification of our

model where  $\lambda > 0$ . Similar to the Downsian model, our model has every consumer  $i$  positions himself closer to the median voter's ideal position than his opponent, where the best response is to position  $\varepsilon > 0$  closer to the median voter for positions near the median voter. As candidate  $i$ 's opponent deviates significantly from the median voter's ideal policy position, however, candidate  $i$ 's best response is now to increase the distance between his own position and that of his opponent (flatter best response function when  $x_j$  is either high or low). Intuitively, at the more extreme points of the best response function, the ability to increase his probability of winning by targetting voter preferences diminishes quickly due to the concavity of the probability function.<sup>21</sup>

The equilibrium results of the original Downsian model and the Downsian specification of our model remain the same. Thus, the only Nash equilibrium we find is  $x_i^* = x_j^* = m$ , the location of the median voter, and  $k_i^* = k_j^* = l_i^* = l_j^* = 0$ , as no donor has any incentive to donate to either candidate. Intuitively, in the Downsian specification of our model, as candidates approach the median voter donations to each candidate effectively disappear. Every candidate has incentive to position at the median voter to maximize his vote share, as well as incentive to deny campaign contributions to his opponent, as described in corollary 4. Furthermore, lowering  $c$ , the marginal cost of donations only exacerbates this effect.

**Wittman (1983) specification** ( $\gamma = 1$ ). In the Wittman (1983) specification, instead of maximizing the probability of winning the election, every candidate  $i$  maximizes his expected policy outcome. As a result, candidates position themselves closer to their ideal policy position rather than the median voter's ideal (as in the Downsian (1957) model). When we set  $\gamma = 1$  and  $\lambda = 0$ , we can obtain both the original model and results as presented by Wittman. Allowing for donations to affect the probability of winning the election ( $\lambda > 0$ ), we obtain the results in figure A2.

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<sup>21</sup>At these parts of the best response function, candidate  $i$  obtains a higher probability of winning the election by distancing himself from his opponent and enabling donors to contribute to his own campaign (as donors will contribute approximately zero when candidates position next to one another).

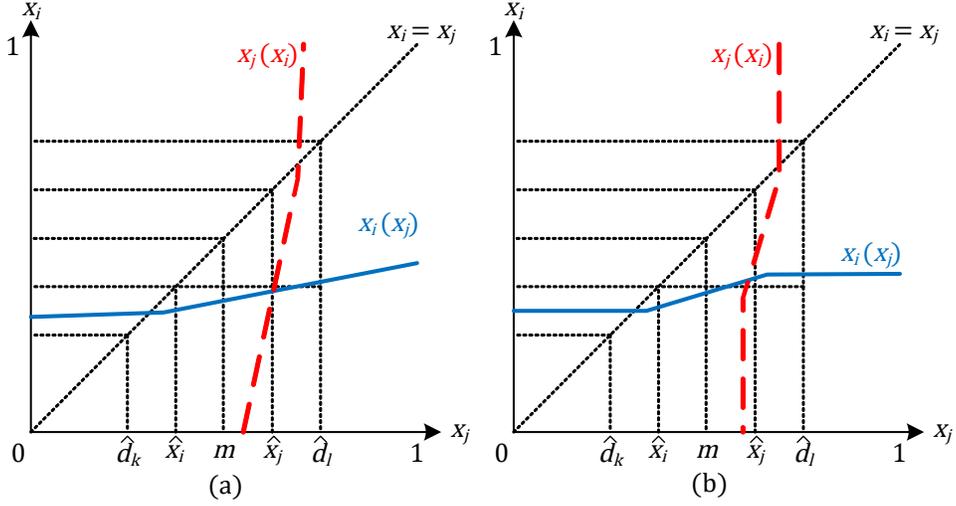


Figure A2. Wittman specification with  $\lambda = 0$  and  $\lambda > 0$ , respectively.

In figure A2(a), we have the original Wittman (1983) model where  $\lambda = 0$ . Every candidate  $i$  positions himself close to his ideal policy position, i.e., the best response function  $x_i(x_j)$  lies close to  $\hat{x}_i$ . As candidate  $j$  positions himself closer to the median voter, candidate  $i$  responds by increasing his own policy position, but at a much slower rate than seen in the Downsian (1957) specification (flatter best response function). This leads to an equilibrium where candidate policy positions diverge from the median voter and from their own ideal policies. Figure A2(b) contains the Wittman specification of our model where  $\lambda > 0$ . The key difference between the two models happens when  $x_j$  is relatively high. In this case, the best response of candidate  $i$  is to remain even closer to his own ideal policy position since candidate  $j$  positions himself significantly above the median voter; see flat segment in the right-hand side of  $x_i(x_j)$ . The intuition is similar to that of the Downsian specification, as candidate  $i$  receives a larger benefit from campaign contributions rather than targeting voter preferences when his opponent positions himself at extreme locations.

The location of our equilibrium under the Wittman (1983) specification with donations can vary relative to the equilibrium of Wittman's model, itself. Intuitively, by introducing donations we both lower voter sensitivity, and introduce bias for whichever candidate receives more donations, as described in Wittman's (1983) paper. We can reproduce Wittman's results by setting an inverse relationship between Wittman's voter sensitivity parameter  $s$  and our donation effectiveness parameter,  $\lambda$ ; and a proportional relationship between Wittman's bias parameter  $B$  and a combination of our  $\lambda$  and  $N(D_i^\eta - D_j^\eta)$  terms.<sup>22</sup> As  $\lambda$  increases, voter sensitivity decreases, which causes candidates to move closer to their own ideal policy positions; and the bias increases in magnitude,

<sup>22</sup>Term  $N(D_i^\eta - D_j^\eta)$  is our normally distributed contribution to the probability that candidate  $i$  wins the election based on their received donations.

which causes candidates to move towards whichever candidate the bias favors.<sup>23</sup>

As the marginal cost of donations,  $c$ , decreases, donors contribute more to their preferred candidates. This influx of donations causes both candidates to position closer to each other, since each candidate is able to deny donations to his opponent; as explained in corollary 4. Due to the concavity of the donors' utility functions, having a candidate that is positioned farther away from the donor move closer entails a much larger increase in utility than having a candidate that is close to the donor move away. Thus, candidate  $i$  can significantly reduce the donations that candidate  $j$  receives by moving slightly closer to candidate  $j$ 's policy position while only experiencing a slight decrease in the amount of donations that he receives.<sup>24</sup>

In summary, when the marginal cost of donations,  $c$ , is sufficiently high  $c > c_w$  (low  $c < c_w$ ), our results predict less (more) convergence than in Wittman's model.<sup>25</sup> When  $\lambda = 0$  (as in the original Wittman model), candidates position between their own ideal policy positions and the ideal policy position of the median voter. As  $\lambda$  increases, candidates put less weight on the location of the median voter's ideal policy and more weight on the source of the bias, the donors' ideal policy position. At the extreme case of  $\lambda = 1$ , candidates disregard the median voter's ideal policy position entirely when they receive donations, and position themselves between their own ideal position and the midpoint of the donors' ideal policies,  $\frac{\hat{d}_k + \hat{d}_l}{2}$ .

Other effects are similar to those described in Wittman (1983). If the ideal policy position of either candidate increases (decreases), both candidates increase (decrease) their equilibrium policy position.<sup>26</sup> Intuitively, if candidate  $j$ 's ideal policy position increases, he positions closer to it, which induces candidate  $i$  to increase his own policy position to receive more votes and to deny candidate  $j$  additional donations. Likewise, if either donor increases (decreases) his most preferred policy position, both candidates increase (decrease) their equilibrium policy position.<sup>27</sup>

**Mixed specification** ( $0 < \gamma < 1$ ). Using a mixed specification, we obtain results that fall between the Downsian (1957) and Wittman (1983) specifications. When  $\gamma$  is low ( $\gamma < 0.655$  in our example), we find that policy convergence at the median voter occurs. For values of  $\gamma$  above this threshold, policy positions diverge until they reach those at the Wittman (1983) specification when  $\gamma = 1$ . Interestingly, the best response functions for both candidates show properties of both the Downsian and Wittman models as shown in figure A3. Their behavior depends on each candidate's

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<sup>23</sup>As described in propositions 3B and 4B in Wittman's (1983) paper. The net effect of an increase in  $\lambda$  is ambiguous, as it strongly depends on the symmetry of ideal policy positions, but in general, as  $\lambda$  increases, candidates have stronger incentives to deny donations to their opponent by moving closer to one another; as described in Corollary 4.

<sup>24</sup>We also find that for every cost  $c < \bar{c}$ , no Nash equilibrium exists. Intuitively, as donations become extremely cheap, candidate behavior becomes erratic. Candidates receive large donations for even small deviations from their current positions, and constantly vie for the most donations from their respective donors. This causes no equilibrium to emerge. As a note, for large donations, the concavity property of our normal distribution also breaks down, which could be driving this result.

<sup>25</sup>In our numerical analysis,  $\gamma = 1$ ,  $\lambda = 0.5$ ,  $w = 0$ ,  $\alpha = \beta = 2$ ,  $\hat{d}_k = 0.2$ ,  $\hat{d}_l = 0.8$ ,  $\hat{x}_i = 0.3$ ,  $\hat{x}_j = 0.7$ ,  $\eta = 0.5$ , we obtain that for the value of  $c_w = 0.053$ , the Wittman model and the Wittman specification of our model yield the same equilibrium policy positions for both candidates.

<sup>26</sup>This includes extreme cases where candidates' ideal policies are at the endpoints of the policy line,  $\hat{x}_i = 0$  and  $\hat{x}_j = 1$ .

<sup>27</sup>Once again, this also holds for extreme ideal policies,  $\hat{x}_k = 0$  and  $\hat{x}_l = 1$ .

location relative to their ideal position.

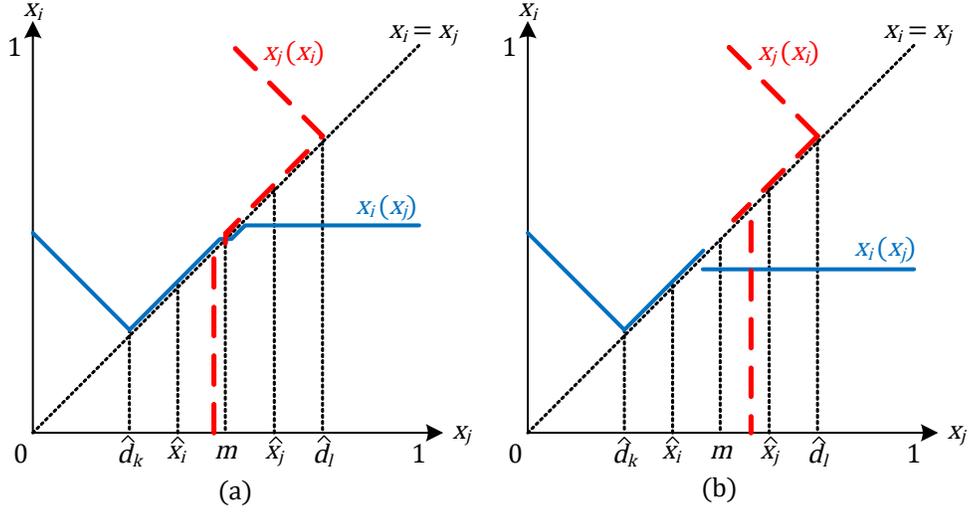


Figure A3. Mixed specification with  $\hat{x}_i < m < \hat{x}_j$  for low and high values of  $\gamma$ .

Without loss of generality, when candidate  $i$  prefers a lower policy position than the median voter and candidate  $j$  prefers a higher policy position than the median voter, i.e.,  $\hat{x}_i < m < \hat{x}_j$ , for low values of  $x_j$ , candidate  $i$ 's best response is to target the voters consistent with the Downsian model as seen in figure A3(a), and he positions himself  $\varepsilon > 0$  closer to the median voter than candidate  $j$ . This occurs up until a policy point above the median voter, where candidate  $i$  no longer increases his position in response to an increase in candidate  $j$ 's position. At this point, candidate  $i$ 's expected policy payoff dominates his preference to maximize his probability of winning the election, and he behaves more in line with the Wittman model. An analogous argument applies for candidate  $j$ 's best response to candidate  $i$ 's position. In this case, both candidates position themselves at exactly the median voter in equilibrium, and policy convergence occurs. In equilibrium, neither donor contributes to either candidate's campaign, and the election is decided by a coin flip.

In figure A3(b), we have the case where  $\gamma$  is large enough to induce policy diversion. The major difference in this case is that both candidates shift from maximizing their probability of winning the election to maximizing their expected policy payoff at a position below (above for candidate  $j$ ) the median voter. This leads to behavior more in line with the Wittman specification rather than the Downsian, where a single Nash equilibrium in pure strategies exists where candidates select different policy positions, ones that are closer to their most ideal position.

## 7.2 Appendix 2 - Comparing candidate positions against donor's ideals

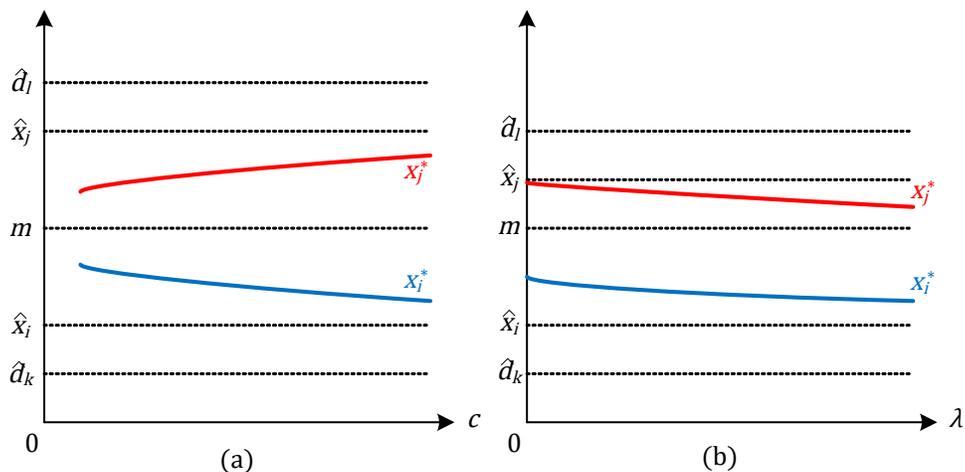


Figure A4. Equilibrium candidate positions as a function of  $c$  and  $\lambda$  when  $\gamma = 1$ .

Figure A4(a) plots candidate equilibrium position as a function of the marginal cost of donations,  $c$ . Starting from the right side of the figure, when  $c$  is large, every candidate receives fewer donations and has little incentive to position closer to his opponent in order to deny him of those donations. As a result, policy divergence is higher with large values of  $c$ . As  $c$  decreases, each candidate positions closer to his rival, as he has strong incentives to deny his rival of the additional donations that are available due to the reduced marginal cost.

Figure A4(b) depicts candidate equilibrium position as a function of the effectiveness of donations,  $\lambda$ . In this figure, candidate  $i$  has an ideal policy position closer to the midpoint between the donors' ideal policy position (donors are slightly asymmetric towards candidate  $i$ ), and thus, candidate  $i$  responds quickly to an increase in  $\lambda$  by moving closer to his own ideal policy position. Candidate  $j$  follows at a slower rate, increasing policy divergence for low values of  $\lambda$ . As  $\lambda$  increases further, candidate  $i$  responds less to further increases (his line becomes flatter), and candidate  $j$  is able to deny more donations to his rival. For high values of  $\lambda$ , we observe decreased policy divergence.

### 7.3 Appendix 3 - Candidate positions with donation constraints

For parameter values of  $\hat{d}_k = 0.2$ ,  $\hat{d}_l = 0.8$ ,  $\hat{x}_i = 0.3$ ,  $\hat{x}_j = 0.7$ ,  $\eta = 0.5$ ,  $\lambda = 0.5$ , and  $a = \beta = 2$ , the following results were obtained:

$\bar{k} = \bar{l}$	Equilibrium Values $(x_i^*, x_j^*)$		
	$c = 0.01$	$c = 0.02$	$c = 0.03$
0.8	(0.346, 0.654)	(0.346, 0.654)	(0.346, 0.654)
0.9	(0.346, 0.654)	(0.346, 0.654)	—
1.2	(0.346, 0.654)	(0.346, 0.654)	—
1.3	(0.346, 0.654)	(0.346, 0.654)	(0.387, 0.613)
1.5	(0.346, 0.654)	(0.346, 0.654)	(0.387, 0.613)
1.6	(0.346, 0.654)	—	(0.387, 0.613)
2.1	(0.346, 0.654)	—	(0.387, 0.613)
2.2	—	—	(0.387, 0.613)
2.5	—	—	(0.387, 0.613)
2.6	—	(0.398, 0.602)	(0.387, 0.613)
4.1	—	(0.398, 0.602)	(0.387, 0.613)
4.2	(0.422, 0.578)	(0.398, 0.602)	(0.387, 0.613)
10	(0.422, 0.578)	(0.398, 0.602)	(0.387, 0.613)

For  $c = 0.01$ , the low marginal cost of donation allows donors to contribute large donations to their respective candidates. When donation constraints are set low,  $\bar{k} < k_1 = 2.1$  in this case, candidates behave as if no donations are received and position themselves at  $(x_i^*, x_j^*) = (0.346, 0.654)$ . For values of  $k_1 < \bar{k} < k_2 = 4.2$ , no equilibrium exists, as candidates leverage constraints on their opponents to position closer to their own ideal policy positions. Lastly, when  $\bar{k} > k_2$ , candidates act as if they were unconstrained and position at  $(x_i^*, x_j^*) = (0.422, 0.578)$ .

As we increase  $c$  to 0.02 or 0.03, we find that the values of  $k_1$  and  $k_2$  decrease. For example, when  $c = 0.02$ , the higher marginal cost of donations causes donors to reduce their contribution levels, and thus the breakpoints for each scenario must also decrease. We find that  $k_1 = 1.5$  and  $k_2 = 2.6$  when  $c = 0.02$ ; and  $k_1 = 0.9$  and  $k_2 = 1.3$  when  $c = 0.03$ . Of note, the fully constrained equilibrium does not change when  $\bar{k} < k_1$  regardless of the value of  $c$  since candidates behave as if they receive no contributions, but as  $c$  decreases, the unconstrained equilibria show reduced policy divergence.

## 7.4 Appendix 4 - Campaign rules among developed democracies

### Americas

Country	Limit on Contributions?	Limit on Spending?	Limit on Indiv. Contributions
Canada	✓	✓	\$1,200
Trin. & Tob.	✓	✓	\$700
USA			-
Uruguay	✓		\$10,300

### European Union / Middle East

Country	Limit on Contributions?	Limit on Spending?	Limit on Indiv. Contributions
Austria		✓	-
Belgium	✓	✓	\$600
Czech R.		✓	-
Denmark			-
Estonia			-
Finland	✓		Varies
France	✓	✓	\$5,400
Germany			-
Greece	✓	✓	\$5,900
Hungary		✓	-
Ireland	✓	✓	\$1,200
Israel	✓	✓	\$115,000
Italy		✓	-
Latvia	✓	✓	\$0
Lith.	✓	✓	\$9,900
Nether.			-
Norway			-
Poland	✓	✓	\$8,400
Portugal	✓	✓	\$0
Slovakia		✓	-
Slovenia	✓	✓	\$18,800
Spain			-
Sweden			-
Switz.			-
UK		✓	-

**Asia / Oceania**

Country	Limit on Contributions?	Limit on Spending?	Limit on Indiv. Contributions
Australia			-
Japan	✓	✓	\$13,300
N. Zealand		✓	-
S. Korea	✓	✓	\$17,600
Taiwan	✓	✓	\$700

**7.5 Appendix 5 - Public Funding Numerical Results**

The presence of public funding in our model behaves qualitatively similar to adding an asymmetry. For example, when  $\gamma$  is low (as in the Downsian Specification), equal public funding levels among candidates retains our equilibrium at the median voter. For even small public funding donation advantages, however, we arrive at situations where no equilibrium in pure strategies exists, much like the cases described in the asymmetry section.

In contrast, when  $\gamma$  is high (as in the Wittman Specification), a public funding advantage does not prevent the emergence of an equilibrium in pure strategies. Under these conditions, both candidates again behave as if an asymmetry were present, positioning closer to the candidate with the public funding advantage, the results of which are shown below in figure A2.

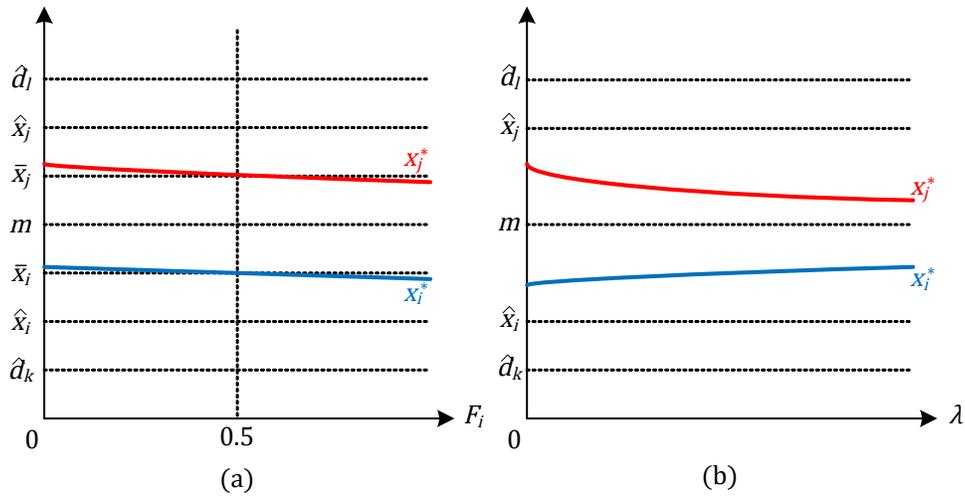


Figure A5. Equilibrium candidate positions as a function of  $F_i$  and  $\lambda$  when  $\gamma = 1$ .

Figure A5(a) depicts candidate equilibrium positions as a function of  $F_i$  while  $F_j$  is held constant at 0.5 and  $\lambda$  is held constant at 0.5. For comparison purposes, we denote point  $\bar{x}_i$  as candidate  $i$ 's equilibrium policy position without public funding. As seen in the figure, for values of  $F_i < 0.5$ , equilibrium positions are skewed upward, towards candidate  $j$ 's ideal. As  $F_i$  increases, however, the

skewness at first disappears at  $F_i = F_j = 0.5$ , and then becomes skewed downward as  $F_i$  increases further above 0.5. Figure A5(b) depicts candidate equilibrium positions as a function of  $\lambda$  with  $F_1 = 0.8$  and  $F_2 = 0.5$ . In this situation, we again observe the increased policy convergence as  $\lambda$  increases, as seen in figure 7b (as an increase in  $\lambda$  increases the effect of private, as well as public donations). However, we do observe an asymmetry in favor of candidate  $i$ , due to their advantage in public funding. As  $\lambda$  increases, candidates shift their priorities from the voters to the donors, but candidate  $i$ 's public donation advantage also shifts both candidates more towards candidate  $i$ 's ideal policy.

## 7.6 Proof of Lemma 2

Assume that donor  $k$  contributes to both candidates  $A$  and  $B$ , i.e.,  $k_i, k_j > 0$  and thus both equations (2) and (3) bind with equality, i.e.,

$$\frac{dp}{dD_i} [u_k(x_i) - u_k(x_j)] = 1 \quad (A1)$$

$$\frac{dp}{dD_j} [u_k(x_i) - u_k(x_j)] = 1 \quad (A2)$$

Setting equations (A1) and (A2) equal to one another and simplifying yields

$$\frac{dp}{dD_i} = \frac{dp}{dD_j} \quad (A3)$$

which cannot hold, since the left side of equation (A3) is positive, while the right side is negative. Therefore, donor  $k$ 's contribution to at least one of the candidates must equal zero. ■

## 7.7 Proof of Corollary 1

First, we show that  $\frac{dk_i^*}{dl_i} = -1$ . Differentiating equation (2) with respect to  $l_i$  yields

$$\overbrace{\frac{d^2p}{dD_i^2}}^{(-)} \left( \frac{dk_i^*}{dl_i} + 1 \right) \overbrace{[u_k(x_i) - u_k(x_j)]}^{(+)} = 0$$

where the only value that can satisfy the above equation is  $\frac{dk_i^*}{dl_i} = -1$ .

Next, we show that  $\frac{dk_i}{dl_j} > 0$ . Using equation (2) with respect to  $k_i^*$  and  $l_j^*$ , we have

$$\frac{dp}{dD_i} [u_k(x_i) - u_k(x_j)] = 1$$

$$\frac{dp}{dD_j} [u_l(x_i) - u_l(x_j)] = 1$$

Setting these two equations equal to one another and rearranging terms yields

$$\frac{dp}{dD_i} [u_k(x_i) - u_k(x_j)] - \frac{dp}{dD_j} [u_l(x_i) - u_l(x_j)] = 0$$

Using the implicit function theorem,

$$\frac{dk_i}{dl_j} = \frac{\overbrace{\frac{d^2p}{dD_j^2} [u_l(x_i) - u_l(x_j)]}^{(-)}}{\underbrace{\frac{d^2p}{dD_i^2} [u_k(x_i) - u_k(x_j)]}_{(-)}} > 0$$

where the signs of  $u_k(x_i) - u_k(x_j)$  and  $u_l(x_i) - u_l(x_j)$  are by definition and the signs of  $\frac{d^2p}{dD_i^2}$  and  $\frac{d^2p}{dD_j^2}$  are due to the concavity and convexity, respectively of  $D_i$  and  $D_j$  on  $p$ . ■

## 7.8 Proof of Corollary 2

Differentiating equation (2) with respect to  $x_i$  and  $x_j$  yields

$$\underbrace{\frac{d^2p}{dD_i^2}}_{(-)} \frac{dk_i^*}{dx_i} \overbrace{[u_k(x_i) - u_k(x_j)]}^{(+)} + \underbrace{\frac{dp}{dD_i}}_{(+)} \frac{du_k(x_i)}{dx_i} = 0 \quad (A4)$$

$$\underbrace{\frac{d^2p}{dD_i^2}}_{(-)} \frac{dk_i^*}{dx_j} \underbrace{[u_k(x_i) - u_k(x_j)]}_{(-)} - \underbrace{\frac{dp}{dD_i}}_{(+)} \frac{du_k(x_j)}{dx_j} = 0 \quad (A5)$$

Since the probability that candidate  $i$  wins the election is increasing and concave, we know that  $\frac{dp}{dD_i} > 0$  and  $\frac{d^2p}{dD_i^2} < 0$ . Likewise, if donor  $k$  is contributing to candidate  $i$ ,  $u_k(x_i) - u_k(x_j) > 0$ . The sign of  $\frac{du_k(x_i)}{dx_i}$  ( $\frac{du_k(x_i)}{dx_j}$ ) can be determined by candidate  $i$ 's ( $j$ 's) position relative to donor  $k$ 's ideal position. If  $x_i < x_k$  ( $x_j < x_k$ ), an increase in candidate  $i$ 's ( $j$ 's) policy position will entail an increase in the utility that donor  $k$  receives from that position, and thus  $\frac{du_k(x_i)}{dx_i} > 0$  ( $\frac{du_k(x_i)}{dx_j} > 0$ ). On the contrary, if  $x_i > x_k$  ( $x_j > x_k$ ), an increase in candidate  $i$ 's ( $j$ 's) policy position will entail a decrease in the utility that donor  $k$  receives from that position, and thus  $\frac{du_k(x_i)}{dx_i} < 0$  ( $\frac{du_k(x_i)}{dx_j} < 0$ ). This leaves  $\frac{dk_i^*}{dx_i}$  ( $\frac{dk_i^*}{dx_j}$ ) as the only unknown sign in the above equation. In order for the equation to hold with equality, it is necessary that  $\frac{dk_i^*}{dx_i}$  ( $\frac{dk_i^*}{dx_j}$ ) have the same (opposite) sign as  $\frac{du_k(x_i)}{dx_i}$  ( $\frac{du_k(x_i)}{dx_j}$ ), i.e.,  $\frac{dk_i^*}{dx_i} > 0$  ( $\frac{dk_i^*}{dx_j} < 0$ ) if  $x_i < x_k$  ( $x_j < x_k$ ) and  $\frac{dk_i^*}{dx_i} < 0$  ( $\frac{dk_i^*}{dx_j} > 0$ ) if  $x_i > x_k$  ( $x_j > x_k$ , respectively). ■

## 7.9 Proof of Lemma 4

Equilibrium location pairs  $(x_i^*, x_j^*)$  must satisfy equation (4) for both candidates, which will depend on the signs of each term in those equations. Unambiguously, we know that  $\frac{dp_i}{dD_i} > 0$ , since by assumption, if candidate  $i$  receives more donations from either candidate, their subjective probability of winning the election will increase. Likewise, we have  $\frac{dp_i}{dD_j} < 0$ , as an increase in the amount of donations received by candidate  $j \neq i$  causes candidate  $i$ 's subjective probability to decrease. As shown in Corollary 1, when the subjective probability that candidate  $i$  wins the election is concave and  $x_i < \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ , we have that  $\frac{dk_i^*}{dx_i} > 0$ ,  $\frac{dk_i^*}{dx_j} < 0$ ,  $\frac{dl_i^*}{dx_i} > 0$ , and  $\frac{dl_i^*}{dx_j} < 0$ . In addition, since  $x_i < \hat{x}_i$ , an increase in candidate  $i$ 's position increases the utility he receives if he wins the election, thus  $\frac{dv_i(x_i)}{dx_i} > 0$ . Due to these relationships, the left-hand side of equation (4) is positive, and thus, it cannot hold with equality. Thus, any  $x_i < \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$  cannot be a solution to stage 1. A similar approach when  $x_i > \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$  shows that the left-hand side of equation (3) is unambiguously negative and also cannot solve equation (4).

When  $x_i = \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$  ( $x_i = \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ ), a similar situation occurs. All signs described in the previous paragraph are identical except for the sign that corresponds with  $\min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$  ( $\max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ ). This term is equal to zero, as candidate  $i$  is either receiving the most possible subjective probability contribution by positioning himself at the median voter, the most possible utility by positioning at his own ideal policy, or the most possible donations from the donor with the lower (higher) ideal policy position by positioning at his most preferred policy position. The outcome is the same where the left side of equation (4) is unambiguously positive (negative), and cannot be satisfied when  $x_i = \min\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$  ( $x_i = \max\{\hat{x}_i, \hat{x}_j, \hat{d}_k, \hat{d}_l, m\}$ , respectively). ■

## 7.10 Proof of Corollary 4

We can show the effect of an increase of candidate  $i$ 's policy position,  $x_i$ , graphically below in figure A3,

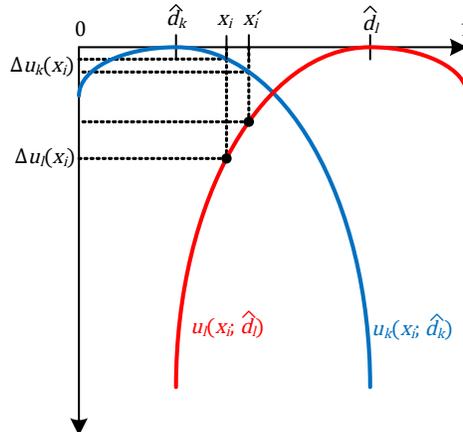


Figure A3. Donor utility functions.

As seen in the above figure, when candidate  $i$  moves from position  $x_i$  to position  $x'_i$  the decrease in utility to donor  $k$ ,  $\Delta u_k(x_i)$  is much less than the gain in utility to donor  $l$ ,  $\Delta u_l(x_i)$ . Holding all other values of equation (2) constant, the resulting increase in candidate  $i$ 's policy position requires both equilibrium donation levels to decrease, but that for donor  $l$  to decrease by a larger amount. ■

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