EconS 301

Homework #8 – Due date: November 29th 2016

Exercise #1 – Monopoly

Consider a monopolist facing inverse demand function $p(q) = 15 - 3q$, where q denotes units of output. Assume that the total cost of this firm is $TC(q) = 5 + 4q$.

a) Find the monopolist's marginal revenue, and its marginal cost.
b) Set marginal revenue equal to marginal cost, to find the optimal output for this monopolist q^M.
c) Find the monopoly price, and profits. Find the consumer surplus, and total welfare.
d) Assume now that the market operated under perfect competition. Find the equilibrium output.
e) Find equilibrium price, profits, consumer surplus, and total welfare, under the perfectly competitive market you analyzed in part (e).
f) Compare your results under a monopoly (part c) and those under a perfectly competitive market (part e).

Exercise #2 – Monopoly with constant elasticity demand curve

Consider a monopolist facing a constant elasticity demand curve $q(p) = 12p^{-3}$.

a) Assume that the total cost function is $TC(q) = 5 + 4q$. Use the inverse elasticity pricing rule (IEPR) to obtain the profit-maximizing price that this monopolist should charge.
b) How would your result in part (a) change if the demand curve changes to $q(p) = 12p^{-5}$, but still assuming the same cost function as in part (a)? Interpret.
c) Assume that the total cost function is $TC(q) = 5 + 4q + 2(q)^2$. Use the inverse elasticity pricing rule (IEPR) to obtain the profit-maximizing price that this monopolist should charge.
d) How would your result in part (c) change if the demand curve changes to $q(p) = 12p^{-5}$, but still assuming the same cost function as in part (c)? Interpret.

Exercise #3 - Multiplant monopoly

Consider a monopolist facing inverse demand function $p(Q) = 10 - Q$, where $Q = q_1 + q_2$ denotes the monopolist's production across two plants, 1 and 2. Assume that total cost in plant 1 is given by $TC_1(q_1) = (3 + 2q_1)q_1$, while that of plant 2 is $TC_2(q_2) = [3 + (2 + d)q_2]q_2$, where parameter $d \geq 0$ represents plant 2's inefficiency to plant 1. When $d = 0$, the total (and marginal) cost of both plants coincide; but when $d > 0$, plant 2 has a higher total and marginal cost than plant 1.

a) Write down the monopolist's joint profit maximization problem $\pi = \pi_1 + \pi_2$.
b) Differentiate with respect to the output in plant 1, q_1, and then in plant 2, q_2. Set each expression equal to zero and use your results to find the optimal production in each plant.
c) How does your result in part (2) change in the inefficiency of plant 2, d. What is the optimal production in each plant if $d = 0$?

d) What is the aggregate production that this monopolist produces? How does Q vary in the inefficiency of plant 2, d?

Exercise #4 - Monopsony with linear supply curve

Consider a firm who is the only employer in a small town (a labor monopsony). It faces an international price $p > 0$ for each unit of output that it produces, its production function is $q = L^{\alpha}$ where $\alpha > 0$, and the labor supply is given by $w(L) = A * L$ where $A > 0$. Intuitively, the firm's production function is concave (when $\alpha < 1$), linear (when $\alpha = 1$), or convex (when $\alpha > 1$).

a) Find the monopsonist marginal revenue product, and its marginal expenditure on labor.
b) Set the marginal revenue product and marginal expenditure on labor equal to each other to find the optimal number of workers that the firm hires.
c) Find the equilibrium wage in this monopsony.
d) Assume now that the labor market was perfectly competitive. Find the optimal number of workers the firm hires, and the equilibrium wage in this setting.
e) Find the deadweight loss of the monopsony.

Exercise #5 – Cournot vs. Stackelberg models of quantity competition

Consider a market with two firms selling a homogeneous good, and competing in quantities. The inverse demand function is given by $p(q_1, q_2) = 1 - (q_1 + q_2)$, and both firms face a common marginal cost $c > 0$.

a) Cournot model. Set up the profit-maximization problem of firm 1.
b) Differentiate with respect to q_1. Solve for q_1 to obtain firm 1's best response function $q_1(q_2)$.
c) Depict firm 1's best response function $q_1(q_2)$. Interpret.
d) Repeat steps (a)-(b) for firm 2, so you obtain firm 2's best response function $q_2(q_1)$.
e) Find the point where the above two best response functions cross each other.
f) What is the equilibrium price? And the equilibrium profits for each firm?
g) *Stackelberg model*. Consider now that firm 1 acts as a leader and firm 2 as the follower. Find the optimal production for firm 1 and 2 in this sequential competition (Stackelberg model). Then find the equilibrium price, and the profits that each firm makes.
h) *Comparison.* Compare the individual output for each firm, price, and profits in the Cournot model (where firms simultaneously select their output levels) and in the Stackelberg model (where firms sequentially choose their output).