EconS 301
Written Assignment #6

Due date: October 27th, 2016.

Exercise #1. Consider a firm with the following Cobb-Douglas product function for labor and capital

$$q = 8L^{1/3}K^{1/2}$$

a) Find the marginal rate of technical substitution, $MRTS$, for this firm.
b) Solve the firm's cost-minimization problem (CMP) for an output level of $q = 100$ units, and assuming that input prices are $w = \$3$ and $r = \$2$ for labor and capital, respectively. Which are the cost-minimizing amount of labor and capital that the firm hires?
- [*Hint*: Set the $MRTS$ found in part (a) equal to the input price ratio. Then, insert your result into the firm's production function evaluated at $q = 100$ units. Last, solve for either L to find the cost-minimizing amount of labor, or for K to find the cost-minimizing amount of capital.]
c) What is the firm's total cost from hiring the optimal units of labor and capital you found in part (b)?
d) Replicate part (b), but now without assuming a specific output level q, or specific values for input prices w and r. Follow the same steps as in part (b), but notice that now you will find that the optimal workers hired, L, is not a number, but a function of q, w and r. This is what we refer as the firm's "demand for labor." A similar argument applies to the optimal amount of capital that the firm hires, K, which we refer as the firm's "demand for capital."
e) Confirm that if we evaluate the expressions you found in part (d) at the same parameter values considered in part (b) —that is an output of $q = 100$ units and input prices of $w = \$3$ and $r = \$2$— we obtain the same optimal units of labor and capital as in part (b).
f) What is the firm's total cost from hiring the optimal units of labor and capital you found in part (d)? As opposed to part (c) where your result was a dollar amount, your result now will be a function of q, w and r.
g) Confirm that if we evaluate the total cost found in part (f) at the same parameter values considered in part (c) —that is an output of $q = 100$ units and input prices of $w = \$3$ and $r = \$2$— we obtain the same total cost as in part (c).

Exercise #2. Repeat your analysis of Exercise #1 for the following linear production function

$$q = 7L + 4K$$

Exercise #3. Repeat your analysis of Exercise #1 for the following fixed-proportion production function

$$q = \min\{2L, 3K\}$$