EconS 301
Written Assignment #3 - *ANSWER KEY*

Exercise #1. Consider a consumer with Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

Assume that the consumer faces a price of \$1 for good 2, and a total income of \$100. The price of good 1 decreases from \$4 to \$2. We next analyze the substitution and income effect of this price change.

a) Find the optimal consumption bundle at the *initial* price of \$4. Label it bundle *A*.
ANSWER: We first find the marginal utilities to use them in the tangency condition for an optimal bundle. In this setting, marginal utilities are

$$MU_{x1} = \frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}} \qquad MU_{x2} = \frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}$$

Hence our tangency condition the MRS is

$$MRS = \frac{MU_{x1}}{MU_{x2}} = \frac{\frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}}}{\frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}} = \frac{x_2}{2x_1}$$

becomes

$$\text{MRS} = \frac{p_1}{p_2} = \frac{4}{1} = \frac{x_2}{2x_1}$$

and solving for x_2, we obtain

$$x_2 = 8x_1$$

We plug this result into the budget constraint $4x_1$ $+x_2$ = 100, as follows

$$4x_1 + 8x_1 = 100$$
$$12x_1 = 100$$

Solving for x_1, we find $x_1 = 8.33$. We can now use this optimal amount of good 1, x_1 = 8.33, to find the optimal amount of good 2. Using our tangency condition again, we have that

$$x_2 = 8x_1 = 8 * 8.33 = 66.67$$

Thus, the consumption bundle under initial prices is *A*=(8.33,66.67).

b) Find the optimal consumption bundle at the *final* price of \$2. Label it bundle *C*.
ANSWER: we found MRS in part (a),

$$MRS = \frac{MU_{x1}}{MU_{x2}} = \frac{\frac{1}{3} x_1^{-\frac{2}{3}} x_2^{\frac{2}{3}}}{\frac{2}{3} x_1^{\frac{1}{3}} x_2^{-\frac{1}{3}}} = \frac{x_2}{2x_1}$$

We have that the tangency condition

$$MRS = \frac{p_1}{p_2} = \frac{2}{1} = \frac{x_2}{2x_1}$$

and solving for x_2, we obtain

$$x_2 = 4x_1$$

We plug this result into the budget constraint $4x_1$ +x_2 = 100, as follows

$$2x_1 + 4x_1 = 100$$

$$6x_1 = 100$$

Solving for x_1

$$x_1 = 16.67$$

We can now use this optimal amount of good 1, $x_1 =$ 16.67, to find the optimal amount of good 2, using our tangency condition again

$$x_2 = 4x_1 = 66.7$$

Thus, the consumption bundle under initial prices is *C*= (16.67,66.67)

c) What is the total effect of the price change?
ANSWER: Total effect=bundle C – bundle A=16.67-8.33=8.34

d) We next seek to disentangle the total effect you found in part (c) into the substitution and income effects. In order to do that, let us start by finding the decomposition bundle. Label it bundle *B*. [*Hint*: Recall that the decomposition bundle must satisfy two conditions: (1) it must generate the same utility level as the initial bundle *A*; and (2) we must have a that the slope of the consumer's indifference curve, *MRS*, coincides with the new price ratio.]
ANSWER: we need to find utility of A first

$$U_a = 8.33^{\frac{2}{3}} * 66.67^{\frac{1}{3}} = 33.38$$

MRS= final price, which is $x_2 = 4x_1$

Hence, bundle B is

$$U_b = 33.38 = x_1^{\frac{2}{3}} * x_2^{\frac{1}{3}} = x_1^{\frac{1}{3}} * \left(4^{\frac{2}{3}} x_1^{\frac{2}{3}}\right) = 2.52x_1$$

We solve $x_1 = 13.25$

We can now use this optimal amount of good 1, $x1$ = 13.25, to find the optimal amount of good 2, using our tangency condition again

$$x_2 = 4x_1 = 52.98$$

Thus, the consumption bundle under initial prices is B= (13.25,52.98)

e) Write the amount of good 1 that this individual consumes on bundles *A*, *B* and *C*. What is the increase in consumption of good 1 due to the substitution effect? What is due to the income effect?
ANSWER
IE=C-B=16.67-13.25=3.42

SE=B-A=13.25-8.33=4.92

f) Using the sign of the income effect, what can you say about good 1? Is it a normal, or an inferior good?
ANSWER It is a normal good because IE>0

Exercise #2. Consider a consumer with the following quasi-linear utility function

$$u(x_1, x_2) = x_1^2 + 5x_2$$

Assume that the consumer faces a price of \$1 for good 2, and a total income of \$120. The price of good 1 decreases from \$4 to \$2. We next analyze the substitution and income effect of this price change.

a) Find the optimal consumption bundle at the *initial* price of \$4. Label it bundle *A*.
ANSWER: We first find the marginal utilities to use them in the tangency condition for an optimal bundle. In this setting, marginal utilities are

$$MU_{x1} = 2x_1 \qquad\qquad MU_{x2} = 5$$

Hence our tangency condition

$$MRS = \frac{MU_{x1}}{MU_{x2}} = \frac{2x_1}{5}$$

becomes

$$\text{MRS} = \frac{p_1}{p_2}, \qquad \text{or} \quad \frac{2x_1}{5} = \frac{4}{1}$$

and solving for x_1, $x_1 = 10$.

We plug $x_1 = 10$ into the budget constraint $4x_1 + x_2 = 120$, as follows

$$40 + x_2 = 120$$
$$x_2 = 80$$

Thus, the consumption bundle under initial prices is *A*=(10,80).

b) Find the optimal consumption bundle at the *final* price of \$2. Label it bundle *C*.
ANSWER: we found MRS in part (a),

$$MRS = \frac{2x_1}{5}$$

Hence our tangency condition

$$MRS = \frac{p_1}{p_2}, \qquad \text{or} \quad \frac{2x_1}{5} = \frac{2}{1}$$

We solve for x_1

$$\frac{2x_1}{2} = 5$$

and solving for x_1, $x_1 = 5$.

We plug x_1 into the budget constraint $2x_1 + x_2 = 120$, as follows

$$10 + x_2 = 120$$
$$x_2 = 110$$

Thus, the consumption bundle under initial prices is *C*=(5,110)

c) What is the total effect of the price change?
ANSWER: Total effect=Bundle C – Bundle A=5-10=-5

d) We next seek to disentangle the total effect you found in part (c) into the substitution and income effects. In order to do that, let us start by finding the decomposition bundle. Label it bundle *B*. [*Hint*: Recall that the decomposition bundle must satisfy two conditions: (1) it must generate the same utility level as the initial bundle *A*; and (2) we must have a that the slope of the consumer's indifference curve, *MRS*, coincides with the new price ratio.]
ANSWER: We first calculate the utility of A first.

$$U_A = 10^2 + 5 * 80 = 500$$

Then, the utility of bundle B must also be 500, that is, $x_1^2 + 5x_2 = 500$

Second, we set MRS= final price. From part (b) of the exercise, we know that this entails $x_1 = 5$

Plugging $x_1 = 5$ into the utility condition of bundle B that we found above, $x_1^2 + 5x_2 = 500$, we obtain that

$$5^2 + 5x_2 = 500$$

which, solving for x_2 yields

$$x_2 = 95$$

Thus, the consumption bundle under initial prices is B=(5,95)

e) Write the amount of good 1 that this individual consumes on bundles *A*, *B* and *C*. What is the increase in consumption of good 1 due to the substitution effect? What is due to the income effect?
ANSWER: IE=bundle C-bundle B=5-5=0
SE=bundle B-bundle A=0-5=-5

f) Using the sign of the income effect, what can you say about good 1? Is it a normal, or an inferior good?
ANSWER: The IE is zero, implying that the good is neither normal nor inferior.

Exercise #3. Consider a consumer with Cobb-Douglas utility function

$$u(x_1 x_2) = x_1^{1/2} x_2^{1/2}$$

Assume that the consumer faces a price of $1 for good 2, and a total income of $150. However, unlike in previous exercises, we now observe that the price of good 1 *increases* from $2 to $3. We next analyze the substitution and income effect of this price change.

a) Find the optimal consumption bundle at the *initial* price of $2. Label it bundle *A*.
ANSWER: We first find the marginal utilities to use them in the tangency condition for an optimal bundle. In this setting, marginal utilities are

$$MU_{x1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} \qquad MU_{x2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}$$

Hence our tangency condition the MRS is

$$MRS = \frac{MU_{x1}}{MU_{x2}} = \frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = \frac{x_2}{x_1}$$

becomes

$$MRS = \frac{p_1}{p_2}, \qquad \text{or} \quad \frac{x_2}{x_1} = \frac{2}{1}$$

and solving for x_2, we obtain

$$x_2 = 2x_1$$

We plug this result into the budget constraint 4x_1 +x_2 = 150, as follows

$$2x_1 + 2x_1 = 150$$
$$4x_1 = 150$$

Solving for x_1, we find $x_1 = 37.5$. We can now use this optimal amount of good 1, x_1 = 37.5, to find the optimal amount of good 2. Using our tangency condition again, we have that

$$x_2 = 2x_1 = 2 * 37.5 = 75$$

Thus, the consumption bundle under initial prices is *A*=(37.5,75).

b) Find the optimal consumption bundle at the *final* price of \$3. Label it bundle *C*.
ANSWER: we found MRS in part (a),

$$MRS = \frac{MU_{x1}}{MU_{x2}} = \frac{\frac{1}{2}x_1^{-\frac{1}{2}}x_2^{\frac{1}{2}}}{\frac{1}{2}x_1^{\frac{1}{2}}x_2^{-\frac{1}{2}}} = \frac{x_2}{x_1}$$

We have that the tangency condition

$$MRS = \frac{p_1}{p_2} = \frac{3}{1} = \frac{x_2}{x_1}$$

and solving for x_2, we obtain

$$x_2 = 3x_1$$

We plug this result into the budget constraint 3x_1 +x_2 = 150, as follows

$$3x_1 + 3x_1 = 150$$
$$6x_1 = 150$$

Solving for x_1, we find $x_1 = 25$. We can now use this optimal amount of good 1, x_1 = 25, to find the optimal amount of good 2. Using our tangency condition again, we have that

$$x_2 = 3x_1 = 3 * 25 = 75$$

Thus, the consumption bundle under initial prices is *C*=(25,75).

c) What is the total effect of the price change?
ANSWER: Total effect=Bundle C – Bundle A=25-37.5=-12.5
Note that the total effect is negative in this case since we are analyzing an increase in the price of good 1.

d) We next seek to disentangle the total effect you found in part (c) into the substitution and income effects. In order to do that, let us start by finding the decomposition bundle. Label it bundle *B*. [*Hint*: Recall that the decomposition bundle must satisfy two conditions: (1) it must generate the same utility level as the initial bundle *A*; and (2) we must have a that the slope of the consumer's indifference curve, *MRS*, coincides with the new price ratio.]

ANSWER: We need to find utility of A first

$$U_A = 37.5^{\frac{1}{2}} * 75^{\frac{1}{2}} = 53$$

Hence, we need that bundle B satisfies

$$x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 53$$

In addition, we know that at bundle B, MRS= final price ratio. From the previous parts of this exercise, we know that this condition entails $x_2 = 3x_1$.
Hence, bundle B is

$$x_1^{\frac{1}{2}} * x_2^{\frac{1}{2}} = x_1^{\frac{1}{2}} * \left(3^{\frac{1}{2}} x_1^{\frac{1}{2}}\right) = 1.7321x_1 = 53$$

We solve for x_1 to obtain $x_1 = 30.6$.
We can now use this optimal amount of good 1, x_1 = 37.48 to find the optimal amount of good 2, using our tangency condition again

$$x_2 = 3x_1 = 91.85$$

Thus, the decomposition bundle is *B*= (30.6,91.85)

e) Write the amount of good 1 that this individual consumes on bundles *A*, *B* and *C*. What is the increase in consumption of good 1 due to the substitution effect? What is due to the income effect?
ANSWER:
Income Effect= Bundle C- Bundle B=25-30.6=-5.6
Substitution effect= Bundle B-Bundle A= 30.6-37.5=-6.9
In words, an increase in the price of good 1 produces a reduction in its consumption of 6.9 units due to the SE, and of 5.6 units due to the IE, for a total effect of 12.5.

f) Using the sign of the income effect, what can you say about good 1? Is it a normal, or an inferior good?
ANSWER: Normal good, because IE goes in the opposite direction as the price change. That is, an increase in the price of good 1 makes the consumer poorer in purchasing power, thus reducing his consumption of this good. (This is the opposite of what we found for price decreases: a decrease in the price of good 1 increases the consumer's purchasing power, leading him to increase his consumption of that good, ultimately producing a positive IE. In that case, too, the price change and the IE moved in opposite directions for normal goods.)