EconS 301

Written Assignment #2 – *Answer Key*

Exercise #1

Part 1. Assume p_1 =\$1 and p_2 =\$2, and the consumer has income $I = \$10$. Find the consumer's budget constraint. Find utility-maximizing consumption bundles for the following utility functions:

(a) $u(x_1 x_2) = x_1 x_2$

(b) $u(x_1, x_2) = \sqrt{x_1} + 10x_2$

Answer:

(a) We first find the marginal utilities to use them in the tangency condition for an optimal bundle. In this setting, marginal utilities are

$$MU_{x_1} = x_2$$
$$MU_{x_2} = x_1$$

Hence our tangency condition

$$\frac{MU_{x_1}}{P_1} = \frac{MU_{x_2}}{P_2}$$

becomes

$$\frac{x_2}{1} = \frac{x_1}{2}$$

and solving for x_2

$$x_2 = \frac{x_1}{2}$$

We plug this result into the budget constraint $x_1 + 2x_2 = 10$ as follows

$$x_1 + 2\left(\frac{x_1}{2}\right) = 10 \text{ or } 2x_1 = 10$$

Solving for x_1, we find $x_1 = 5$.

We can now use this optimal amount of good 1, $x_1 = 5$, to find the optimal amount of good 2, using our tangency condition again

$$x_2 = \frac{x_1}{2} = \frac{5}{2} = 2.5$$

Thus, to maximize our utility, our consumption bundle is $\left(5, \frac{5}{2}\right)$.

(b) Again, we first find our marginal utilities to use them in our tangency condition for the optimal bundle. Thus our marginal utilities are

$$MU_{x_1} = \frac{1}{2\sqrt{x_1}}$$

$$MU_{x_2} = 10$$

Hence, our tangency condition $\frac{MU_{x_1}}{P_1} = \frac{MU_{x_2}}{P_2}$ becomes

$$\frac{\frac{1}{2\sqrt{x_1}}}{1} = \frac{10}{2}$$

Rearranging, we obtain

$$\frac{1}{2\sqrt{x_1}} = 5$$

and solving for x_1,

$$x_1 = \frac{1}{100}$$

We then plug that into our budget constraint $x_1 + 2x_2 = 10$ and solve for x_2.

$$1\left(\frac{1}{100}\right) + 2x_2 = 10 \quad \text{or} \quad 2x_2 = \frac{999}{100}$$

Solving for x_2 we find

$$x_2 = \frac{999}{200}$$

Thus to maximize our utility, our optimal bundle is $\left(\frac{1}{100}, \frac{999}{200}\right)$.

Exercise #2

Consider a consumer with Cobb-Douglas utility function

$$u(x_1 x_2) = x_1^{1/3} x_2^{2/3}$$

Assume that the consumer faces a price of \$10 for good 1, a price of \$20 for good 2, and a total income of \$100.

a) Find the optimal consumption bundle that solves this consumer's utility maximization problem (UMP).
b) Which utility level he can reach from the optimal bundle you found in part (a)?
c) Assume now that the consumer seeks to reach the utility level you found in part (b). Find the optimal consumption bundle that solves his expenditure minimization problem (EMP).
d) Does the optimal bundle found in part (a) coincide with that found in part (c). Explain.

Answer:

(a) First we must find our marginal utilities, so we can use them with the tangency condition.

$$MU_{x_1} = \frac{1}{3} x_1^{-2/3} x_2^{2/3}$$
$$MU_{x_2} = \frac{2}{3} x_1^{1/3} x_2^{-1/3}$$

Hence, our tangency condition $\frac{MU_{x_1}}{P_1} = \frac{MU_{x_2}}{P_2}$ becomes

$$\frac{\frac{1}{3}x_1^{-2/3}x_2^{2/3}}{10} = \frac{\frac{2}{3}x_1^{1/3}x_2^{-1/3}}{20}$$

and solving for x_1

$$\frac{2}{3}x_1^{-2/3}x_2^{2/3} = \frac{2}{3}x_1^{1/3}x_2^{-1/3}$$

or

$$\frac{x_2^{2/3}}{x_1^{2/3}} = \frac{x_1^{1/3}}{x_2^{1/3}}$$

which ultimately simplifies to $x_1 = x_2$.

We plug this expression into the budget constraint $10x_1 + 20x_2 = 100$ as follows

$$10x_1 + 20x_1 = 100 \quad \text{or} \quad 3x_1 = 100$$

Solving for x_1, we find

$$x_1 = \frac{10}{3}$$

We now use this optimal amount of good 1, to find the optimal amount of good 2 using our tangency condition.

$$x_1 = x_2 = \frac{10}{3}$$

Thus, to maximize our utility, our optimal bundle is $\left(\frac{10}{3}, \frac{10}{3}\right)$.

(b) We find the utility by plugging in our optimal bundle into our utility function.
$u(x_1 x_2) = x_1^{1/3}x_2^{2/3}$

$$u(x_1, x_2) = \left(\frac{10}{3}\right)^{1/3}\left(\frac{10}{3}\right)^{2/3} = \frac{10}{3}$$

(c) To solve for the EMP, we are now trying to minimize the expenditure while still maintaining the same utility function as above. To solve for the EMP, we will use our tangency condition; however, this time we will put prices on top of the equation.

We must first find our marginal utilities to plug into our tangency condition. Our marginal utilities are

$$MU_{x_1} = \frac{1}{3}x_1^{-2/3}x_2^{2/3}$$
$$MU_{x_2} = \frac{2}{3}x_1^{1/3}x_2^{-1/3}$$

Hence our tangency condition $\frac{MU_{x_1}}{P_1} = \frac{MU_{x_2}}{P_2}$ becomes

$$\frac{\frac{1}{3}x_1^{-2/3}x_2^{2/3}}{10} = \frac{\frac{2}{3}x_1^{1/3}x_2^{-1/3}}{20}$$

Rearranging, we obtain

$$\frac{x_1^{2/3}}{x_2^{2/3}} = \frac{x_2^{1/3}}{x_1^{1/3}}$$

which simplifies to $x_1 = x_2$.

We plug this result into the utility function $\frac{10}{3} = x_1^{1/3}x_2^{2/3}$ as follows

$$\frac{10}{3} = x_1^{1/3}x_1^{1/3}$$

Solving for x_1, we find

$$x_1 = \frac{10}{3}$$

Since we found our optimal amount of good 1, we can plug this into our tangency condition to find x_2.

$$x_1 = x_2 = \frac{10}{3}$$

Thus, to maintain our utility found in part b, our optimal consumption bundle is $\left(\frac{10}{3}, \frac{10}{3}\right)$.

(d) Yes, these bundles coincide since we are trying to find the same utility level in both the UMP and the EMP. The UMP shows the highest utility we can reach given our budget, and the EMP shows the cheapest bundle we can buy to reach a certain utility. Since we assume they are trying to reach the same utility level, they are equal because we used all of our budget to reach this utility (UMP) and there is no cheaper combination to reach that utility level (EMP).

Exercise #3

Consider a consumer with Cobb-Douglas utility function

$$u(x_1 x_2) = x_1^{1/2}x_2^{1/2}$$

Assume that the consumer faces a price p_1 for good 1, a price p_2 for good 2, and a total income of $\$I$.

a) Find the optimal consumption bundle that solves this consumer's utility maximization problem (UMP).

b) Does the optimal consumption of good 1 found in part (a) increase or decrease in its own price p_1? Does it increase or decrease in the price of good 2, p_2? And, does it increase or decrease in income, $\$I$?
c) Does the optimal consumption of good 2 found in part (a) increase or decrease in its own price p_2? Does it increase or decrease in the price of good 1, p_1? And, does it increase or decrease in income, $\$I$?
d) Find the optimal consumption bundle that solves this consumer's expenditure minimization problem (EMP), assuming that he seeks to reach a "utility target" of $\bar{u}$.
e) Does the optimal consumption of good 1 found in part (d) increase or decrease in its own price p_1? Does it increase or decrease in the price of good 2, p_2? And, does it increase or decrease in the utility target the consumer seeks to reach, $\bar{u}$?
f) Does the optimal consumption of good 2 found in part (d) increase or decrease in its own price p_2? Does it increase or decrease in the price of good 1, p_1? And, does it increase or decrease in the utility target the consumer seeks to reach, $\bar{u}$?

Answer:

(a) We will first find our marginal utilities, so we can use them with our tangency conditions. In this setting, our marginal utilities are

$$MU_{x_1} = \frac{1}{2}x_1^{-1/2}x_2^{1/2}$$
$$MU_{x_2} = \frac{1}{2}x_1^{1/2}x_2^{-1/2}$$

Hence our tangency condition $\frac{MU_{x_1}}{P_1} = \frac{MU_{x_2}}{P_2}$ becomes

$$\frac{\frac{1}{2}x_1^{-1/2}x_2^{1/2}}{P_1} = \frac{\frac{1}{2}x_1^{1/2}x_2^{-1/2}}{P_2}$$

Rearranging, we obtain

$$\frac{P_2 x_2^{1/2}}{2x_1^{1/2}} = \frac{P_1 x_1^{1/2}}{2x_{\_2}^{1/2}}$$

And solving for x_1

$$x_1 = \frac{P_2 x_2}{P_1}$$

We plug this result into our budget constraint $P_1x_1 + P_2x_2 = I$ as follows

$$P_1\frac{P_2x_2}{P_1} + P_2x_2 = I \quad \text{or} \quad 2P_2x_2 = I$$

Solving for x_2, we find

$$x_2 = \frac{I}{2P_2}$$

We can now use our optimal amount of good 2, $x_2 = \frac{I}{2P_2}$, to find the optimal amount of good 1 using the tangency condition

$$x_1 = \frac{P_2 \frac{I}{2P_2}}{P_1} = \frac{I}{2P_1}$$

Thus, to maximize utility, our optimal consumption bundle is $\left(\frac{I}{2P_1}, \frac{I}{2P_2}\right)$.

(b) We see that P_1 is in the denominator. Hence, as P_1 goes up, the quantity consumed of x_1 goes down. This tells us it is a normal good. We can see that P_2 is in the numerator. As P_2 goes up, the quantity consumed of x_1 goes up. This tells us that x_1 and x_2 are substitutes. Finally, as I goes up, x_1 goes up. Once again, telling us it is a normal good.

(c) Similarly, as P_2 goes up, x_2 goes down. Again, we see x_2 is a normal good. As P_1 goes up, x_2 goes up. Thus tells us x_1 and x_2 are substitutes. Finally, as I goes up, x_2 goes up.

(d) Again, we are trying to solve for the EMP, so we are trying to minimize our expenditure while still maintaining the utility we found above. To do this, we will use our tangency condition with prices on top.

We will first find our marginal utilities, so we can plug those into our tangency condition. Our marginal utilities are

$$MU_{x_1} = \frac{1}{2} x_1^{-1/2} x_2^{1/2}$$
$$MU_{x_2} = \frac{1}{2} x_1^{1/2} x_2^{-1/2}$$

Hence our tangency condition $\frac{MU_{x_1}}{P_1} = \frac{MU_{x_2}}{P_2}$ becomes

$$\frac{\frac{1}{2} x_1^{-1/2} x_2^{1/2}}{P_1} = \frac{\frac{1}{2} x_1^{1/2} x_2^{-1/2}}{P_2}$$

Rearranging,

$$P_1 \frac{1}{2} x_1^{1/2} x_2^{-1/2} = P_2 \frac{1}{2} x_1^{-1/2} x_2^{1/2}$$

$$\frac{P_1 x_1^{1/2}}{2x_2^{1/2}} = \frac{P_2 x_2^{1/2}}{2x_1^{1/2}}$$

and solving for x_1

$$x_1 = \frac{P_2 x_2}{P_1}$$

We plug this optimal consumption of good 1 into our utility function $u(x_1 x_2) = x_1^{1/2} x_2^{1/2}$ as follows

$$\bar{u} = \left(\frac{P_2 x_2}{P_1}\right)^{1/2} x_2^{1/2} \quad \text{or} \quad \bar{u} = x_2 (p_2)^{1/2} (p_1)^{-1/2}$$

Solving for x_2, we find

$$x_2 = \bar{u}(p_1)^{1/2}(p_2)^{-1/2} \text{ or } x_2 = \frac{\bar{u}(p_1)^{1/2}}{(p_2)^{1/2}}$$

Since we found our optimal consumption of good 2, $x_2 = \frac{\bar{u}(p_1)^{1/2}}{(p_2)^{1/2}}$, we can plug this into our tangency condition to find x_1.

$$x_1 = \frac{P_2 \frac{\bar{u}(p_1)^{1/2}}{(p_2)^{1/2}}}{P_1} \quad \text{or} \quad x_1 = \bar{u}(p_2)^1(p_1)^{1/2}(p_2)^{-1/2}(p_1)^{-1}$$

Solving for x_1, we find

$$x_1 = \bar{u}(p_2)^{1/2}(p_1)^{-1/2} \text{ or } x_1 = \frac{\bar{u}(p_2)^{1/2}}{(p_1)^{1/2}}$$

Thus, to reach utility, $\bar{u}$, our optimal consumption bundle is $\left(\frac{\bar{u}(p_2)^{1/2}}{(p_1)^{1/2}}, \frac{\bar{u}(p_1)^{1/2}}{(p_2)^{1/2}}\right)$

(e) This will be very similar to part (c). As P_1 goes up, consumption of x_1 goes down because P_1 is in the denominator. Thus x_1 is a normal good. As P_2 goes up, x_1 goes up. Thus x_1 and x_2 are substitutes. Finally, as U goes up, x_2 goes up. Thus telling us that x_1 is a normal good because as our income rises, we choose to buy more.

(f) Similarly, as P_2 goes up, x_2 consumption goes down. So we know that x_2 is considered a normal good. As P_1 goes up, x_2 goes up. Confirming as we saw in part e that x_1 and x_2 are substitutes. Finally, as U goes up, x_2 consumption goes up. Thus x_2 is a normal good.