ECONS 301 – Homework #1

Answer Key

Exercise #1 (Supply and demand). Suppose that the demand and supply for milk in the European Union (EU) is given by

$$p = 120 - 0.7Q^d \quad \text{and} \quad p = 3 + 0.2Q^s$$

where the quantity is in millions of liters and the price is in cents per liter. Assume that the EU does not import or export milk.

a) Find the market equilibrium quantity, and the equilibrium price.
b) Find the consumer and producer surplus at the market equilibrium that you found in part (a).
c) Assume now that European farmers successfully lobby for a price floor of 36 cents per liter. What is the new quantity sold in the market?
d) Find the consumer and producer surplus after the price floor.
e) What is the deadweight loss from introducing the price floor?
f) If the EU authorities were to buy the surplus milk from farmers at the price floor of 36 cents per liter, how much would they spend in millions of euros? [Recall that 1 Euro=100 cents.]

ANSWER:

a) Set the equations equal to each other, and then solve for Q:

$3+0.2Q = 120 - 0.7Q$

$0.9Q = 117$

Q* = 130

Substitute the value of Q=130 that we found into either of the equations above to solve for *p*. (The answers should be the same if you use the demand or the supply curve, as we show in the two columns below.)

p=120−0.7(130)	p=3+0.2(130)
p=120−91	p=3+26
P* = 29	**p* = 29**

b) Calculate the area of the triangle made by the supply and demand curves. Above the equilibrium price is consumer surplus, below is producer surplus:

Consumer surplus:	Producer surplus:
½ (120 – 29) x 130 = **5,915**	½ (29 – 3) x 130 = **1,690**

c) If p=36 is given by the price floor, let us substitute it into the demand equation:

36 = 120−0.7Qd

$0.7Qd = 84$

$\boldsymbol{Qd} = \mathbf{120}$

d) For consumer surplus, calculate the triangle above the price line (of p=36) and below the demand curve (for a graphical representation, see figure at the end of the exercise):

(120–36) x ½ x 120 = **5,040**

For producer surplus, calculate the area under the price line and above the supply curve, then subtract the area of deadweight loss (see DWL calculation in part e).

((36–3) x ½ x 120)–35 = **1,945**

e) Deadweight loss is the area of the triangle made by the change in quantity and change in price

(Q* - Q`) x (p` - p*) x ½; that is,

(130–120) x (36–29) x ½ = **35**

f) First, find the amount of surplus quantity that is available. This is where we substitute the price floor amount into the supply equation, and subtract the amount purchased by consumers:

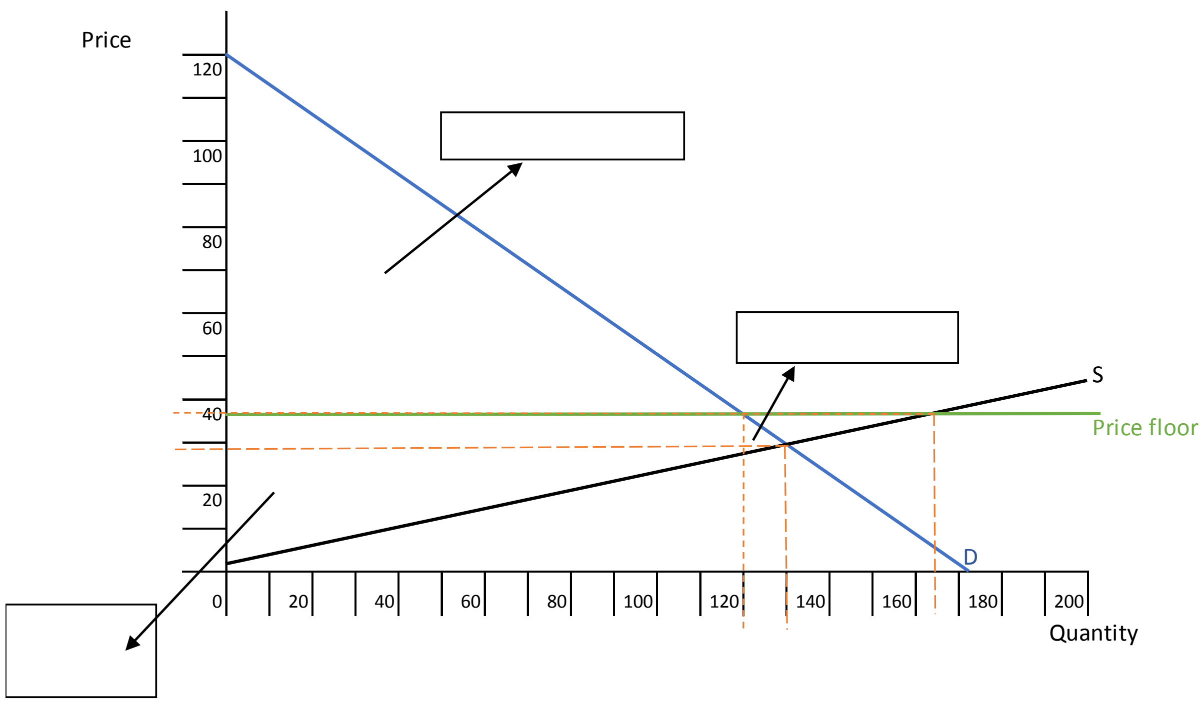
36 = 3 + 0.2Qs

33 = 0.2Qs

$Qs = 165$

Surplus = Qs – Qd = 165 – 120 = 45

Money spent = 36 cents x 45 million liters = 0.36 x 45,000,000 = €16,200,000



Exercise #2 (Indifference curves and MRS). For each of the following utility functions, find the marginal rate of substitution function, *MRS*, and plot the indifference curve for which the consumer reaches a utility level of $u = 10$.

(a) $u(x_1, x_2) = x_1 x_2$

(b) $u\ (x_1 x_2) = 2x_2$

(c) $u(x_1 x_2) = x_1 + x_2$

(d) $u(x_1 x_2) = \min\{x_1, 2x_2\}$

(e) $u(x_1 x_2) = x_2 + x_1^2$

ANSWER:

We use the fact that the *MRS* equals the ratio of the marginal utilities, or $= \frac{MU_1}{MU_2}$. In each case, we first calculate the marginal utilities using derivative properties, and then we find their ratio.

(a) Using $u(x_1, x_2) = x_1 x_2$, we have that marginal utilities are

$MU_1 = \frac{\partial(x_1 x_2)}{\partial x_1} = x_2$ (holding x_2 as a constant)

$MU_2 = \frac{\partial(x_1 x_2)}{\partial x_2} = x_1$ (holding x_1 as a constant)

Therefore, the MRS is

$$MRS = \frac{MU_1}{MU_2} = \frac{x_2}{x_1}$$

which decreases in x_1, thus indicating that indifference curves are bowed in towards the origin (i.e., indifference curves become flatter as we move rightward).

(b) Taking now $u\ (x_1, x_2) = 2x_2$, marginal utilities are

$MU_1 = \frac{\partial(2x_2)}{\partial x_1} = 0$ (there is no x_1 in the utility function).

$MU_2 = \frac{\partial(2x_2)}{\partial x_2} = 2$ (2 is the constant coefficient of x_2)

Therefore, the MRS is

$$MRS = \frac{MU_1}{MU_2} = \frac{0}{2} = 0$$

which is constant in x_1, thus indicating that indifference curves are flat (i.e., the slope of the indifference curves does not change in x_1). The indifference map can be depicted as flat lines, with increasing utility levels as we move north (higher amounts of good 2). Intuitively, the utility function does not contain good 1, indicating that this individual only cares about good 2.

(c) We now take utility function $u(x_1, x_2) = x_1 + x_2$, which implies that marginal utilities are

$$MU_1 = \frac{\partial(x_1+x_2)}{\partial x_1} = 1 \text{ (1 is the hidden coefficient of } x_1\text{)}$$

$$MU_2 = \frac{\partial(x_1+x_2)}{\partial x_2} = 1 \text{ (1 is the hidden coefficient of } x_2\text{)}$$

Therefore, the MRS is

$$MRS = \frac{MU_1}{MU_2} = \frac{1}{1} = 1$$

which is constant in x_1, thus indicating that indifference curves have a constant slope of -1 (i.e., the slope of the indifference curves does not change as we move rightward).

(d) Take $u(x_1, x_2) = \min\{x_1, 2x_2\}$. The marginal utilities depend on whether $x_1 < 2x_2$, or $x_1 > 2x_2$.

If $x_1 < 2x_2$ (before the kink in the indifference curve, i.e., at its vertical segment), then

$$MU_1 = \frac{\partial(\min\{x_1,2x_2\})}{\partial x_1} = 1 \quad \text{and} \quad MU_2 = \frac{\partial(\min\{x_1,2x_2\})}{\partial x_2} = 0$$

Therefore, the MRS is

$$MRS = \frac{MU_1}{MU_2} = \frac{1}{0} = \infty$$

If, instead, $x_1 > 2x_2$ (to the right-hand of the kink in the indifference curve, i.e., at its flat segment) then

$$MU_1 = \frac{\partial(\min\{x_1,2x_2\})}{\partial x_1} = 0 \quad \text{and} \quad MU_2 = \frac{\partial(\min\{x_1,2x_2\})}{\partial x_2} = 2$$

Therefore, the MRS is

$$MRS = \frac{MU_1}{MU_2} = \frac{0}{2} = 0$$

Finally, if $x_1 = 2x_2$ (exactly at the kink of the indifference curve), the *MRS* is undefined, as described in class.

(e) Take now utility function $u(x_1, x_2) = x_2 + x_1^2$. Marginal utilities are

$$MU_1 = \frac{\partial(x_2+x_1^2)}{\partial x_1} = 2x_1 \qquad \text{and} \qquad MU_2 = \frac{\partial(x_2+x_1^2)}{\partial x_2} = 1$$

Therefore, the MRS is

$$MRS = \frac{MU_1}{MU_2} = \frac{2x_1}{1} = 2x_1$$

which is increasing in x_1, thus indicating that indifference curves become steeper as good 1 increases (i.e., as we move rightward). Graphically, indifference curves are bowed away from the origin.