Time Inconsistent Preferences

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Advanced Microeconomics II - Washington State University

Time Inconsistent Preferences - Tadelis 8.3.4

- Previously, we assumed that players maximize their discounted sum of payoffs.
  - This is typically done using *exponential discounting*, where every future period is multiplied by δ^t.
- We can show that using exponential discounting, an individual's consumption choices are consistent across time periods.

Time Inconsistent Preferences

- Consider a player with $u(x) = \ln(x)$ who needs to allocate a fixed budget K across three periods. Assume that prices all equal 1. His maximization problem is

$$\max_{x_2, x_3} \quad \ln(\underbrace{K - x_2 - x_3}_{x_1}) + \delta \ln(x_2) + \delta^2 \ln(x_3)$$

with first-order conditions

$$\begin{aligned} -\frac{1}{K - x_2 - x_3} + \frac{\delta}{x_2} &= 0 \\ -\frac{1}{K - x_2 - x_3} + \frac{\delta^2}{x_3} &= 0 \end{aligned}$$

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- Solving, we find

$$x_1^* = K\frac{1}{1+\delta+\delta^2}$$
$$x_2^* = K\frac{\delta}{1+\delta+\delta^2}$$
$$x_2^* = K\frac{\delta^2}{1+\delta+\delta^2}$$

- But what if the player made his choices sequentially? Would he reach the same equilibrium? Let's try backward induction.

Time Inconsistent Preferences

- In period 2, the player solves

$$\max_{x_2} \quad \ln(x_2) + \delta \ln(\underbrace{K_2 - x_2}_{x_3})$$

where

$$K_2 = K - x_1^* = K - K\frac{1}{1+\delta+\delta^2} = K\frac{\delta+\delta^2}{1+\delta+\delta^2}$$

Solving, we find

$$x_2^* = \frac{K_2}{1+\delta} = K\frac{\delta}{1+\delta+\delta^2}$$

Time Inconsistent Preferences

- Thus, regardless of his previous actions, the player will choose the same consumption amounts sequentially as he will simultaneously. Thus, this player has *time consistent preferences*.
    - This is a useful property of exponential discounting.
- What about other types of discounting, namely hyperbolic discounting.
    - In this case, a player uses the discount rate δ as seen in exponential discounting, but he uses an additional discount factor $\beta \in (0, 1)$ to discount all of the future compared to present consumption. Our previous example would look as follows under hyperbolic discounting:

$$\max_{x_2, x_3} \quad \ln(\underbrace{K - x_2 - x_3}_{x_1}) + \beta\delta \ln(x_2) + \beta\delta^2 \ln(x_3)$$

Time Inconsistent Preferences

- Intuitively, if the player is looking toward the future, the discount factor he uses between periods $t = 1$ and $t = 2$ is stronger than the one he uses between periods $t = 2$ and $t = 3$.
- Hyperbolic discounting will cause problems with self-control, in that a player will plan to do one thing but later choose to revise his plan.
  - This is studied frequently in behavior economics. In laboratory setting, it is frequently found that individuals do not consume "rationally." Robby Rosenman's module in Applied Microeconomics touches on this very topic, as well as other similar ones.

Time Inconsistent Preferences

- Let $\delta = 1$ for simplicity. We know that a rational person using exponential discounting will equalize his consumption across the three periods, i.e., $x_i^* = \frac{K}{3}$ $i = 1, 2, 3$. Let's look at the case where a player uses hyperbolic discounting, and for simplicity, we'll assume that $\beta = \frac{1}{2}$.
- Our simultaneous problem becomes

$$\max_{x_2, x_3} \quad \ln(\underbrace{K - x_2 - x_3}_{x_1}) + \frac{1}{2}\ln(x_2) + \frac{1}{2}\ln(x_3)$$

with solution

$$\begin{aligned} x_1^* &= \frac{1}{2}K \\ x_2^* &= \frac{1}{4}K \\ x_3^* &= \frac{1}{4}K \end{aligned}$$

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- Checking to see if the player's sequential solution is the same, we use backward induction. Our period 2 problem becomes

$$\max_{x_2} \quad \ln(x_2) + \frac{1}{2}\ln(\underbrace{K - x_1 - x_2}_{x_3})$$

This yields best-response functions of

$$\begin{aligned} x_2(x_1) &= \frac{2}{3}(K - x_1) \\ x_3(x_1) &= \frac{1}{3}(K - x_1) \end{aligned}$$

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- We can substitute these best-response functions back into our period 1 problem to obtain

$$\max_{x_1} \quad \ln(x_1) + \frac{1}{2}\ln\left(\frac{2}{3}(K - x_1)\right) + \frac{1}{2}\ln\left(\frac{1}{3}(K - x_1)\right)$$

with solution

$$\begin{aligned} x_1^* &= \frac{1}{2}K \\ x_2^* &= \frac{1}{3}K \\ x_3^* &= \frac{1}{6}K \end{aligned}$$

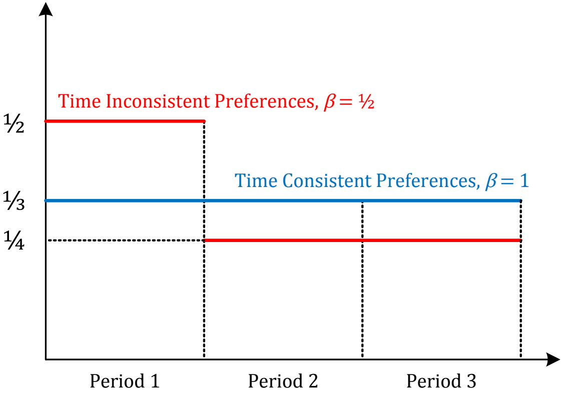
  - Note: This differs from Tadelis' solution. He has an error in his FOCs. (This is why we check them!)

Time Inconsistent Preferences

- This results in a very different solution than our simultaneous benchmark, $x_1^* = \frac{1}{2}K$, $x_2^* = \frac{1}{4}K$, $x_3^* = \frac{1}{4}K$.
- In this situation, the player consumes the same amount as he would normally in period 1, but in period 2, he becomes impatient once again, and overconsumes what he would have wanted to in period 1. We refer to this as the player having *time inconsistent preferences*.

Time Inconsistent Preferences

- Comparing time consistent with time inconsistent preferences in the simultaneous game ($\delta = 1$):



Time Inconsistent Preferences

- Comparing time consistent with time inconsistent preferences in the sequential game ($\delta = 1$):