Subjective probability theory

1. Reading: section 6.F in MWG.
2. So far we were assuming that probabilities were objective and observable.
3. This not the case many times. Instead, people might hold probabilitic beliefs about the likelihood of a certain event: *subjective probabilities*.

Subjective probability theory

1. Can we deduce subjective probabilities from actual behavior? Yes!
2. Imagine a decision maker who prefers a gamble

$$(\$1 \text{ in state } 1, \$0 \text{ in state } 2) \succsim (\$0 \text{ in state } 1, \$1 \text{ in state } 2)$$

3. If the value of money is the same across states, then he must be assigning a higher subjective probability to state 1 than to state 2.

Subjective probability theory

1. Before we can obtain a subjective EU theorem, we must start with some definitions...
2. First, we define state s preferences, $\succsim_s$, on state s lotteries $F_s(\cdot)$ by

$$F_s(\cdot) \succsim F_s'(\cdot) \text{ if } \int u_s(x_s)dF_s(x_s) \geq \int u_s(x_s)dF_s'(x_s)$$

3. Hence, the state preferences $(\succsim_1, \succsim_2, ..., \succsim_S)$ on state lotteries $(F_1, F_2, ..., F_S)$ are **state uniform** if

$$\succsim_s = \succsim_{s'} \text{ for any two states } s \text{ and } s'$$

4. That is, if for any two states, s and s', the ranking of lotteries coincides $F_s(\cdot) \succsim F_s'(\cdot)$ for any two lotteries $F_s(\cdot)$ and $F_s'(\cdot)$.
5. Alternative interpretation, for any two states, s and s', the ranking of expected utilities from playing two lotteries $F_s(\cdot)$ and $F_s'(\cdot)$ coincide, i.e., $\int u_s(x_s)dF_s(x_s) \geq \int u_s(x_s)dF_s'(x_s)$

Subjective probability theory

1. With state uniformity, $u_s(\cdot)$ and $u_{s'}(\cdot)$ can differ only by an increasing linear transformation.
2. That is, there is a utility function $u(\cdot)$ such that

$$\begin{aligned} u_s(\cdot) &= \pi_s u(\cdot) + \beta_s, \text{ and} \\ u_{s'}(\cdot) &= \pi_{s'} u(\cdot) + \beta_{s'} \end{aligned}$$

for every state s and s', and for every $\pi_s, \pi_{s'} > 0$ and $\beta_s, \beta_{s'} > 0$.

   1. In words, the ranking between the expected utility of state s and s' remains unaffected.

Subjective probability theory

1. Given state uniformity...
2. **Subjective EU Theorem:**
   1. Suppose that $\succsim$ satisfies continuity and the extended IA. Suppose, in addition, that the derived state preferences are state uniform.

Then, there are probabilities $(\pi_1, \pi_2, ..., \pi_S) >> 0$ and a utility function $u(\cdot)$ on amounts of money such that for any

$$(x_1, x_2, ..., x_S) \succsim (x'_1, x'_2, ..., x'_S) \text{ iff } \sum_s \pi_s u(x_s) \geq \sum_s \pi_s u(x'_s)$$

1. **Ellsberg paradox: (anomaly)**
    1. An urn contains 300 balls: among these 100 are red, 200 are either blue or green.
    2. We first present the following two gambles to a group of students, asking each of them to choose either gamble A or B.
    3. *Gamble A*: \$1000 if the ball is red.
    4. *Gamble B*: \$1000 if the ball is blue.

Subjective probability theory

1. **Ellsberg paradox: (anomaly)**
   1. We next present the following two gambles to the same group of students, asking each of them to choose either gamble C or D.
   2. *Gamble C*: $1000 if the ball is *not* red.
   3. *Gamble D*: $1000 if the ball is *not* blue.
2. Common choices: people choose A to B, and C to D.
   1. But this choices violate subjective EU theory! Let us see why...

Subjective probability theory

1. **Ellsberg paradox: (anomaly)**
   1. We know that

$$\begin{aligned} p(\text{Red}) &= 1 - p(\text{not Red}), \text{ and} \qquad (1) \\ p(\text{Blue}) &= 1 - p(\text{not Blue}) \end{aligned}$$

   2. If gamble A is preferred to B, then we must have

$$p(\text{Red})u(\$1000) > p(\text{Blue})u(\$1000) \implies p(\text{Red}) > p(\text{Blue}) \qquad (2)$$

   3. And if gamble C is preferred to D, then we must have

$$\begin{aligned} p(\text{Not Red})u(\$1000) &> p(\text{Not Blue})u(\$1000) \qquad (3) \\ \text{That is, } p(\text{Not Red}) &> p(\text{Not Blue}) \end{aligned}$$

   4. But expressions (1), (2) and (3) are incompatible.