Let us consider an oligopoly game where two firms compete in quantities. Market demand is given by the expression $p = 1 - q_1 - q_2$, and firms have incomplete information about their marginal costs. In particular, firm 2 privately knows whether its marginal costs are $MC_2 = \begin{cases} 0 \text{ with probability } 1/2 \\ 1/4 \text{ with probability } 1/2 \end{cases}$

On the other hand, firm 1 does not know firm 2’s cost structure. Firm 1’s marginal costs are $MC_1 = 0$, and this information is common knowledge among the firms (firm 2 also knows it).

Find the Bayesian Nash equilibrium of this oligopoly game, specifying how much every firm produces.

**Firm 2.** First, let us focus on Firm 2, the informed player in this game, as we usually do when solving for the BNE of games of incomplete information.

When firm 2 has low costs, its profits are
\[
Profits^L_2 = (1 - q_1 - q_2^L)q_2^L = q_2^L - q_1 q_2^L (q_2^L)^2
\]

Differentiating with respect to $q_2^L$, we can obtain firm 2’s best response function when experiencing low costs, $BRF^L_2(q_1)$.
\[
1 - q_1 - 2q_2^L = 0 \implies q_2^L(q_1) = \frac{1 - q_1}{2} \quad (BRF^L_2(q_1))
\]

On the other hand, when firm 2 has high costs ($MC = 1/4$), its profits are
\[
Profits^H_2 = (1 - q_1 - q_2^H)q_2^H - \frac{1}{4}q_2^H = q_2^H - q_1 q_2^H (q_2^H)^2 - \frac{1}{4}q_2^H
\]

Differentiating with respect to $q_2^H$, we can obtain firm 2’s best response function when experiencing high costs, $BRF^H_2(q_1)$.
\[
1 - q_1 - 2q_2^H - \frac{1}{4} = 0 \implies q_2^H(q_1) = \frac{3 - q_1}{2} = \frac{3}{8} - \frac{q_1}{2} \quad (BRF^H_2(q_1))
\]
Firm 1. Let us now analyze Firm 1 (the uninformed player in this game). First note that its profits must be expressed in expected terms, since firm 1 does not know whether firm 2 has low or high costs.

\[
Profits_1 = \begin{cases} 
\frac{1}{2} (1 - q_1 - q^L_2) q_1 & \text{if low costs} \\
\frac{1}{2} (1 - q_1 - q^H_2) q_1 & \text{if high costs}
\end{cases}
\]

Rearranging,

\[
Profits_1 = \left( \frac{1}{2} - \frac{q_1}{2} - \frac{q^L_2}{2} + \frac{1}{2} - \frac{q_1}{2} - \frac{q^H_2}{2} \right) q_1 = \left( 1 - q_1 - \frac{q^L_2}{2} - \frac{q^H_2}{2} \right) q_1
\]

\[
= q_1 - (q_1)^2 - \frac{q^L_2}{2} q_1 - \frac{q^H_2}{2} q_1
\]

Differentiating with respect to \( q_1 \), we can obtain firm 1’s best response function, \( BRF_1(q^L_2, q^H_2) \). Note that we do not have to differentiate for the case of low and high costs, since firm 1 does not observe such information. In particular,

\[
1 - 2q_1 - \frac{q^L_2}{2} - \frac{q^H_2}{2} = 0 \implies q_1 (q^L_2, q^H_2) = \frac{1}{2} - \frac{q^L_2}{2} - \frac{q^H_2}{2}
\]

(BRF \((q^L_2, q^H_2)\))

- After finding the best response functions for both types of Firm 2 and for the unique type of Firm 1 we are ready to plug the first two BRFs into the latter. Specifically,

\[
q_1 = \frac{1}{2} - \frac{1-q_1}{2} - \frac{3 - q_1}{2}
\]

which simplifies to

\[
q_1 = \frac{1}{16} + \frac{8}{16} q_1
\]

And solving for \( q_1 \), we find \( q_1 = \frac{1}{8} \).

- With this information, it is easy to find the particular level of production for firm 2 when experiencing low marginal costs,

\[
q^L_2 (q_1) = \frac{1 - q_1}{2} = \frac{1 - \frac{1}{8}}{2} = \frac{7}{16}
\]

- As well as the level of production for firm 2 when experiencing high marginal costs,

\[
q^H_2 (q_1) = \frac{3 - q_1}{2} = \frac{3 - \frac{1}{8}}{2} = \frac{5}{16}
\]

- Therefore, the Bayesian Nash equilibrium of this oligopoly game with incomplete information about firm 2’s marginal costs prescribes the following production levels

\[
(q_1, q^L_2, q^H_2) = \left( \frac{3}{8}, \frac{7}{16}, \frac{5}{16} \right)
\]
EconS 424: Strategy and Game Theory

Oligopoly games with incomplete information about market demand*

March 11, 2015

Let us consider an oligopoly game where two firms compete in quantities. Both firms have the same marginal costs, $MC = $1, but they are asymmetrically informed about the actual state of market demand. In particular, Firm 2 does not know what is the actual state of demand, but knows that it is distributed with the following probability distribution

$p(Q) = \begin{cases} 
10 - Q & \text{with probability } 1/2 \\
5 - Q & \text{with probability } 1/2 
\end{cases}$

On the other hand, Firm 1 knows the actual state of market demand, and Firm 2 knows that Firm 1 knows this information (i.e., it is common knowledge among the players).

**Firm 1.** First, let us focus on Firm 1, the informed player in this game, as we usually do when solving for the BNE of games of incomplete information.

When firm 1 observes a high demand market its profits are

$Profits_1 = (10 - q_1^H - q_2)q_1^H$

Differentiating with respect to $q_1^H$, we can obtain Firm 1’s best response function when experiencing low costs, $BRF_1^H(q_2)$.

$10 - 2q_1^H - q_2 - 1 = 0 \implies q_1^H(q_2) = 4.5 - \frac{q_2}{2}$

On the other hand, when firm 1 observes a low demand market its profits are

$Profits_1 = (5 - q_1^L - q_2)q_1^L - 1q_1^L = 5q_1^L - (q_1^L)^2 - q_2q_1^L - 1q_1^L$

Differentiating with respect to $q_1^L$, we can obtain Firm 1’s best response function when experiencing high costs, $BRF_1^L(q_2)$.

$5 - 2q_1^L - q_2 - 1 = 0 \implies q_1^L(q_2) = 2 - \frac{q_2}{2}$

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**Firm 2.** Let us now analyze Firm 2 (the uninformed player in this game). First note that its profits must be expressed in expected terms, since firm 2 does not know whether market demand is high or low.

\[ \text{Profits}_2 = \frac{1}{2} \left[ (10 - q_1^H - q_2)q_2 - 1q_2 \right] + \frac{1}{2} \left[ (5 - q_1^L - q_2)q_2 - 1q_2 \right] \]

Rearranging,

\[ \text{Profits}_2 = \frac{1}{2} \left[ 10q_2 - q_1^H q_2 - (q_2)^2 - q_2 \right] + \frac{1}{2} \left[ 5q_2 - q_1^L q_2 - (q_2)^2 - q_2 \right] \]

Differentiating with respect to \( q_2 \), we can obtain firm 2’s best response function, \( BRF_2(q_1^L, q_1^H) \).

- After finding the best response functions for both types of Firm 1 and for the unique type of Firm 2 we are ready to plug the first two BRFs into the latter. Specifically,

\[ q_2 (q_1^L, q_1^H) = \frac{13 - q_1^L - q_1^H}{4} = 3.25 - 0.25 \left( q_1^L + q_1^H \right) \]

which simplifies to

\[ q_2 = 3.25 - 1.625 + 0.25q_2 \]

And solving for \( q_2 \), we find \( q_2 = 2.167 \).

- With this information, it is easy to find the particular level of production for firm 1 when experiencing low market demand,

\[ q_1^L (q_2) = 2 - \frac{q_2}{2} = 2 - \frac{2.167}{2} = 0.916 \]

- As well as the level of production for firm 1 when experiencing high market demand,

\[ q_1^H (q_2) = 4.5 - \frac{q_2}{2} = 4.5 - \frac{2.167}{2} = 3.4167 \]

- Therefore, the Bayesian Nash equilibrium (BNE) of this oligopoly game with incomplete information about market demand prescribes the following production levels

\[ (q_1^H, q_1^L, q_2) = (3.416, 0.916, 2.167) \]