EconS 424: Strategy and Game Theory
Oligopoly games with incomplete information about firms' cost structure*

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Let us consider an oligopoly game where two firms compete in quantities. Market demand is given by the expression $p = 1 - q_1 - q_2$, and firms have incomplete information about their marginal costs. In particular, firm 2 privately knows whether its marginal costs are

$$MC_2 = \begin{cases} 0 \text{ with probability } 1/2 \\ 1/4 \text{ with probability } 1/2 \end{cases}$$

On the other hand, firm 1 does not know firm 2's cost structure. Firm 1's marginal costs are $MC_1 = 0$, and this information is common knowledge among the firms (firm 2 also knows it).

Find the Bayesian Nash equilibrium of this oligopoly game, specifying how much every firm produces.

Firm 2. First, let us focus on Firm 2, the informed player in this game, as we usually do when solving for the BNE of games of incomplete information.

When firm 2 has low costs, its profits are

$$Profits_2^L = (1 - q_1 - q_2^L)q_2^L = q_2^L - q_1 q_2^L \left(q_2^L\right)^2$$

Differentiating with respect to q_2^L, we can obtain firm 2's best response function when experiencing low costs, $BRF_2^L(q_1)$.

$$1 - q_1 - 2q_2^L = 0 \Longrightarrow q_2^L(q_1) = \frac{1 - q_1}{2} \qquad (BRF_2^L(q_1))$$

On the other hand, when firm 2 has high costs ($MC = \frac{1}{4}$), its profits are

$$Profits_2^H = (1 - q_1 - q_2^H)q_2^H - \frac{1}{4}q_2^H = q_2^H - q_1 q_2^H \left(q_2^H\right)^2 - \frac{1}{4}q_2^H$$

Differentiating with respect to q_2^H, we can obtain firm 2's best response function when experiencing high costs, $BRF_2^H(q_1)$.

$$1 - q_1 - 2q_2^H - \frac{1}{4} = 0 \Longrightarrow q_2^H(q_1) = \frac{\frac{3}{4} - q_1}{2} = \frac{3}{8} - \frac{q_1}{2} \qquad (BRF_2^H(q_1))$$

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Firm 1. Let us now analyze Firm 1 (the uninformed player in this game). First note that its profits must be expressed in expected terms, since firm 1 does not know whether firm 2 has low or high costs.

$$Profits_1 = \underbrace{\frac{1}{2}(1 - q_1 - q_2^L)q_1}_{\text{if low costs}} + \underbrace{\frac{1}{2}(1 - q_1 - q_2^H)q_1}_{\text{if high costs}}$$

Rearranging,

$$Profits_1 = \left(\frac{1}{2} - \frac{q_1}{2} - \frac{q_2^L}{2} + \frac{1}{2} - \frac{q_1}{2} - \frac{q_2^H}{2}\right) q_1 = \left(1 - q_1 - \frac{q_2^L}{2} - \frac{q_2^H}{2}\right) q_1$$

$$= q_1 - (q_1)^2 - \frac{q_2^L}{2} q_1 - \frac{q_2^H}{2} q_1$$

Differentiating with respect to q_1, we can obtain firm 1's best response function, $BRF_1(q_2^L, q_2^H)$. Note that we do not have to differentiate for the case of low and high costs, since firm 1 does not observe such information). In particular,

$$1 - 2q_1 - \frac{q_2^L}{2} - \frac{q_2^H}{2} = 0 \Longrightarrow q_1\left(q_2^L, q_2^H\right) = \frac{1}{2} - \frac{q_2^L}{2} - \frac{q_2^H}{2} \qquad (BRF_1(q_2^L, q_2^H))$$

- After finding the best response functions for both types of Firm 2 and for the unique type of Firm 1 we are ready to plug the first two BRFs into the latter. Specifically,

$$q_1 = \frac{1}{2} - \frac{\frac{1-q_1}{2}}{2} - \frac{\frac{3}{8} - \frac{q_1}{2}}{2}$$

  which simplifies to

$$q_1 = \frac{1}{16} + \frac{8}{16} q_1$$

  And solving for q_1, we find $q_1 = \frac{1}{8}$.

- With this information, it is easy to find the particular level of production for firm 2 when experiencing *low* marginal costs,

$$q_2^L\left(q_1\right) = \frac{1 - q_1}{2} = \frac{1 - \frac{1}{8}}{2} = \frac{7}{16}$$

- As well as the level of production for firm 2 when experiencing *high* marginal costs,

$$q_2^H\left(q_1\right) = \frac{3}{8} - \frac{\frac{1}{8}}{2} = \frac{5}{16}$$

- Therefore, the Bayesian Nash equilibrium of this oligopoly game with incomplete information about firm 2's marginal costs prescribes the following production levels

$$\left(q_1, q_2^L, q_2^H\right) = \left(\frac{3}{8}, \frac{7}{16}, \frac{5}{16}\right)$$

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Oligopoly games with incomplete information about market demand*

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Let us consider an oligopoly game where two firms compete in quantities. Both firms have the same marginal costs, $MC = \$1$, but they are asymmetrically informed about the actual state of market demand. In particular, Firm 2 does not know what is the actual state of demand, but knows that it is distributed with the following probability distribution

$$p(Q) = \begin{cases} 10 - Q \text{ with probability } 1/2 \\ 5 - Q \text{ with probability } 1/2 \end{cases}$$

On the other hand, firm 1 knows the actual state of market demand, and firm 2 knows that firm 1 knows this information (i.e., it is common knowledge among the players).

Firm 1. First, let us focus on Firm 1, the informed player in this game, as we usually do when solving for the BNE of games of incomplete information.

When firm 1 observes a high demand market its profits are

$$\begin{aligned} Profits_1 &= (10 - Q)q_1^H - 1q_1^H \\ &= (10 - q_1^H - q_2)q_1^H - q_1^H \\ &= 10q_1^H - \left(q_1^H\right)^2 - q_2q_1^H - 1q_1^H \end{aligned}$$

Differentiating with respect to q_1^H, we can obtain firm 1's best response function when experiencing low costs, $BRF_1^H(q_2)$.

$$10 - 2q_1^H - q_2 - 1 = 0 \Longrightarrow q_1^H(q_2) = 4.5 - \frac{q_2}{2} \qquad (BRF_1^H(q_2))$$

On the other hand, when firm 1 observes a low demand market its profits are

$$Profits_1 = (5 - q_1^L - q_2)q_1^L - 1q_1^L = 5q_1^L - \left(q_1^L\right)^2 - q_2q_1^L - 1q_1^L$$

Differentiating with respect to q_1^L, we can obtain firm 1's best response function when experiencing high costs, $BRF_1^L(q_2)$.

$$5 - 2q_1^L - q_2 - 1 = 0 \Longrightarrow q_1^L(q_2) = 2 - \frac{q_2}{2} \qquad (BRF_1^L(q_2))$$

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Firm 2. Let us now analyze Firm 2 (the uninformed player in this game). First note that its profits must be expressed in expected terms, since firm 2 does not know whether market demand is high or low.

$$Profits_2 = \frac{1}{2}\underbrace{\left[(10 - q_1^H - q_2)q_2 - 1q_2\right]}_{\text{demand is high}} + \frac{1}{2}\underbrace{\left[(5 - q_1^L - q_2)q_2 - 1q_2\right]}_{\text{demand is low}}$$

Rearranging,

$$Profits_2 = \frac{1}{2}\left[10q_2 - q_1^H q_2 - (q_2)^2 - q_2\right] + \frac{1}{2}\left[5q_2 - q_1^L q_2 - (q_2)^2 - q_2\right]$$

Differentiating with respect to q_2, we can obtain firm 2's best response function, $BRF_2(q_1^L, q_1^H)$. Note that we do not have to differentiate for the case of low and high demand, since firm 2 does not observe such information). In particular,

$$\frac{1}{2}\left[10 - q_1^H - 2q_2 - 1\right] + \frac{1}{2}\left[5 - q_1^L - 2q_2 - 1\right] = 0$$

Rearraging,

$$13 - q_1^H - 4q_2 - q_1^L = 0$$

And solving for q_2,

$$q_2\left(q_1^L, q_1^H\right) = \frac{13 - q_1^L - q_1^H}{4} = 3.25 - 0.25\left(q_1^L + q_1^H\right) \qquad \left(BRF_2\left(q_1^L, q_1^H\right)\right)$$

- After finding the best response functions for both types of Firm 1 and for the unique type of Firm 2 we are ready to plug the first two BRFs into the latter. Specifically,

$$q_2 = 3.25 - 0.25\left(\underbrace{\left[2 - \frac{q_2}{2}\right]}_{q_1^L} + \underbrace{\left[4.5 - \frac{q_2}{2}\right]}_{q_1^H}\right)$$

which simplifies to

$$q_2 = 3.25 - 1.625 + 0.25q_2$$

And solving for q_2, we find $q_2 = 2.167$.

- With this information, it is easy to find the particular level of production for firm 1 when experiencing *low* market demand,

$$q_1^L\left(q_2\right) = 2 - \frac{q_2}{2} = 2 - \frac{2.167}{2} = 0.916$$

- As well as the level of production for firm 1 when experiencing *high* market demand,

$$q_1^H\left(q_2\right) = 4.5 - \frac{q_2}{2} = 4.5 - \frac{2.167}{2} = 3.4167$$

- Therefore, the Bayesian Nash equilibrium (BNE) of this oligopoly game with incomplete information about market demand prescribes the following production levels

$$\left(q_1^H, q_1^L, q_2\right) = (3.416, 0.916, 2.167)$$