

Repeated games: *Overlapping generations*

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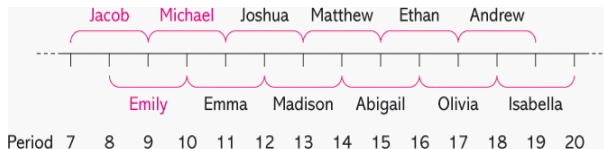
Strategy and Game Theory - Washington State University

Chapter 15

- 1 Chapter 15 (Harrington) - Cooperation in infinitely lived institutions.
- 2 So far individuals interacting in an infinitely repeated game knew there were some chances they were going to meet each other again.
 - 1 i.e., cooperation was sustained by the "shadow of the future" hanging over future encounters.
- 3 But in some cases individuals know for sure they won't see each other again.
 - 1 Why do people cooperate then?
- 4 In this chapter we will examine cooperation in institutions where
 - 1 individuals are finitely lived, but
 - 2 the institution lasts forever.

Chapter 15

- 1 An infinitely lived institution can be understood as an overlapping generations model in macroeconomics.
 - 1 That is, at any stage some people are young, some are middle-aged, some are old.
 - 2 Importantly, when the old die in the following period, the population is replenished by newborns.



- 3 Hence, the institution lives forever.
- 2 How can we sustain cooperation in these settings?

Chapter 15

① Another potentially problematic setting:

- ① People interact only one period: Businessmen A and B meet only once.
 - ② If I am businessman A, how I am going to discipline B (playing a punishment strategy, as in the GTS) if I never meet businessman B again?
 - ③ Although one person cannot discipline another, society at large might be able to perform that function.
 - ④ For example, if information about past encounters is observed by other people who will interact with businessman B in the future, they can punish him for acting improperly towards A.
- ② We will describe how to sustain cooperation in these settings.

Overlapping generations and tribal defense

- 1 Consider a nation, a village, or tribe with $N \geq 2$ members.
- 2 Each member decides:
 - 1 whether to exert effort defending the group (public project), at a private cost of 10, or
 - 2 shirk.
- 3 Every member obtains a benefit of 6 units for every individual who exerts effort.
- 4 Hence, if m members exert effort, my utility is

$$u_i(s_i, m) = \begin{cases} 6(m + \overbrace{1}^{\text{Me!}}) \overbrace{-10}^{\text{Cost}} & \text{if } s_i = \text{exert effort} \\ 6m & \text{if } s_i = \text{no effort} \end{cases}$$

Overlapping generations and tribal defense

- 1 Given utility

$$u_i(s_i, m) = \begin{cases} 6(m+1) - 10 & \text{if } s_i = \text{exert effort} \\ 6m & \text{if } s_i = \text{no effort} \end{cases}$$

it is immediate to show that exerting effort is a strictly dominated strategy.

- 2 In particular,

$$6(m+1) - 10 < 6m \iff 6m - 4 < 6m \iff -4 < 0$$

which holds for any value of m .

- 3 That is, I have incentives to free-ride (shirk) regardless of the number of individuals who end up exerting effort.
- 4 Hence, the psNE of the unrepeated game has $s_i = \text{no effort}$ for every player $i \in N$.

Overlapping generations and tribal defense

- 1 For simplicity, let's solve this game as we know so far: when players **interact infinitely often** (they never die).
- 2 In this case, we can design the following modified GTS:
 - 1 At $t = 1$, exert effort (cooperate)
 - 2 At $t > 1$, exert effort if all players exerted effort in all previous periods...
 - 1 but temporarily revert to no effort for one period if any player deviates from exerting effort in previous periods.
 - 2 Then, after one period of reversion (punishment), go back to the cooperative outcome, i.e., exert effort.

Overlapping generations and tribal defense

- ① After a history of cooperation, my payoff if I keep cooperating is:

$$(6N - 10) + \delta(6N - 10) + \delta^2(6N - 10) + \dots$$

- ② While my payoff from deviating to no effort is:

$$\underbrace{6(N - 1)}_{\substack{\text{you are not coop} \\ \text{while all other } (N-1) \\ \text{members cooperate}}} + \underbrace{\delta 0}_{\text{punishment in psNE}} + \underbrace{\delta^2(6N - 10)}_{\text{go back to coop}} + \dots$$

Overlapping generations and tribal defense

- 1 Comparing these payoffs, cooperation can be sustained as the SPNE of the infinitely repeated game if:

$$\begin{aligned} & (6N - 10) + \delta(6N - 10) + \delta^2(6N - 10) + \dots \\ \geq & 6(N - 1) + \delta 0 + \delta^2(6N - 10) + \dots \end{aligned}$$

Rearranging,

$$6N - 10 + \delta(6N - 10) \geq 6N - 6$$

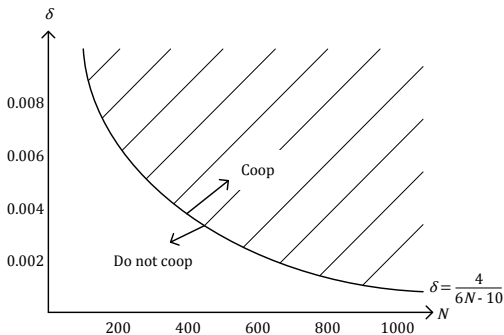
Hence,

$$\delta \geq \frac{4}{6N - 10}$$

- 2 Figure of this cutoff for δ (next slide)

Overlapping generations and tribal defense

- Minimal discount factor supporting cooperation in the Overlapping Generation-Tribal defense game, as a function of the population size, N



- Cooperation is easier to sustain the larger the population is.

Overlapping generations and tribal defense

- 1 But, what if players **do not interact infinitely often**?
- 2 You live during T periods only, and there are N members in total.
- 3 At any period T , there are $\frac{N}{T}$ members currently alive in this generation T .
 - 1 *Example:* $N = 100$ and $T = 4$ years, then $\frac{N}{T} = \frac{100}{4} = 25$ members are children, 25 are teenagers, 25 are adults, and 25 are seniors.

Overlapping generations and tribal defense

- 1 Then, at any period,

$$N - \frac{N}{T} \text{ people are younger than age } T$$

$$= \frac{NT - N}{T} = \left(\frac{T - 1}{T} \right) N$$

- 1 In the previous example where $N = 100$ and $T = 4$ years, $\left(\frac{T-1}{T} \right) N = \left(\frac{3-1}{4} \right) 100 = 75$ individuals are younger than the maximum age any member in the population reaches.
- 2 In particular, 25 members are children, 25 are teenagers, and 25 are adults.

Overlapping generations and tribal defense

- 1 Let us now analyze how to support cooperation in this setting.
- 2 Consider the following strategy
 - 1 At the *last* period of your life (period T), you don't exert any effort (e.g., retirement for seniors).
 - 1 How would I be disciplined otherwise? When we meet them in the afterlife?
 - 2 During all previous $T - 1$ periods, you exert effort, but if someone deviates from this strategy:
 - 1 you revert to the psNE of the stage game during one period (temporary punishment), and
 - 2 move to the cooperative outcome (exerting effort) afterwards.

Overlapping generations and tribal defense

- 1 In order to show that such strategy can be sustained as SPNE of the game, we must show that it is optimal for:
 - 1 the individual who is in the last period (T) of his life (of course!).
 - 2 the individual who is in the penultimate period ($T - 1$) of his life.
 - 3 the individual who is in period $T - 2$ of his life.
 - 4 the individual who is in period $T - 3$ of his life, etc.

Overlapping generations and tribal defense

- Payoff in penultimate period of life, i.e., $T - 1$:
- Payoff from cooperating:

$$\underbrace{\left[6 \underbrace{\left(\frac{T-1}{T} \right) N - 10}_m \right]}_{\text{effort}} + \delta \underbrace{\left[6 \underbrace{\left(\frac{T-1}{T} \right) N}_m \right]}_{\substack{\text{no effort, he is a senior} \\ \text{but } m \text{ is unaffected,} \\ \text{thanks to the newborns!}}$$

- Payoff from deviating:

$$6 \left[\underbrace{\left(\frac{T-1}{T} \right) N - 1}_m, \text{ without you} \right] + \underbrace{\delta \cdot 0}_{\substack{\text{punished during} \\ \text{retirement!}}}$$

Overlapping generations and tribal defense

- Comparing,

$$6 \left(\frac{T-1}{T} \right) N - 10 + \delta \left[6 \left(\frac{T-1}{T} \right) N \right] \geq 6 \left(\frac{T-1}{T} \right) N - 6$$

$$\delta \geq \frac{4}{6 \left(\frac{T-1}{T} \right) N} = \frac{2}{3 \left(\frac{T-1}{T} \right) N} \quad (\text{Condition 1})$$

Overlapping generations and tribal defense

- Player who is still two periods from retirement, i.e., $T - 2$ (Teenager):
- Payoff from cooperation:

$$\underbrace{\left[6 \left(\frac{T-1}{T} \right) N - 10 \right]}_{\text{effort as a teenager}} + \delta \underbrace{\left[6 \left(\frac{T-1}{T} \right) N - 10 \right]}_{\text{effort as an adult}} + \delta^2 \underbrace{\left[6 \left(\frac{T-1}{T} \right) N \right]}_{\text{no effort as a senior}}$$

- Payoff from deviating to no effort:

$$\underbrace{6 \left[\left(\frac{T-1}{T} \right) N - 1 \right]}_{\text{I shirk as a teenager...}} + \underbrace{\delta \cdot 0}_{\text{punished as an adult...}} + \delta^2 \underbrace{\left[6 \left(\frac{T-1}{T} \right) N \right]}_{\text{but enjoy life as a senior!}}$$

Overlapping generations and tribal defense

- Let's compare the payoffs.
 - First, note that last period payoffs were the same. Hence, we don't even write them in our payoff comparison.

$$\cancel{6 \left(\frac{T-1}{T} \right) N - 10} + \delta \left[6 \left(\frac{T-1}{T} \right) N - 10 \right] \geq \cancel{6 \left(\frac{T-1}{T} \right) N - 6}$$

$$\implies \delta \left[6 \left(\frac{T-1}{T} \right) N - 10 \right] \geq 10 - 6$$

$$\implies \delta \geq \frac{4}{6 \left(\frac{T-1}{T} \right) N - 10}$$

$$= \frac{2}{3 \left(\frac{T-1}{T} \right) N - 5} \quad \text{(Condition 3)}$$

Overlapping generations and tribal defense

- Similarly for individuals in previous periods, e.g., $T - 3$:
- Payoff from cooperation:

$$\begin{aligned} & \underbrace{\left[6 \left(\frac{T-1}{T} \right) N - 10 \right]}_{\text{effort as a child}} + \delta \underbrace{\left[6 \left(\frac{T-1}{T} \right) N - 10 \right]}_{\text{effort as a teenager}} \\ & + \delta^2 \underbrace{\left[6 \left(\frac{T-1}{T} \right) N - 10 \right]}_{\text{effort as an adult}} + \delta^3 \underbrace{\left[6 \left(\frac{T-1}{T} \right) N \right]}_{\text{no effort as a senior}} \end{aligned}$$

Overlapping generations and tribal defense

- Payoff from deviating to no effort:

$$\begin{aligned} & \underbrace{6 \left[\left(\frac{T-1}{T} \right) N - 1 \right]}_{\text{I shirk as a child}} + \underbrace{\delta \cdot 0}_{\text{punished as a teenager}} \\ & + \delta^2 \underbrace{\left[6 \left(\frac{T-1}{T} \right) N - 10 \right]}_{\text{effort as an adult}} + \delta^3 \underbrace{\left[6 \left(\frac{T-1}{T} \right) N \right]}_{\text{no effort as a senior}} \end{aligned}$$

Overlapping generations and tribal defense

- Comparing
 - Note that the last two period payoffs were the same. Hence, we don't need to write it down in our payoff comparison.

$$\cancel{6 \left(\frac{T-1}{T} \right) N - 10} + \delta \left[\cancel{6 \left(\frac{T-1}{T} \right) N - 10} \right] \geq \cancel{6 \left(\frac{T-1}{T} \right) N - 6}$$

$$\implies \delta \left[\cancel{6 \left(\frac{T-1}{T} \right) N - 10} \right] \geq 10 - 6$$

$$\implies \delta \geq \frac{4}{6 \left(\frac{T-1}{T} \right) N - 10} = \frac{2}{3 \left(\frac{T-1}{T} \right) N - 5}$$

- (Coincides with our above Condition 2)

Overlapping generations and tribal defense

- ① In sum, this strategy profile is a SPNE if both conditions

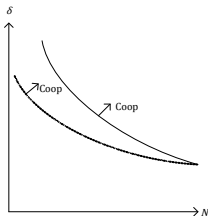
$$\underbrace{\delta \geq \frac{2}{3 \left(\frac{T-1}{T}\right) N - 5}}_{\text{Condition 2}} \quad \text{and} \quad \underbrace{\delta \geq \frac{2}{3 \left(\frac{T-1}{T}\right) N}}_{\text{Condition 1}} \quad \text{hold}$$

- ② But note that one condition is more restrictive than another one since...

$$\delta \geq \frac{2}{3 \left(\frac{T-1}{T}\right) N - 5} > \frac{2}{3 \left(\frac{T-1}{T}\right) N}$$

Overlapping generations and tribal defense

- Plotting both cutoffs for different values of N , we obtain:



- Solid Line:** Cutoff for the player in her $T - 2$ period of life

$$\delta \geq \frac{2}{3 \left(\frac{T-1}{T} \right) N - 5} \quad (\text{Teenager})$$

- Dashed Line:** Cutoff for the player in her $T - 1$ period of life

$$\delta \geq \frac{2}{3 \left(\frac{T-1}{T} \right) N} \quad (\text{Adult})$$

Overlapping generations and tribal defense

1 Intuition:

- 1 the temptation to cheat is weaker for someone in her penultimate period of life, because...
 - 2 cheating today would result in her foregoing the "retirement benefit" of $6 \left(\frac{T-1}{T} \right) N$ in the following period (her retirement years).
- 2 In other words, the real challenge is inducing people to sacrifice when they are further away from receiving their retirement benefit.
 - 1 In our model, this implied that the condition to induce an individual to cooperate in period $T - 2$, i.e., $\delta \geq \frac{2}{3 \left(\frac{T-1}{T} \right) N - 5}$,
 - 2 was more demanding than the similar condition for an individual in period $T - 1$, i.e., $\delta \geq \frac{2}{3 \left(\frac{T-1}{T} \right) N}$.

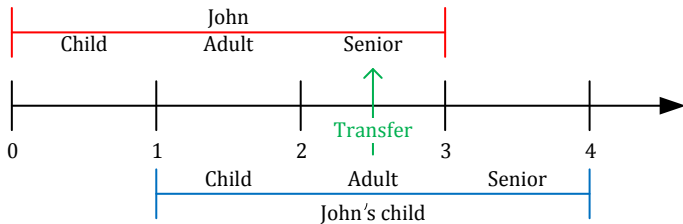
Overlapping generations and tribal defense

- 1 Cooperation can then be supported as a SPNE of the infinitely repeated game:
 - 1 even if agents do not live forever,
 - 2 but the institution is infinitely lived, so that younger individuals entering the population can punish players who previously defected.
- 2 *Check your understanding* exercise 15.1:
 - 1 Same exercise as tribal defense, but...
 - 2 suppose that punishment lasts as long as the lifetime of the person who shirks.
 - 3 That is, if a person shirks in period t of her life (when she was supposed to work), then everyone shirks for the rest $T - t$ periods.
 - 4 Find the conditions on δ that sustain cooperation.

Taking care of elderly parents

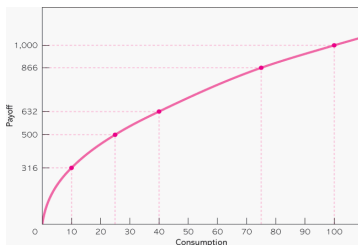
- 1 Let us now consider a variation in the above OLG model.
- 2 People live for 3 stages: youth, adult and senior.
- 3 People only generate income as adults, for an amount of \$100.
 - 1 and they have a child.
- 4 They cannot generate any income as seniors, and therefore they rely on the generosity (transfers) of adults.
 - 1 For simplicity, we assume that grandchildren cannot make intergenerational transfers to their grandparents!
- 5 How can cooperation be sustained in the SPNE of the game?

Taking care of elderly parents



Taking care of elderly parents

- 1 Before we proceed with a particular strategy, we also consider that utility is concave in money...



suggesting that additional amounts of money provide smaller increments in utility, e.g., $u(x) = 100 \cdot \sqrt{x}$

Taking care of elderly parents

- 1 Consider the following strategy:
 - 1 Transfer \$25 to your elderly parent if she helped her parents before, but...
 - 2 Transfer \$0 to your elderly parent if she didn't help her parents before.
- 2 The essence of this intergenerational norm is that:
 - 1 a person has an obligation to take care of a parent, **unless** that parent was negligent with respect to his or her parent, in which case neglect is the punishment.

Taking care of elderly parents

- ① If I cooperate (sticking to this intergenerational norm) my payoffs are

$$866 + \delta 500$$

- ① where 866 is my utility after transferring \$25 to my elderly parents, i.e., utility from $\$100 - \$25 = \$75$ ($100 \cdot \sqrt{75} = 866$),
- ② and 500 is the utility from the \$25 that my children will give me tomorrow (when I become an elderly, $100 \cdot \sqrt{25} = 500$).
- ② If, in contrast, I deviate (making no transfers to my elderly parents today), my payoffs are

$$1,000 + \delta 0$$

- ① where 1,000 is the utility from keeping all my income (\$100) without making any transfer ($100 \cdot \sqrt{100} = 1000$), and
- ② and 0 represents that I won't be receiving any transfer from my children (since my kids observe I was negligent with their grandpa).

Taking care of elderly parents

- 1 Comparing these payoffs, cooperation can be sustained in the SPNE if

$$866 + \delta 500 \geq 1,000 + \delta 0$$

and solving for δ , we obtain

$$\delta \geq \frac{134}{500} = 0.268$$

Taking care of elderly parents

Conclusions:

- 1 When there is no inheritance to act as a lure, the elderly parent cannot punish the adult for failing to take care of him.
- 2 In this context, the disciplining device lies not with the elderly parent, but with her grandchild!
- 3 Elderly parents are taken care of "even by the selfish child," since otherwise they will be punished by their own children later on.

Cooperation in large populations

- 1 Let us now move to the second question in this chapter:
 - 1 How to support cooperation when players interact only once?
 - 2 *Example:* eBay
- 2 Buyers and sellers have incentives to be fraudulent since they will rarely meet again.
- 3 How to promote cooperation in this setting?
 - 1 Feedback system.

- 1 Let's start with a description of the game.
- 2 Consider a seller who can sell three types of goods

True Quality	Seller's Cost	Buyer's Value
Excellent	13	30
Very good	8	15
Shoddy	2	0

at only three possible prices: \$5, \$10 and \$20.

- 3 Before clicking on "Buy It Now" the buyer observes the price and the seller's feedback score.
- 4 If the buyer chooses not to buy, his payoff is zero.

- 1 If the buyer buys the product, payoffs are

True Quality	Price	Seller's Cost	Seller's Payoff	Buyer's Value	Buyer's Payoff
Excellent	20	13	7	30	10
Excellent	10	13	-3	30	20
Excellent	5	13	-8	30	25
Very good	20	8	12	15	-5
Very good	10	8	2	15	5
Very good	5	8	-3	15	10
Shoddy	20	2	18	0	-20
Shoddy	10	2	8	0	-10
Shoddy	5	2	3	0	-5

- 2 **Example:** a good of excellent quality sold at a price of \$20, provides a net payoff of $20-13=7$ to the seller, and a net payoff of $30-20=10$ to the buyer.

- 1 There are an infinite number of periods, but a particular buyer and seller meet only once.
- 2 Consider the following strategy:
- 3 **Seller:**
 - 1 If I don't have negative comments, then choose *Excellent* quality and charge a price of \$20.
 - 2 If I have one negative comment, then choose *Very good* quality and charge a price of \$10.
 - 3 If I have two or more negative comments, then choose *Shoddy* quality and charge a price of \$5.

① Buyer's buying strategy:

- ① If the seller doesn't have negative comments, then Buy.
- ② If the seller has one negative comment, then Buy only if the price is 10 or lower.
- ③ If the seller has two or more negative comments, then Don't buy.

② Buyer's feedback strategy (in case she buys):

- ① Provide positive feedback if:
 - ① the quality of the product was Excellent, or
 - ② the quality of the product was Very good and its price was 10 or lower.
- ② Provide negative feedback if:
 - ① the quality of the product was Very good but the price was \$20, or
 - ② the quality of the product was Shoddy.

- ① Given the above strategy, the buyer expects:
 - ① Excellent quality from a seller with **no** negative comments,
 - ② Very good quality from a seller with **only one** negative comment, and
 - ③ Shoddy quality from a seller with **two or more** negative comments.
- ② Let's start checking that this strategy is optimal for the buyer, then we will move to the seller.

1 Checking the Buyer's buying strategy:

- 1 If the seller has no negative feedback, then the buyer expects the good to be of Excellent quality, and
- 2 therefore buys regardless of price (see table).

True Quality	Price	Seller's Cost	Seller's Payoff	Buyer's Value	Buyer's Payoff
Excellent	20	13	7	30	10
Excellent	10	13	-3	30	20
Excellent	5	13	-8	30	25
Very good	20	8	12	15	-5
Very good	10	8	2	15	5
Very good	5	8	-3	15	10
Shoddy	20	2	18	0	-20
Shoddy	10	2	8	0	-10
Shoddy	5	2	3	0	-5

1 Checking the Buyer's buying strategy:

- 1 If the seller has only one negative comment, then the buyer expects the good to be of Very good quality, and
- 2 he should buy only if the price is \$10 or lower (see table).

True Quality	Price	Seller's Cost	Seller's Payoff	Buyer's Value	Buyer's Payoff
Excellent	20	13	7	30	10
Excellent	10	13	-3	30	20
Excellent	5	13	-8	30	25
Very good	20	8	12	15	-5
Very good	10	8	2	15	5
Very good	5	8	-3	15	10
Shoddy	20	2	18	0	-20
Shoddy	10	2	8	0	-10
Shoddy	5	2	3	0	-5

1 Checking the Buyer's buying strategy:

- 1 If the seller has two or more negative comments, the buyer expects the good to be of Shoddy quality (zero value), and
- 2 he does not buy, regardless of the price (see table).

True Quality	Price	Seller's Cost	Seller's Payoff	Buyer's Value	Buyer's Payoff
Excellent	20	13	7	30	10
Excellent	10	13	-3	30	20
Excellent	5	13	-8	30	25
Very good	20	8	12	15	-5
Very good	10	8	2	15	5
Very good	5	8	-3	15	10
Shoddy	20	2	18	0	-20
Shoddy	10	2	8	0	-10
Shoddy	5	2	3	0	-5



① **Checking the Buyer's feedback strategy:**

- ① Since providing feedback is assumed to be costless...
- ② it is optimal for the buyer to provide truthful feedback.
- ③ (We will comment on this later on).

① Checking the Seller's strategy:

- ① When the seller has **two or more negative comments**, he can anticipate that the buyer:
 - ① will infer that the good is of Shoddy quality, and hence won't buy, regardless of the quality the seller reports and regardless of his pricing strategy.
 - ② Then offering Shoddy quality (as prescribed) is as good as offering any other type, since the seller won't be able to sell any unit.

① Checking the Seller's strategy:

- ① When the seller has **one negative comment**, the buyer anticipates him to offer Very good quality.
 - ① If he offers this quality at an equilibrium price of \$10, his profit is \$2 (see table), entailing a positive comment from this buyer.
 - ② In this case, he can anticipate earning a profit stream of 2, i.e., $\frac{2}{1-\delta}$.
 - ③ By instead charging a price of \$5, he still makes the sale but obtaining lower profits.
 - ④ By instead charging a price of \$20, he doesn't make the sale and gets zero profit. (Neither option is interesting)

1 Checking the Seller's strategy:

- 1 When the seller has one negative comment (continues):

True Quality	Price	Seller's Cost	Seller's Payoff	Buyer's Value	Buyer's Payoff
Excellent	20	13	7	30	10
Excellent	10	13	-3	30	20
Excellent	5	13	-8	30	25
Very good	20	8	12	15	-5
Very good	10	8	2	15	5
Very good	5	8	-3	15	10
Shoddy	20	2	18	0	-20
Shoddy	10	2	8	0	-10
Shoddy	5	2	3	0	-5

1 Checking the Seller's strategy:

- 1 When the seller has one negative comment (continues):
 - 1 The only interesting deviation is to offering Shoddy quality at a price of \$10.
 - 2 This raises his profit today to \$8 (see table), but...
 - 3 at the expense of increasing the number of negative comments to two, yielding no sales thereafter.
 - 4 Hence, this seller is willing to act as prescribed if

$$\frac{2}{1-\delta} \geq 8 \iff \delta \geq \frac{3}{4}$$

1 Checking the Seller's strategy:

- 1 When the seller has one negative comment (continues):

True Quality	Price	Seller's Cost	Seller's Payoff	Buyer's Value	Buyer's Payoff
Excellent	20	13	7	30	10
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A green arrow points to the 'Very good' row with a price of 5. A green box highlights the 'Very good' and 'Shoddy' rows. A green circle highlights the 'Seller's Payoff' value of 2 for the 'Very good' row with a price of 10, and another green circle highlights the 'Seller's Payoff' value of 8 for the 'Shoddy' row with a price of 10. A green arrow points from the circle around 2 to the circle around 8.

1 Checking the Seller's strategy:

- 1 Let us now examine the seller with **no negative comments**:
 - 1 Equilibrium prescribes him offering Excellent quality at a price of \$20, yielding a profit of 7 today.
 - 2 Good reputation is maintained, yielding a stream of \$7 profits thereafter, i.e., $\frac{7}{1-\delta}$.
 - 3 The best deviation is to a Shoddy quality, with profits of 18 (since both Shoddy and Very good trigger a negative comment from the current customer).
 - 4 Such negative comment makes the seller move to a situation similar to that analyzed above (with one negative comment) with payoffs $\frac{2}{1-\delta}$.
 - 5 Hence, he behaves as prescribed if

$$\frac{7}{1-\delta} \geq 18 + \delta \frac{2}{1-\delta} \iff \delta \geq \frac{11}{16}$$

1 Checking the Seller's strategy:

- 1 When the seller has one negative comment (continues):

True Quality	Price	Seller's Cost	Seller's Payoff	Buyer's Value	Buyer's Payoff
Excellent	20	13	7	30	10
Excellent	10	13	-3	30	20
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Very good	5	8	-3	15	10
Shoddy	20	2	18	0	-20
Shoddy	10	2	8	0	-10
Shoddy	5	2	3	0	-5

Annotations: Green arrows point to the first and last rows of the table. A green circle highlights the Seller's Payoff of 7 in the first row, and another green circle highlights the Seller's Payoff of 18 in the eighth row. A green arrow points from the 7 to the 18.

- 1 Hence, this strategy profile is an equilibrium if both $\delta \geq \frac{3}{4} = 0.75$ and $\delta \geq \frac{11}{16} \simeq 0.68$ hold.
- 2 But since $\delta \geq \frac{3}{4} = 0.75$ is more restrictive than $\delta \geq \frac{11}{16} \simeq 0.68$...
 - 1 we can simply say that this strategy profile can be sustained in the SPNE of the game if $\delta \geq \frac{3}{4}$.
- 3 **Intuition:**
 - 1 The feedback score allows the population of buyers to have a "collective memory" so that any of them can learn how a seller behaved in past transactions.
 - 2 The punishment to the seller for misbehaving is therefore provided by future buyers.
 - 3 It is the prospect of those future sales that deters a seller from cheating buyers.