# Screening models in the labor market

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*EconS 503 - Advanced Microeconomics II* - Washington State University

# Screening

- Workers' types are still unobservable.
- Firms often a menu of contracts $(w_H, t_H)$ and $(w_L, t_L)$ where w denotes wages and t represents the task assigned to the worker (we assume the task to

$$u(w, t|\ \theta) = w - c(t, \theta)$$

# Screening

- Similarly as in previous section, $c(0, \theta) = 0$, and

$$\begin{aligned} c_t(t, \theta) &> 0 \\ c_{tt}(t, \theta) &> 0 \end{aligned}$$

- positive and increasing marginal costs from the task and

$$\begin{aligned} c_\theta(t, \theta) &< 0 \\ c_{t\theta}(t, \theta) &< 0 \rightarrow S.C.C \end{aligned}$$

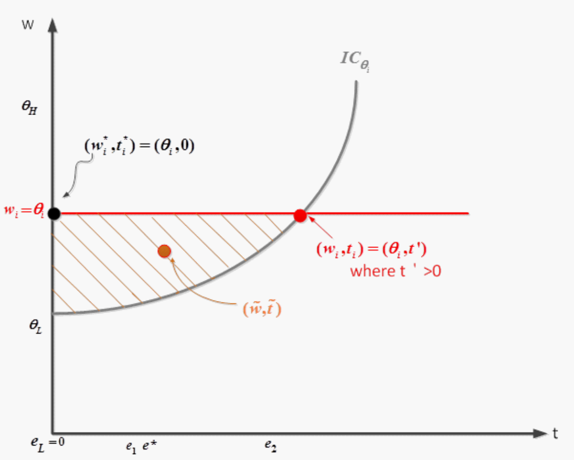
# Benchmark-Observable Types

In any SPNE of the screening game, firms offer $(w_i^*, t_i^*) = (\theta_i, 0)$ to an type-$\theta$ worker and firms earn no profits.

**Proof:**

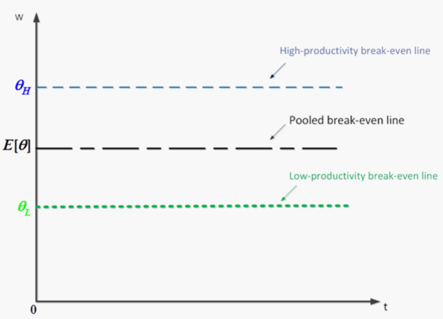
- For a given $t_i = 0$, $w_i > \theta_i$ would lead to losses and $w_i < \theta_i$ would lead to profits (other firms could offer $w_i + \epsilon$)
- For a given $w_i = \theta_i$ any $t_i^{'} > 0$ cannot be part of the equilibrium, as any competing firm could offer a $(w, t)$-pair in the shaded region, which would be accepted by the worker, such as point $(\tilde{w}, \tilde{t})$ in the following figure.

# Benchmark-Observable Types



# Unobservable Types

- Let us start depicting the break-even lines of contracts, $(w, t)$-pairs, that would yield zero profits if they attract: only high-productive workers, only low-productive workers, or both types of workers.



# Unobservable Types

- **Firms make no profits.**
- **Proof (1st part, Separating).**
- Contracts $(w_L, t_L) \neq (w_{,H}, t_H)$ induce separation. If a firm obtains positive profits, e.g., $w_H < \theta_H$ and $w_L < \theta_L$, then another firm could offer a new pair of contracts $(w_L + \epsilon, t_L)$ and $(w_H + \epsilon, t_H)$. These new contracts are accepted by all low-productive workers and all high-productive workers, respectively. In addition, since $\epsilon \to 0$, such contract offer is profitable for the firm.
- However, a similar argument would apply for rival firms, successively approaching wages to marginal productivity, i.e., $w_i = \theta_i$ for all types $i = H, L$.
  - Ultimately, firms make no profits.

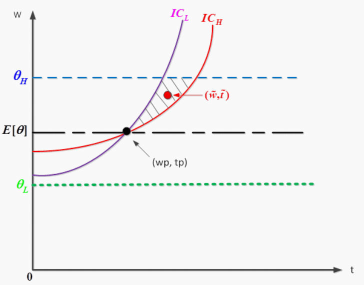
**Proof (2nd part, Pooling):**

- Contracts
$$(w_L, t_L) = (w_H, t_H) = (w_p, t_p)$$
and induce a pooling of all workers (same argument as above, but with a unique contract that attracts all workers). That is, any firm could offer $(w_p + \epsilon, t_p)$, attract all workers, and make larger profits (by making $\epsilon \to 0$).
- Then no firm makes profits and $(w_p, t_p)$ must lies on the pooling break-even line.

# Unobservable Types

No pooling equilibrium exists.

**Proof (easy):** By contradiction, assume a pooling equilibrium contract $(w_p, t_p)$ exists, as in the next figure.




Either firm could deviate by offering a contract on the shaded area, such as $(\tilde{w}, \tilde{t})$, which only attracts the high-productive worker, and allows a profit margin of $(\theta_H - \tilde{w}) > 0$.

# Unobservable types

- Summary of what we did thus far:
  - Firms make no profits.
  - No pooling equilibrium exists.
  - Let's then analyze the separating equilibrium next.

# Separating SPNE-Salaries

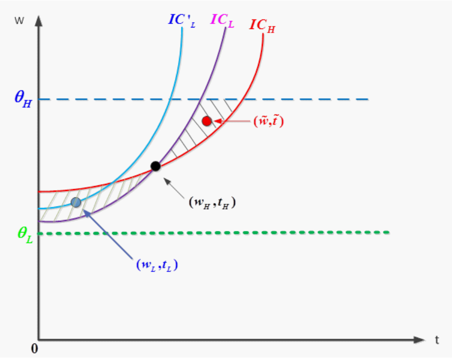
- In the SPNE, contracts $(w_H, t_H)$ and $(w_L, t_L)$ must satisfy $w_H = \theta_H$ and $w_L = \theta_L$, yielding zero profits. (we will analyze tasks later)

**Proof:**

- *(Low types)* If a firm offers $w_L < \theta_L$, then other firms can earn profits by offering $(w_L, t_L) = (\tilde{w}, t_L)$ where $\theta_L > \tilde{w} > w_L$.
  - All low-ability workers accept it, leaving a positive margin to the firm since $\tilde{w} < \theta_L$.
  - However, this cannot be an equilibrium as other firms would have the incentives to further increase $\tilde{w}$ closer to $\theta_L$, ultimately stopping at exactly $w_L = \theta_L$, as we needed to show.

# Separating SPNE-Salaries

**Proof (cont'd):** *(High types)* If a firm offers $w_H < \theta_H$, then $(w_L, t_L)$ contract must lie on the dashed LHS region of the next figure: this guarantee that L-type doesn't profitably deviate to $(w_H, t_H)$, nor H-type is tempted to choose $(w_L, t_L)$.



# Separating SPNE-Salaries

- However, any firm could offer a contract such as $(\tilde{w}, \tilde{t})$ on the shaded area. By doing so, it attracts all H-type workers and none of the L-types.
- This argument applies for all $w_H < \theta_H$, implying that $w_H = \theta_H$.
  - That is, once $w_H = \theta_H$ we cannot find other contracts that are preferable for the H-type and disliked by the L-type.

# Separating SPNE-Tasks

- So far we identified that that wages satisfy

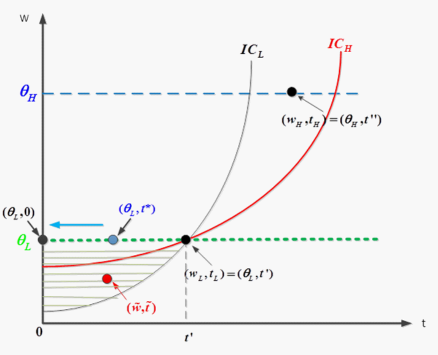
$$w_H = \theta_H \quad \text{and} \quad w_L = \theta_L,$$

- But what about the tasks $t_H$ and $t_L$ in contracts $(w_H, t_H)$ and $(w_L, t_L)$?

# Separating SPNE-Tasks

**Low type** $(w_L, t_L) = (\theta_L, 0)$

- We are just claiming that $t_L = 0$ since we already knew that $w_L = \theta_L$.
- Let's prove it by contradiction: Can we have $t_L = t' > 0$?



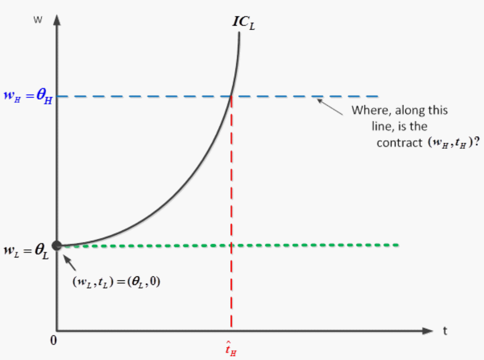
# Separating SPNE-Tasks

- If a firm offers $(w_L, t_L) = (\theta_L, t')$, other firms could attract all low-productive workers by offering a contract on the shaded region, such as $(\tilde{w}, \tilde{t})$, and obtain positive profits from all workers (low or high ability).
- Strictly speaking, once we proved $w_L = \theta_L$, the above argument would imply that competing firms would offer a contract $(\tilde{w}, \tilde{t}) = (\theta_L, t^*)$, where $t^* < t'$, along the horizontal line of $w_L = \theta_L$ moving leftward.
  - This argument holds until you reach the axis, i.e., $(w_L, t_L) = (\theta_L, 0)$.
  - Hence, the low-productive worker receives the same contract $(w_L, t_L) = (\theta_L, 0)$ as under complete information (observable types, our initial benchmark).

# Separating SPNE-Tasks

## High type

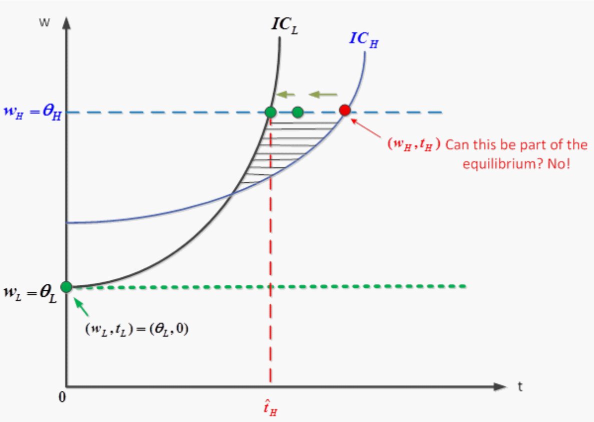
- Once we determined $(w_L, t_L) = (\theta_L, 0)$ for the low types, and the salary $w_H = \theta_H$ to the high type, we only need to find his task $t_H$.



# Separating SPNE-Tasks

- Any contract $(w_H, t_H) = (\theta_H, \hat{t_H})$, or with $t_H \geq \hat{t_H}$, prevents L-types from choosing it.
- Any contract with $w_H = \theta_H$ but with $t_H > \hat{t_H}$ cannot be part of the equilibrium:
  - Competing firms could offer a contract $(w_H, t_H) = (\tilde{w}, \tilde{t})$, as in the shaded area of the next figure, attracting only high-productive workers and making positive profits.

# Separating SPNE-Tasks




Hence, only $t_H = \hat{t_H}$ can be part of the separating equilibrium.

# Separating SPNE-Tasks

- Reductions in the task until $\hat{t_H}$ attract all high-productive workers.
  - Firms' competition will thus successively reduce $t_H$ until $\hat{t_H}$.
- We cannot move below $\hat{t_H}$ (left of this cutoff).
  - Otherwise, both high- and low-productive workers would be attracted, thus not achieving separation (self-selection).

# Separating SPNE - Summary

Therefore, we can summarize the separating SPNE as follows:

- Firms offer the menu of contracts
  - $(w_L, t_L) = (\theta_L, 0)$ and
  - $(w_H, t_H) = (\theta_H, \hat{t_H})$, where $\hat{t_H}$ solves

$$\theta_H - c(\hat{t_H}, \theta_L) = \theta_L - c(0, \theta_L).$$

  - [As a remark, this condition can be further simplified to $\theta_H = \theta_L + c(\hat{t_H}, \theta_L)$ since $c(0, \theta) = 0$ for all $\theta$ by assumption.]
- Low-productive workers accept contract $(w_L, t_L) = (\theta_L, 0)$.
- High-productive workers accept contract $(w_H, t_H) = (\theta_H, \hat{t_H})$.

# Separating SPNE - Summary

**Practice**:
If $c(t,\theta) = \frac{t^2}{\theta}$, where $\theta = \{1,2\}$, we can easily find contracts $(w_L, t_L) = (1,0)$ and $(w_H, t_H) = (2, \hat{t_H})$, where the task $\hat{t_H}$ is found with

$$\theta_H - c(\hat{t_H}, \theta_H) = \theta_L - c(0, \theta_L)$$

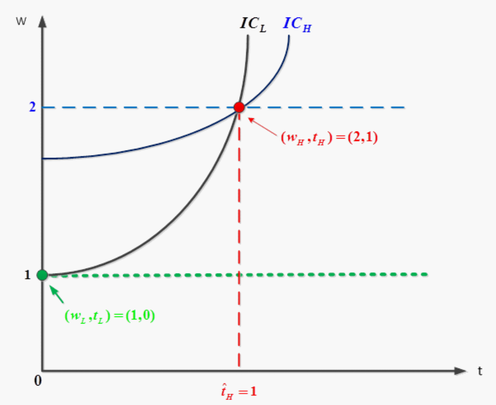
since $c(0,\theta) = 0$ for all $\theta$,

$$2 - \frac{(\hat{t_H})^2}{2} = 1 - 0 \Longleftrightarrow 4 - (\hat{t_H})^2 = 2$$

$$\Longleftrightarrow 2 = (\hat{t_H})^2 \Leftrightarrow \hat{t_H} = \sqrt{2} \cong 1.42$$

# Separating SPNE-Tasks

Summary of our numerical example:




Plese note that $\hat{t_H}$ should be placed at $\hat{t_H} = 1.42$, not at $\hat{t_H} = 1$.

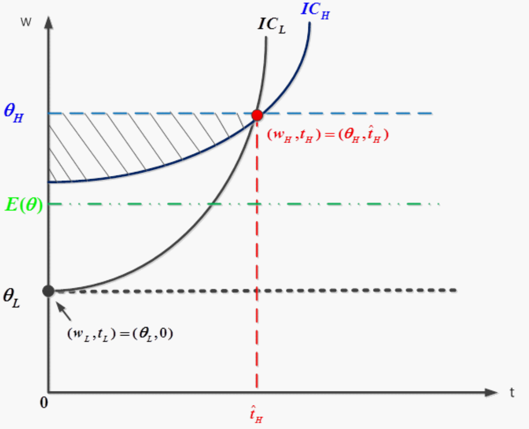
# Separating SPNE - When does it exist?

- When the horizontal intercept's of the high-productive worker's indifference-curve lies above $E(\theta)$.
  - That is, if his utility function is $u_H = w_H - c(t_H, \theta_H)$, solving for $w_H$ we obtain

$$w_H = u_H + c(t_H, \theta_H),$$

    and since $c(0, \theta_H) = 0$ by assumption, the horizontal intercept is $u$.
  - We hence need that $u_H > E(\theta)$.
- Why do we need this condition (see next figure)?

# Separating SPNE - When does it exist?



# Separating SPNE - When does it exist?

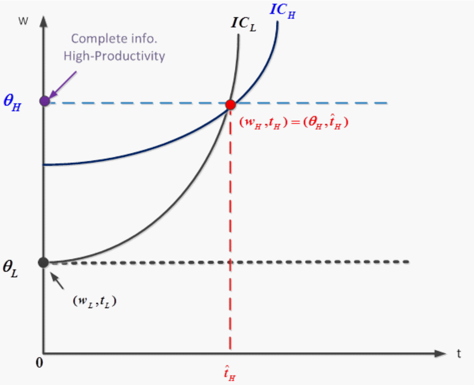
- Firms cannot increase their profits by offering an alternative contract different from $(w_L, t_L)$ and $(w_H, t_H)$, attracting only high- or only low-productivity workers
  - Let's look at the above figure.
- However, firms could attract *both* types of workers, by offering $(w, t)$-pairs in the shaded region.
  - But doing so is only profitable for the firm if $w < E(\theta)$, which is not the case here.
  - How would that figure look like?
  - In that case, a separating SPNE does not exist, but a pooling SPNE does.

# Payoff Comparision relative to Complete Information

- We can now compare the equilibrium payoff that each type of worker obtains in this setting (incomplete information with the firm using screening to distinguish workers' types) against two benchmarks:
  - **Complete information**, where the firm can observe workers' types, implying $w_L = \theta_L$ and $w_H = \theta_H$; and
  - **Incomplete information without screening** (or if screening was banned).

# Payoff Comparision relative to Complete Information

- *Low-productivity worker*: under complete info, he would receive $w_L = \theta_L$ and no need of doing unproductive tasks. Hence, he is as well-off as under the separating SPNE.

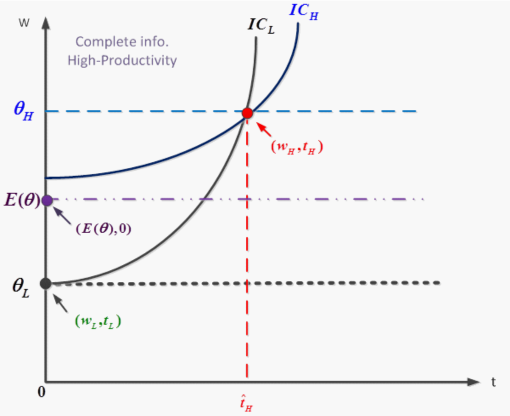


# Payoff Comparision relative to Complete Information

- *High-productivity worker*:
  - Under complete info, he would receive $w_H = \theta_H$ and no need to execute unproductive tasks.
  - His utility level would be higher than in the separating SPNE, with an indifference curve passing through point $(\theta_H, 0)$ on the vertical axis.
  - That is, high-ability workers engage in unproductive tasks simply to separate themselves from low-ability workers.

# Payoff Comparision relative to No Screening

Under no screening, uninformed firms have to offer a unique contract $(w, t) = (E[\theta], 0)$, on the vertical axis of the next figure.



# Payoff Comparision relative to No Screening

- *Low-productivity worker*: he is better off if screening is not available.
- *High-productivity worker*: he is worse off if screening is not available.

Alternatively, screening must make the high-ability worker better-off, otherwise a separating PBE would not exist.