EconS 301
Review Session #6 – Chapter 8: Cost Curves

8.12. Consider a production function with two inputs, labor and capital, given by Q = $(\sqrt{L} + \sqrt{K})^2$. The marginal products associated with this production function are as follows:

$$MP_L = \left[L^{\frac{1}{2}} + K^{\frac{1}{2}}\right] L^{\frac{-1}{2}}$$
$$MP_K = [L^{\frac{1}{2}} + K^{\frac{1}{2}}] K^{\frac{-1}{2}}$$

Let w = 2 and r = 1

a) Suppose the firm is required to produce Q units of output. Show how the cost-minimizing quantity of labor depends on the quantity Q. Show how the cost-minimizing quantity of capital depends on quantity Q.

Starting with the tangency condition we have

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$
$$\frac{\left[L^{1/2} + K^{1/2}\right] L^{-1/2}}{\left[L^{1/2} + K^{1/2}\right] K^{-1/2}} = \frac{2}{1}$$
$$\frac{K}{L} = 4$$
$$K = 4L$$

Plugging this into the total cost function yields

$$Q = \left[L^{1/2} + (4L)^{1/2}\right]^2$$
$$Q = \left[3L^{1/2}\right]^2$$
$$Q = 9L$$
$$L = \frac{Q}{9}$$

Inserting this back into the solution for K above gives

$$K = \frac{4Q}{9}$$

b) Find the equation of the firm’s long-run total cost curve.

$$TC = 2\left(\frac{Q}{9}\right) + \frac{4Q}{9}$$
$$TC = \frac{2Q}{3}$$

c) Find the equation of the firm's long-run average cost curve.

$$AC = \frac{TC}{Q} = \left(\frac{2Q}{3}\right) \Big/ Q$$

$$AC = \frac{2}{3}$$

d) Find the solution to the firm's short-run cost-minimization problem when capital is fixed at a quantity of 9 units (i.e. $\bar{K} = 9$).

When $Q \leq 9$ the firm needs no labor. If $Q > 9$ the firm must hire labor. Setting $\bar{K} = 9$ and plugging in for capital in the production function yields

$$Q = \left[L^{1/2} + 9^{1/2}\right]^2$$

$$Q^{1/2} = L^{1/2} + 3$$

$$L^{1/2} = Q^{1/2} - 3$$

$$L = \left[Q^{1/2} - 3\right]^2$$

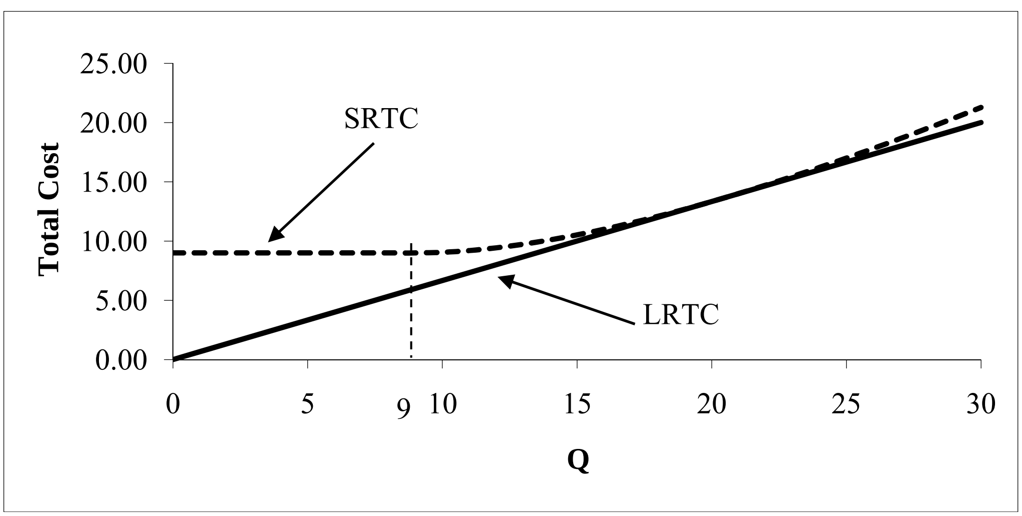
Thus,

$$L = \begin{cases} \left[Q^{1/2} - 3\right]^2 & \text{if } Q > 9 \\ 0 & \text{if } Q \leq 9 \end{cases}$$

e) Find the short-run total cost curve, and graph it along with the long-run total cost curve.

$$TC = \begin{cases} 2\left(Q^{1/2} - 3\right)^2 + 9 & \text{when } Q > 9 \\ 9 & \text{when } Q \leq 9 \end{cases}$$

Graphically, short-run and long-run total cost are shown in the following figure.



f) Find the associated short-run average cost curve.

$$AC = \frac{TC}{Q} = \begin{cases} \dfrac{2\left(Q^{1/2}-3\right)^2+9}{Q} & \text{if } Q > 9 \\ \dfrac{9}{Q} & \text{if } Q \le 9 \end{cases}$$

8.14. A hat manufacturing firm has the following production function with capital and labor being the inputs: Q=min(4L, 7K)—that is it has a fixed-proportions production function. If *w* is the cost of a unit of labor and *r* is the cost of a unit of capital, derive the firm's long-run total cost curve and average cost curve in terms of the input prices and Q.

The fixed proportions production function implies that for the firm to be at a cost minimizing optimum, $4L = 7K$ and both of these equal *Q*. Therefore, *L* = *Q*/4 and *K* = *Q*/7. So the firm's total cost is $wL + rK = wQ/4 + rQ/7 = [\frac{w}{4} + \frac{r}{7}]Q$.

The average cost curve is $LRAC = TC/Q = \frac{w}{4} + \frac{r}{7}$. Note that this average cost curve is independent of Q and is simply a straight line.

8.19. Consider a production function of three inputs, labor, capital, and materials, given by Q=LKM. The marginal products associated with this production function are as followsL MP_L=KM, MP_K=LM, and MP_M=LK. Let *w* = 5, *r* =1, and *m* = 2, where *m* is the price per unit of materials.

a) Suppose that the firm is required to produce Q units of output. Show how the cost-minimizing quantity of labor depends on the quantity Q. Show how the cost-minimizing quantity of capital depends on the quantity Q. Show how the cost-minimizing quantity of materials depends on the quantity Q.

Equating the bang for the buck between labor and capital implies

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$
$$\frac{KM}{LM} = \frac{5}{1}$$
$$K = 5L$$

Equating the bang for the buck between labor and materials implies

$$\frac{MP_L}{MP_M} = \frac{w}{m}$$

$$\frac{KM}{KL} = \frac{5}{2}$$

$$M = \frac{5L}{2}$$

Plugging these into the production function yields

$$Q = L(5L)\left(\frac{5L}{2}\right)$$

$$Q = \frac{25L^3}{2}$$

$$L^3 = \frac{2Q}{25}$$

$$L = \left(\frac{2Q}{25}\right)^{1/3}$$

Substituting into the tangency condition results above implies

$$K = 5\left(\frac{2Q}{25}\right)^{1/3}$$

and

$$M = \frac{5}{2}\left(\frac{2Q}{25}\right)^{1/3}$$

b) Find the equation of the firm's long-run total cost curve.

$$TC = 5\left(\frac{2Q}{25}\right)^{1/3} + 5\left(\frac{2Q}{25}\right)^{1/3} + 2\left(\frac{5}{2}\right)\left(\frac{2Q}{25}\right)^{1/3}$$

$$TC = 15\left(\frac{2Q}{25}\right)^{1/3}$$

c) Find the equation of the firm's long-run average cost curve.

$$AC = \frac{TC}{Q} = \frac{15}{Q}\left(\frac{2Q}{25}\right)^{1/3}$$

d) Suppose that the firm is required to produce Q units of output, but that its capital is fixed at a quantity of 50 units (i.e. $\overline{K} = 50$). Show how the cost-minimizing quantity of labor depends on the quantity Q. Show how the cost-minimizing quantity of materials depends on the quantity Q.

Beginning with the tangency condition

$$\frac{MP_L}{MP_M} = \frac{w}{m}$$

$$\frac{KM}{KL} = \frac{5}{2}$$

$$M = \frac{5L}{2}$$

Setting $\bar{K} = 50$ and substituting into the production function yields

$$Q = L(50)\left(\frac{5L}{2}\right)$$

$$Q = 125L^2$$

$$L = \sqrt{\frac{Q}{125}}$$

Substituting this result into the tangency condition result above implies

$$M = \frac{5\sqrt{\frac{Q}{125}}}{2}$$

$$M = \sqrt{\frac{Q}{20}}$$

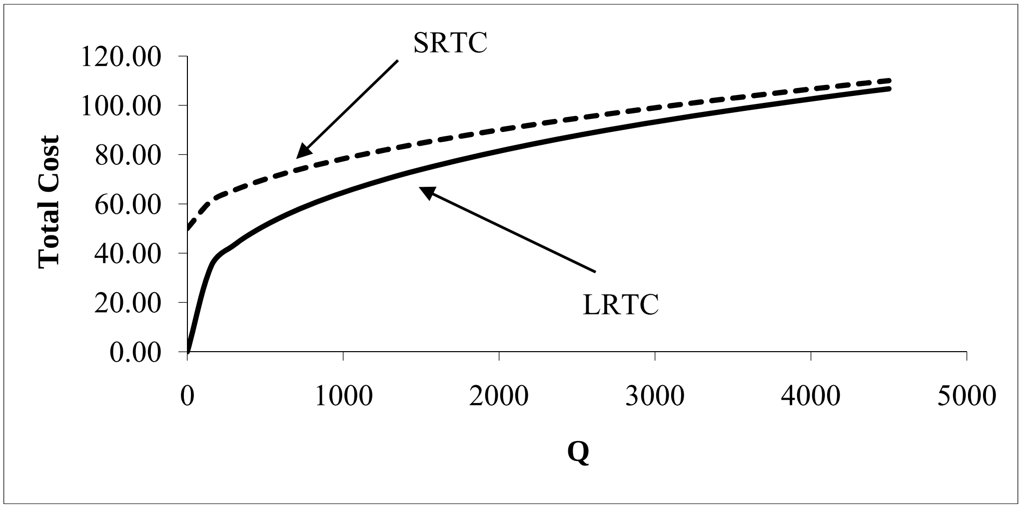
e) Find the equation of the short-run total cost curve when capital is fixed at a quantity of 50 units (i.e. $\bar{K} = 50$) and graph it along with the long-run total cost curve.

In the short run,

$$TC = 5\sqrt{\frac{Q}{125}} + 50 + 2\sqrt{\frac{Q}{20}}$$

$$TC = 2\sqrt{\frac{Q}{5}} + 50$$

Graphically, short-run and long-run total cost curves are shown in the following figure.



f) Find the equation of the associated short-run average cost curve.

Short run average cost is given by

$$AC = \frac{TC}{Q} = \frac{2\sqrt{\frac{Q}{5}} + 50}{Q}$$