

EconS 301 – Intermediate Microeconomics

Review Session #11 – Chapter 14: Strategy and Game Theory

- 1) Asahi and Kirin are the two largest sellers of beer in Japan. These two firms compete head to head in dry beer category in Japan. The following table shows the profit (in millions of yen) that each firm earns when it charges different prices for its beer:

		<i>Kirin</i>			
		¥630	¥660	¥690	¥720
<i>Asahi</i>	¥630	180, 180	184, 178	185, 175	186, 173
	¥660	178, 184	183, 183	192, 182	194, 180
	¥690	175, 185	182, 192	191, 191	198, 190
	¥720	173, 186	180, 194	190, 198	196, 196

- a) Does Asahi have a dominant strategy? Does Kirin?
- b) Both Asahi and Kirin have a dominated strategy: Find and identify it.
- c) Assume that Asahi and Kirin will not play the dominated strategy you identified in part (b) (i.e., cross out the dominated strategy for each firm in the table.) Having eliminated the dominated strategy, show that Asahi and Kirin now have another dominated strategy.
- d) Assume that Asahi and Kirin will not play the dominated strategy you identified in part (c). Having eliminated this dominated strategy, determine whether A and Kirin now have a dominant strategy.
- e) What is the Nash equilibrium in this game?

Answer

- a) Neither player has a dominant strategy in this game.
- b) In this game, Asahi has a dominated strategy, ¥720 is dominated by ¥690, and Kirin has a dominated strategy, ¥720 is dominated by ¥690. Assuming neither player will play these dominated strategies we can remove them from the game. The reduced game is

		<i>Kirin</i>		
		¥630	¥660	¥690
<i>Asahi</i>	¥630	180, 180	184, 178	185, 175
	¥660	178, 184	183, 183	192, 182
	¥690	175, 185	182, 192	191, 191

- c) Now that we have eliminated a dominated strategy from the original game, both players now have a dominated strategy in the reduced game. Asahi has a dominated strategy, ¥690 is dominated by ¥660, and Kirin has a dominated strategy, ¥690 is dominated by ¥660. Assuming neither player will play these dominated strategies we can remove them from the game. The reduced game is

		<i>Kirin</i>	
		¥630	¥660
<i>Asahi</i>	¥630	180, 180	184, 178
	¥660	178, 184	183, 183

- d) Now that we have eliminated another dominated strategy from the original game, both players have a dominant strategy to choose ¥630.
- e) Based on the analysis above, the Nash equilibrium in this game has both players choosing ¥630.

2) Consider the following game, where $x > 0$:

		<i>Firm 2</i>	
		High Price	Low Price
<i>Firm 1</i>	High Price	140, 140	20, 160
	Low Price	$90 + x, 90 - x$	50, 50

- a) For what values of x do both firms have a dominant strategy? What is the Nash equilibrium (or equilibria) in these cases?
- b) For what values of x does only one firm have a dominant strategy? What is the Nash equilibrium (or equilibria) in these cases?
- c) Are there any values of x such that neither firm has a dominant strategy? Ignoring mixed strategies, is there a Nash equilibrium in such cases?

Answer

- a) For $x > 50$, both firms have a dominant strategy. The unique NE is (*Low, Low*).
- b) For $40 < x < 50$ only Firm 2 has a dominant strategy. The unique NE in this case is still (*Low, Low*).
- c) For $x < 40$, there are zero dominant strategies, and no NE exists.

3) In a World Series game, Kerry Wood is pitching and Alex Rodriguez is batting. The count on Rodriguez is 3 balls and 2 strikes. Wood has to decide whether to throw a fastball or a curveball. Rodriguez has to decide whether to swing or not swing. If Wood throws a fastball and Rodriguez doesn't swing, the pitch will almost certainly be a strike, and Rodriguez will be out. If Rodriguez does swing, however, there is a strong likelihood that he will get a hit. If Wood throws a curve and Rodriguez swings, there is a strong likelihood that Rodriguez will strike out. But if Wood throws a curve and Rodriguez doesn't swing, there is a good chance that it will be ball four and Rodriguez will walk (assume that a walk is as good as a hit in this instance).

The following table shows the payoffs from each pair of choices that the two players can make:

		<i>Alex Rodriguez</i>	
		Swing	Do Not Swing
<i>Kerry Wood</i>	Fastball	-100, 100	100, -100
	Curveball	100, -100	-100, 100

- a) Is there a Nash equilibrium in pure strategies game?
- b) Is there a mixed strategy Nash equilibrium in game? If so, what is it?

Answer

- a) There are no Nash equilibria in pure strategies in this game.
- b) This game does have a Nash equilibrium in mixed strategies. To find the mixed strategy equilibrium, we need to find the probabilities with which, for example, Rodriguez plays “Swing” and “Do Not Swing” so that Wood’s payoff from playing “Fast Ball” and “Curve Ball” are the same. Letting the probability of “Swing” be P and “Do Not Swing” be $1 - P$, Wood’s expected payoff if he chooses “Fast Ball” is $-100P + 100(1 - P)$. His expected payoff if he chooses “Curve Ball” is $100P - 100(1 - P)$. Equating these yields:

$$\begin{aligned}
 -100P + 100(1 - P) &= 100P - 100(1 - P) \\
 100 - 200P &= 200P - 100 \\
 P &= 0.50
 \end{aligned}$$

Thus, in equilibrium Rodriguez should play “Swing” with a 50% probability and “Do Not Swing” with a 50% probability. Since the problem is symmetric, Wood should play “Fast Ball” with a 50% probability and “Curve Ball” with a 50% probability.

- 4) In the mid-1990s, Value Jet wanted to enter the market serving routes that would compete head to head with Delta Airlines in Atlanta. Value Jet knew that Delta might respond in one of two ways: Delta could price war or it could be "accommodating," keep price at a high level. Value Jet had to decide whether it would enter on a small scale or on a large scale. Annual profits (in millions of dollars) associated with strategy are summarized in the following table (where the first number is the payoff to Value Jet and the second the payoff to Delta):

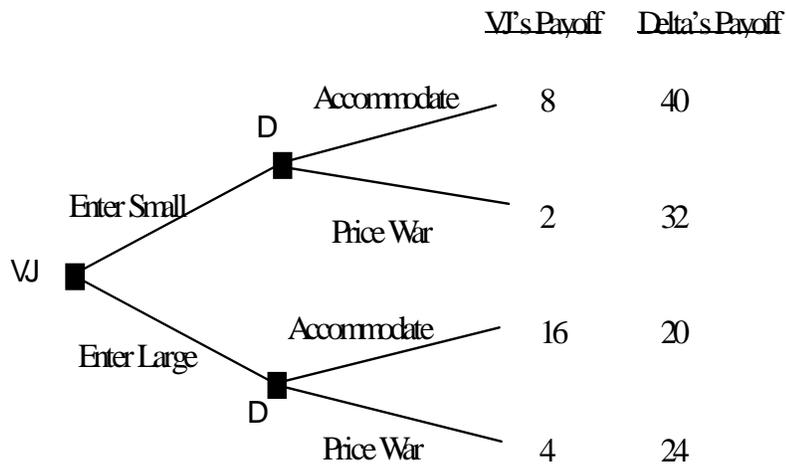
		Delta	
		Accommodate (Price High)	Price Low (Price War)
Value Jet	Enter on Small Scale	8, 40	2, 32
	Enter on Large Scale	16, 20	4, 24

- a) If Value Jet and Delta choose their strategies simultaneously, what strategies would the two firms choose at the Nash equilibrium, and what would be the payoff for Value Jet? Explain.
- b) As it turned out, Value Jet decided to move first, entering on a small scale. It communicated this information by issuing a public statement announcing that it had limited aspirations in this marketplace and had no plans to grow beyond its initial small size. Analyze the sequential game in which Value Jet chooses "small" or large in the first stage and then Delta accommodates or starts a price war in the second stage. Did Value Jet enhance its profit by moving first and entering on a small scale? If so, how much more did it earn with this strategy? If not, explain why not? (Hint: Draw the game tree)

Answer

a) Value Jet has a dominant strategy to enter large. Given that, Delta would respond by launching a price war. Thus there is a unique pure strategy Nash equilibrium in which Value Jet enters large and Delta starts a price war. Value Jet's payoff is \$4 zillion.

b)



The game tree above models the two stages of the game. The payoffs are the same as in the matrix in (a). If VJ (Value Jet) builds low, D (Delta) will accommodate (preferring 40 over 32). Thus, D will get 8 if it enters small. If VJ (Value Jet) builds large, D (Delta) will start a price war (preferring 24 over 20). Thus, D will get 4 if it enters small. Value Jet's optimal strategy is to build small and for Delta to accommodate. Value Jet will then receive \$8 zillion. Value Jet has increased its profit from \$4 zillion in (a) to \$8 zillion in (b), so by moving first, gains an extra \$4 zillion in profit.

- 5) Besanko, Inc. and Braeutigam, Ltd. compete in the high-grade carbon fiber market. Both firms sell identical grades of carbon fiber, a commodity product that will sell at a common market price. The challenge for each firm is to decide upon a capacity expansion strategy. The following problem pertains to this choice.
- Suppose it is well known that long-run market demand in this industry will be robust. In light of that, the payoffs associated with various capacity expansion strategies that Besanko and Braeutigam might pursue are shown in the following table. What are the Nash equilibrium capacity choices for each firm if both firms make their capacity choices simultaneously?
 - Again, suppose that the table gives the payoffs to each firm under various capacity scenarios, but now suppose that Besanko can commit in advance to a capacity strategy. That is, it can choose no expansion, modest expansion, or major expansion. Braeutigam observes this choice and makes a choice of its own (no expansion or modest expansion). What is the equilibrium in this sequential-move capacity game?

		Braeutigam	
		No Expansion	Modest Expansion
Besanko	No Expansion	\$1,013, \$1,013	\$844, \$1,125
	Modest Expansion	\$1,125, \$844	\$900, \$900
	Major Expansion	\$1,013, \$506	\$675, \$450

Answer

- As we see below (where squares represent Besanko's best response and circles represent Braeutigam's), the Nash equilibrium is for each player to choose MODEST EXPANSION.

		Braeutigam	
		No Expansion	Modest Expansion
Besanko	No Expansion	\$ 1,013	\$ 844
	Modest Expansion	\$ 1,125	\$ 900
	Major Expansion	\$ 1,013	\$ 675
		\$ 1,013	\$ 450

b) If Besanko moves first, then he will commit to a MAJOR EXPANSION, and Braeutigam will choose NO EXPANSION. Here's why:

- If Besanko chooses NO EXPANSION, Braeutigam's best response is MODEST EXPANSION. Besanko's payoff is \$844.
- If Besanko chooses MODEST EXPANSION, Braeutigam's best response is MODEST EXPANSION. Besanko's payoff is \$900.
- If Besanko chooses MAJOR EXPANSION, Braeutigam's best response is NO EXPANSION. Besanko's payoff is \$1,013.

Besanko does best when he choose MAJOR EXPANSION, putting Braeutigam in a position in which it is optimal for him to choose NO EXPANSION.