EconS 301 – Intermediate Microeconomics
Review Session #10 – Chapter 13: Market Structure and Competition

Exercise 13.2. A homogeneous products duopoly faces a market demand function given by $P = 300 - 3Q$, where $Q = Q_1 + Q_2$. Both firms have constant marginal cost MC =100.

a) What is Firm 1's profit-maximizing quantity, given that Firm 2 produces an output of 50 units per year? What is Firm 1's profit-maximizing quantity when Firm 2 produces 20 units per year?

With two firms, demand is given by $P = 300 - 3Q_1 - 3Q_2$. If $Q_2 = 50$, then $P = 300 - 3Q_1 - 150$ or $P = 150 - 3Q_1$. Setting $MR = MC$ implies

$$150 - 6Q_1 = 100$$
$$Q_1 = 8.33$$

If $Q_2 = 20$, then $P = 240 - 3Q_1$. Setting $MR = MC$ implies

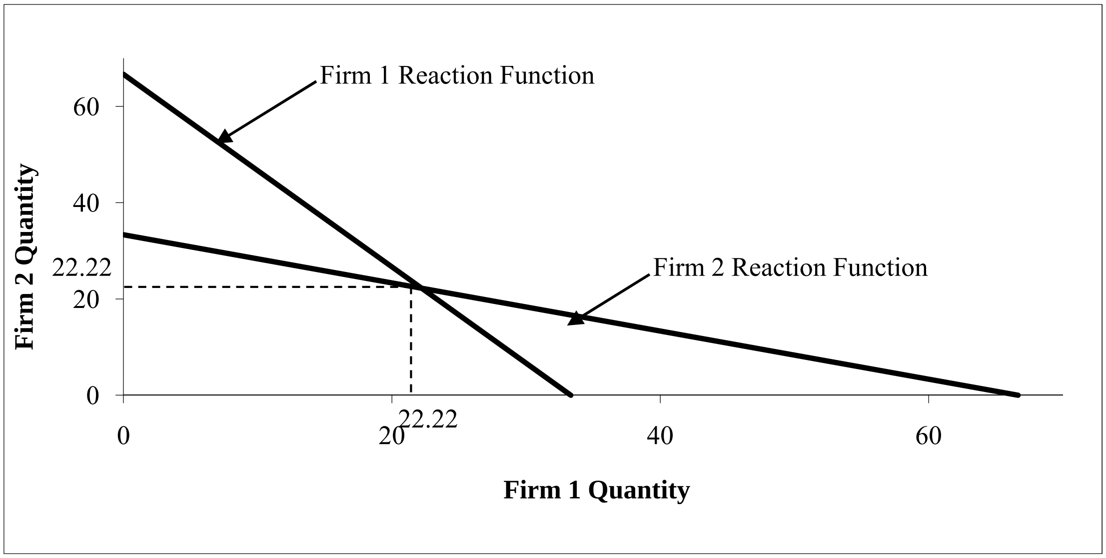
$$240 - 6Q_1 = 100$$
$$Q_1 = 23.33$$

b) Derive the equation of each firm's reaction curve and then graph these curves.

For Firm 1, $P = (300 - 3Q_2) - 3Q_1$. Setting $MR = MC$ implies

$$(300 - 3Q_2) - 6Q_1 = 100$$
$$Q_1 = 33.33 - 0.5Q_2$$

Since the marginal costs are the same for both firms, symmetry implies $Q_2 = 33.33 - 0.5Q_1$. Graphically, these reaction functions appear as



c) What is the Cournot equilibrium quantity per firm and price in this market?

Because of symmetry, in equilibrium both firms will choose the same level of output. Thus, we can set $Q_1 = Q_2$ and solve

$$Q_2 = 33.33 - 0.5Q_2$$
$$Q_2 = 22.22$$

Since both firms will choose the same level of output, both firms will produce 22.22 units. Price can be found by substituting the quantity for each firm into market demand. This implies price will be $P = 300 - 3(44.44) = 166.67$.

d) What would the equilibrium price in this market be if it were perfectly competitive?

If this market were perfectly competitive, then equilibrium would occur at the point where $P = MC = 100$.

$$300 - 3Q = 100$$
$$200 = 3Q$$
$$Q = \frac{200}{3}$$

e) What would the equilibrium price in this market be if the two firms colluded to set the monopoly price?

If the firms colluded to set the monopoly price, then

$$300 - 6Q = 100$$
$$Q = 33.33$$

At this quantity, market price will be $P = 300 - 3(^{200}/_{6}) = 200$.

f) What is the Bertrand equilibrium price in this market?

If the firms acted as Bertrand oligopolists, the equilibrium would coincide with the perfectly competitive equilibrium of $P = 100$.

g) What are the Cournot equilibrium quantities and industry price when one firm has a marginal cost of 100 but the other firm has a marginal cost of 90?

Suppose Firm 1 has $MC = 100$ and Firm 2 has $MC = 90$. For Firm 1, $P = (300 - 3Q_2) - 3Q_1$. Setting $MR = MC$ implies

$$(300 - 3Q_2) - 6Q_1 = 100$$
$$Q_1 = 33.33 - 0.5Q_2$$

For Firm 2, $P = (300 - 3Q_1) - 3Q_2$. Setting $MR = MC$ implies

$$(300 - 3Q_1) - 6Q_2 = 90$$
$$Q_2 = 35 - 0.5Q_1$$

Solving these two reaction functions simultaneously yields $Q_1 = 21.11$ and $Q_2 = 24.44$. With these quantities, market price will be $P = 163.36$.

Exercise 13.3. Zack and Andon compete in the peanut market. Zack is very efficient at producing nuts with a low marginal cost C_z=1; Andon, however, has a constant marginal cost C_a=10. If the market demand for nuts is $P = 100 - Q$, find the Cournot equilibrium price and the quantity and profit level for each competitor.

For Zack, $MR_Z = MC_Z$ implies $100 - 2q_Z - q_A = 1$, so Zack's reaction function is $q_Z = ½*(99 - q_A)$. Similarly, $MR_A = MC_A$ implies $100 - 2q_A - q_Z = 10$ so Andon's reaction function is $q_A = ½*(90 - q_Z)$. Solving these two equations in two unknowns yields $q_Z = 36$ and $q_A = 27$. The market price is $P = 100 - (36 + 27) = 37$. Zack earns $\pi_Z = (37 - 1)*36 = 1296$ and Andon earns $\pi_A = (36 - 10)*27 = 702$.

Exercise 13.6. Suppose that demand for cruise ship vacations is given by $P = 1200 - 5Q$, where Q is the total number of passengers when the market price is P/

a) The market initially consists of only three sellers, Alpha Travel, Beta Worldwide, and Chi Cruiseline. Each seller has the same marginal cost of $300 per passenger. Find the symmetric Cournot equilibrium price and output for each seller.

Consider first the problem of Alpha Travel. It produces until $MR_A = MC_A$ or $1200 - 5(Q_B + Q_C) - 10Q_A = 300$. Thus its reaction function is

$$Q_A = 90 - 0.5(Q_B + Q_C).$$

Symmetry implies that in equilibrium $Q_A = Q_B = Q_C$, so we can solve to find that $Q_i = 45$ for each firm. Thus the equilibrium price is $P = 525$.

b) Now suppose that Beta Worldwide and Chi Cruiseline announce their intention to merge into a single firm. They claim that their merger will allow them to achieve cost savings so that their marginal cost is less than \$300 per passenger. Supposing that the merged firm, BetaChi, has a marginal cost of c < \$300, while Alpha Travel's marginal cost remains \$300, for what values of c would the merger raise consumer surplus relative to part (a).

Reconsidering Alpha's profit-maximization problem, we now have that $MR_A = MC_A$ or $1200 - 5Q_{BC} - 10Q_A = 300$. Thus its reaction function is

$$Q_A = 90 - 0.5Q_{BC}.$$

The merged firm will produce until $MR_{BC} = MC_{BC}$ or $1200 - 5Q_A - 10Q_{BC} = c$, so its reaction function is

$$Q_{BC} = 120 - 0.1c - 0.5Q_A$$

Solving these two equations as a function of c yields $Q_{BC} = 100 - (2/15)*c$ and $Q_A = 40 + (1/15)*c$. Total output is then $Q = 140 - (1/15)*c$, and so the market price is $P = 500 + ⅓*c$. Put simply, the merger raises consumer surplus only if the price falls; thus consumer surplus rises only when $500 + ⅓*c < 525$, or $c < 75$.

Exercise 13.12. Consider an oligopoly in which firms choose quantities. The inverse market demand curve is given $P = 280 - 2(X + Y)$, where X is the quantity of Firm 1, and Y is the quantity of Firm 2. Each firm has a marginal cost equal to 40.

a) What is the Cournot equilibrium outputs for each firm? What is the market price at the Cournot equilibrium? What is the profit for each firm?

The table below summarizes the answer to this problem. The solution details follow.

	Firm 1 output	**Firm 2 output**	**Market Price**	**Firm 1 Profit**	**Firm 2 Profit**
Cournot	40	40	120	3,200	3,200
Stackelberg with Firm 1 as leader	60	30	100	3,600	1,800

a) Firm 1's marginal revenue is $MR = 280 - 2Y - 4X$. Equating MR to MC gives us:

$$280 - 2Y - 4X = 40,$$

$$240 - 2Y = 4X$$

$$X = 60 - 0.5Y$$

This is Firm 1's reaction function. Firm 2 is identical, and so Firm 2's reaction function will be a mirror image of Firm 1's

$$Y = 60 - 0.5X$$

If we solve these reaction functions simultaneously, we find $X = Y = 40$. At this output, the corresponding market price is $P = 280 - 2(40 + 40) = 120$. Each firm's profit is thus: 120*40 – 40*40 = 3,200.

b) What is the Stackelberg equilibrium, when Firm 1 acts as the leader? What is the market price at the Stackelberg equilibrium? What is the profit for each firm?

To find the Stackelberg equilibrium in which Firm 1 is the leader, we start by writing the expression for Firm 1's total revenue:

$$TR = (280 - 2Y - 2X)X$$

In place of Y, we substitute in Firm 2's reaction function: $Y = 60 - 0.5X$

$$TR = [280 - 2(60 - 0.5X) - 2X]X = (160 - X)X$$

Firm 1's marginal revenue is therefore $MR = 160 - 2X$. Equating marginal revenue to marginal cost gives us:

$$160 - 2X = 40, \text{ or } X = 60.$$

To find Firm 2's output, we plug $X = 60$ back into Firm 2's reaction function:

$$Y = 60 - 0.5(60) = 30.$$

The market price is found by plugging $X = 60$ and $Y = 30$ back into the demand curve: $P = 280 - 2(60 + 30) = 100$.

Thus, at the Stackelberg equilibrium, Firm 1's profit is: 100*60 – 40*60 = 3,600. Firm 2's profit is 100*30 – 40*30 = 1,800.

Exercise 13.18. Apple's iPod has been the portable MP3-player of choice among many gadget enthusiasts. Suppose that Apple has a constant marginal cost of 4 and that market demand is given by $Q = 200 - 2P$.

a) If Apple is a monopolist, find its optimal price and output. What are its profits?

A monopolist sets $MR = MC$ (don't forget to invert the demand curve first!) so $100 - Q^m = 4$. Thus, $Q^m = 96$ and $P^m = 52$. As a monopolist, Apple's profits are $\pi = (52 - 4)*96 = 4608$.

b) Now suppose there is a competitive fringe of 12 price-taking firms, each of whom has a total cost function $TC(q) = 3q^2 + 20q$ with corresponding marginal cost curve $MC = 6q + 20$. Find the supply function of the fringe (hint: a competitive firm supplies along its marginal cost curve above its shutdown price).

Each fringe firm maximizes profits by setting $P = MC = 6q + 20$, so we can derive a single firm's supply curve as $q = (P - 20)/6$, so long as $P > 20$. With the fringe comprising 12 firms, total supply is $Q_{fringe} = 12q$, or

$$Q_{fringe} = \begin{cases} 0 & P \le 20 \\ 2P - 40 & P > 20 \end{cases}$$

c) If Apple operates as the dominant firm facing compeititon from the fringe in this market, now what is its optimal output? How many units will fringe providers sell? What is the market price, and how much profit does Apple earn?

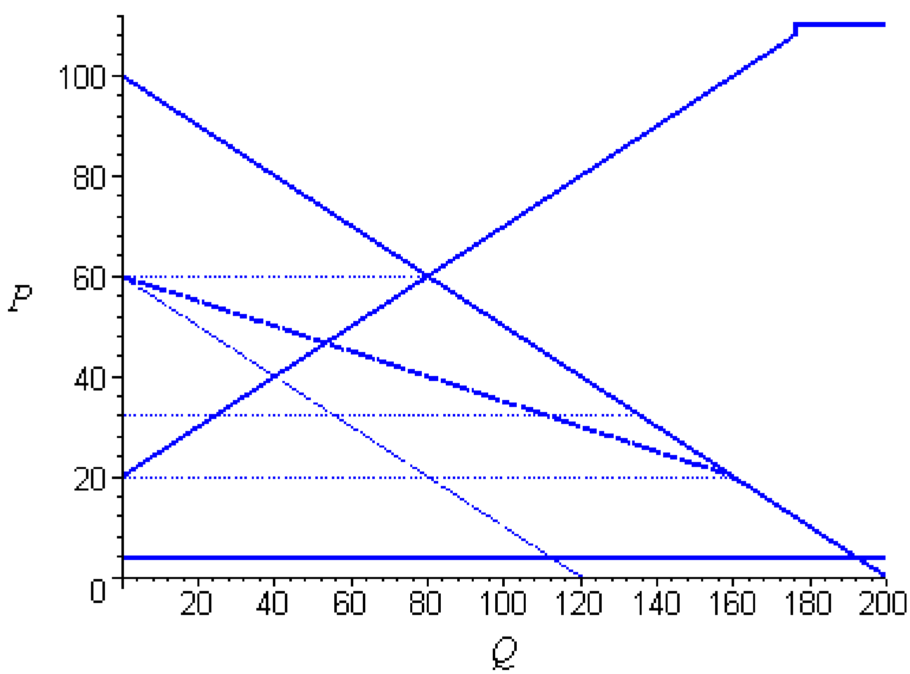
First, find Apple's (denoted *DF* for "dominant firm") residual demand, for $P > 20$: $Q_{DF} = Q_{mkt} - Q_{fringe} = 200 - 2P - (2P - 40) = 240 - 4P$. Inverting, this is $P = 60 - 0.25Q$. So Apple sets $MR = MC$ or

$$60 - 0.5Q_{DF} = 4$$

$$Q_{DF} = 112$$

From the residual demand, Apple's price is $P = 60 - 0.25*112 = 32$. At this price, the fringe supplies $Q_{fringe} = 2P - 40 = 2*32 - 40 = 24$. Apple's profits are $\pi = (32 - 4)*112 = 3136$.

d) Graph your answer from part c



Exercise 13.23. Two firms, Alpha and Bravo, compete in the European chewing gum industry. The products of the two firms are differentiated, and each month the two firms set their prices. The demand functions facing each firm are:

$$Q_A = 150 - 10P_A + 9P_B$$

$$Q_B = 150 - 10P_B + 9P_A$$

where the subscript A denotes the firm Alpha and the subscript B denotes the firm Bravo. Each firm has a constant marginal cost of $7 per unit.

a) Find the equation of the reaction function for each firm.

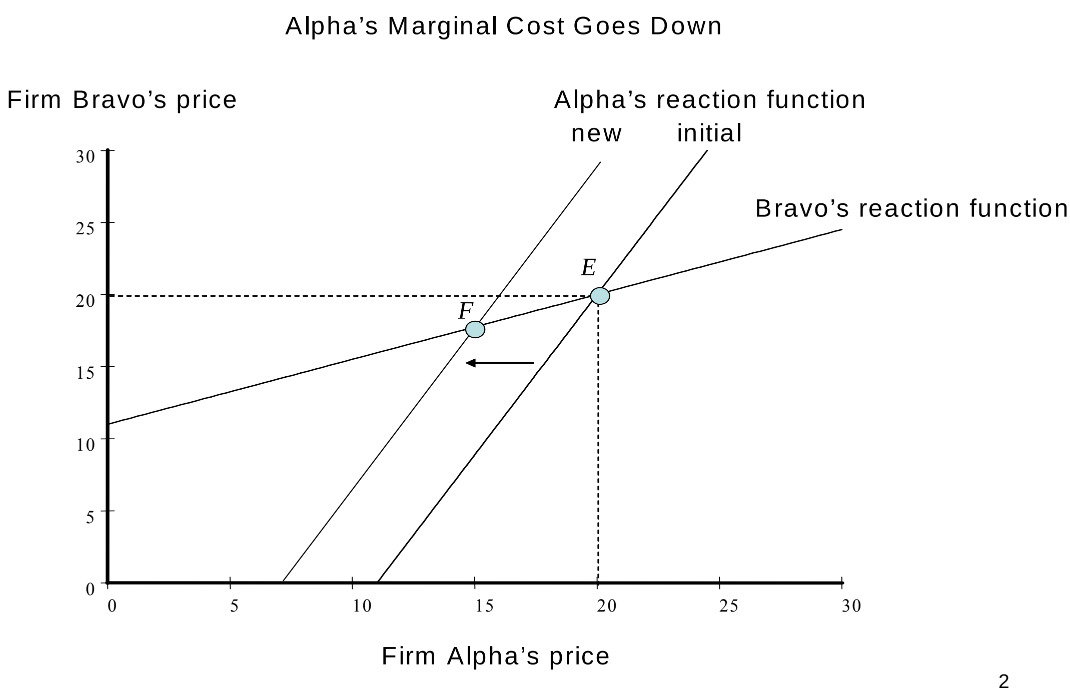
We will first solve for Alpha's reaction function. We begin by solving Alpha's demand function for P_A in terms of Q_A and P_B: $P_A = 15 – (1/10)Q_A + (9/10)P_B$. The corresponding marginal revenue equation is: $MR_A = 14 – (2/10)Q_A + (9/10)P_B$. Equating marginal revenue to marginal cost and solving for Q_A gives us Alpha's profit-maximizing quantity as a function of Bravo's price: $MR_A = MC_A \Rightarrow 15 – (2/10)Q_A + (9/10)P_B = 7$, which gives us: $Q_A = 40 + (9/2)P_B$. Now, substitute this expression for Q_A back into the expression for the demand curve with P_A on the left-hand side and Q_A on the right-hand side: $P_A = 15 – (1/10)[40 + (9/2)P_B] + (9/10)P_B \Rightarrow P_A = 11 + (9/20)P_B$. This is Alpha's reaction function.

b) Find the Bertrand equilibrium price of each firm.

We can find Bravo's reaction function by following steps identical to those followed to derive Alpha's reaction function. Following these steps gives us: $P_B = 11 + (9/20)P_A$. We now have two equations (the two reaction functions) in two unknowns, P_A and P_B. Solving these equations gives us the Bertrand equilibrium prices: $P_A = P_B = 20$.

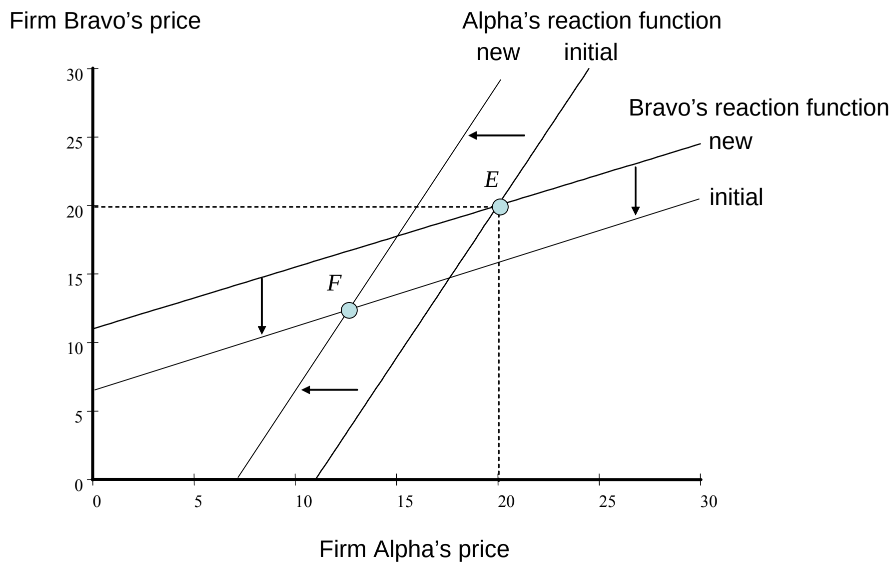
c) Sketch how each firm's reaction function is affected by each of the following changes:

i) Alpha's marginal cost goes down (with Bravo's marginal cost remaining the same).



ii) Alpha and Bravo's marginal cost goes down by the same amount.

Alpha and Bravo's Marginal Cost Go Down by the Same Amount



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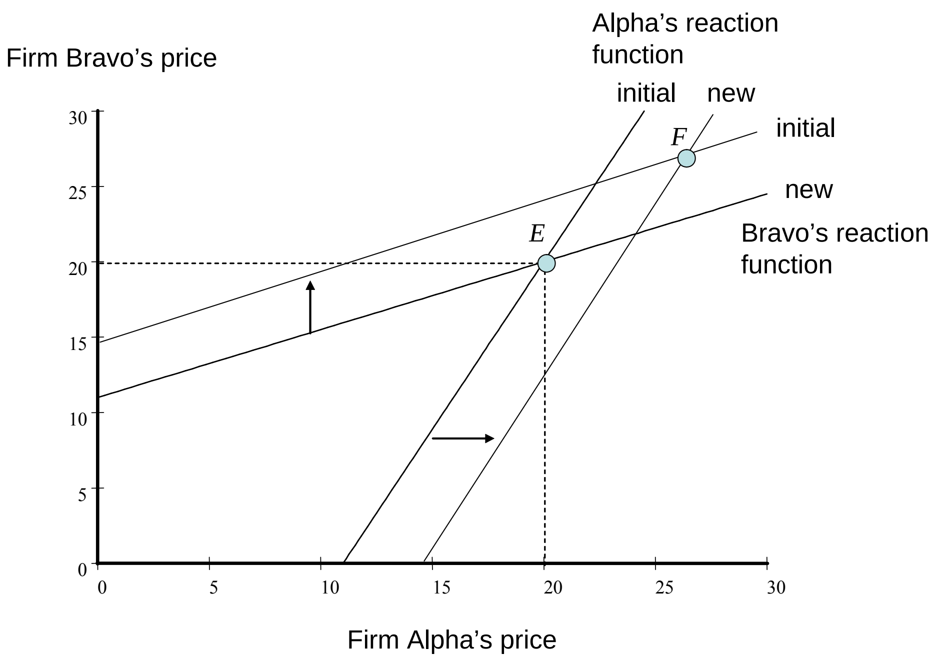
iii) Demand conditions change so that the "150" term in the demand function now becomes larger than 150.

Demand Curve Intercept Goes Up for Each Firm



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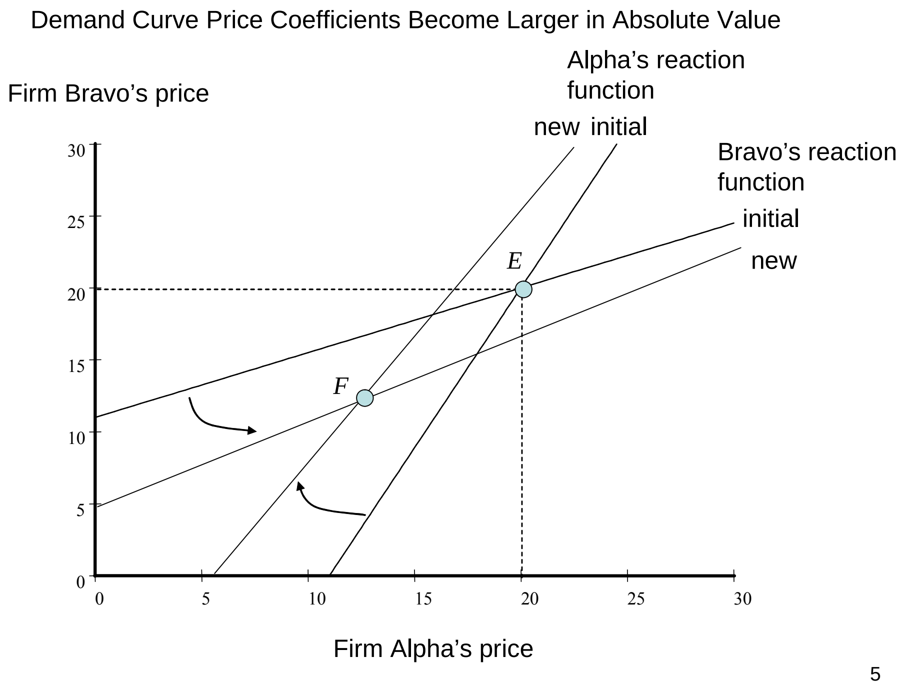
iv) The “10” and “9” terms in each demand function now become larger (e.g., they become “50” and “49” respectively).



d) Explain in words how the Bertrand equilibrium price of each firm is affected by each of the following changes:

i) Alpha’s marginal cost goes down (with Bravo’s marginal cost remaining the same).

Each firm’s equilibrium price goes down.

ii) Alpha and Bravo’s marginal cost goes down by the same amount.

Each firm’s equilibrium price goes down.

iii) Demand conditions change so that the “150” term in the demand function now becomes larger than 150.

Each firm’s equilibrium price goes up.

iv) The “10” and “9” terms in each demand function now become larger (e.g., they become “50” and “49” respectively).

Each firm’s equilibrium price goes down.

Note: The graphs above can be used to explain these changes.

Exercise 13.31. The Thai food restaurant business in Evanston, Illinois, is monopolistically competitive. Suppose that each existing potential restaurant has a total cost function given by $TC = 10Q + 40{,}000$, where Q is the number of patrons per month and TC is the total cost per month. The fixed cost of \$40,000 includes fixed operating expenses (such as salary of the chef), the lease on the building space where the restaurant is located, and interest expenses on the bank loan needed to start the business in the first place.

Currently, there are 10 Thai restaurants in Evanston. Each restaurant faces a demand function given by $Q = \frac{4{,}000{,}000}{N} P^{-5} \overline{P}^{4}$, where P is the price of a typical entrée at the restaurant, $\overline{P}$ is the price of a typical entrée averaged over all other Thai restaurants in Evanston, and N is the total number of restaurants. Each restaurant takes the prices of other Thai restaurants as given when choosing its own price.

a) What is the own-price elasticity of demand facing a typical restaurant?

Each firm faces an own price elasticity of demand of -5.

b) For a typical restaurant, what is the profit-maximizing price of a typical entrée?

Because the demand function exhibits constant elasticity, we can directly solve for a firm's profit-maximizing price by using the inverse elasticity pricing rule:

$(P - MC)/P = -1/\varepsilon_{Q,P}$

From the equation of the total cost function, we see that each firm faces a marginal cost of \$10 per patron. This implies

$(P - 10)/P = -1/(-5)$, or P = \$12.50.

c) At the profit-maximizing price, how many patrons does a typical restaurant serve per month? Given this number of patrons, what is the average total cost of a typical restaurant?

Each firm will charge a price of \$12.50, so therefore each restaurant attracts $(4{,}000{,}000/10)(12.50)^{-5}(12.50)^{4} = 32{,}000$ patrons per month. Given this number of patrons each restaurant has an average total cost given by $10 + 40{,}000/32{,}000 = \$11.25$.

d) What is the long-run equilibrium number of Thai restaurants in the Evanston market?

To find the long-run equilibrium number of firms, we note that when there is an arbitrary number of firms N, the number of patrons going to each restaurant is $(4{,}000{,}000/N)(12.50)^{-5}(12.50)^{4} = 320{,}000/N$. Given this number of patrons, each firm's average total cost is $10 + 40{,}000/(320{,}000/N) = 10 + 0.125N$. Because the demand function is constant elasticity, each firm will charge a price of \$12.50 no matter how many firms are in the market. In a long-run equilibrium, the price of 12.50 must equal each firm's average total cost. Thus: $10 + 0.125N = 12.50$, or 20.