EconS 501 - Micro Theory I[1]
Recitation #9 - Monopoly

Exercise 1

A monopolist faces a market demand curve given by: $Q = 70 - p$.

(a) If the monopolist can produce at constant average and marginal costs of $AC = MC = 6$, what output level will the monopolist choose in order to maximize profits? What is the price at this output level? What are the monopolist's profits?

(b) Assume instead that the monopolist has a cost structure where the total costs are described by:

$$C(Q) = 0.25Q^2 - 5Q + 300$$

With the monopolist facing the same market demand and marginal revenue, what price-quantity combination will be chosen now to maximize profits? What will profits be?

(c) Assume now that a third cost structure explains the monopolist's position, with total costs given by:

$$C(Q) = 0.0133Q^3 - 5Q + 250$$

Again, calculate the monopolist's price-quantity combination that maximizes profits. What will profit be? *Hint*: Set $MC = MR$ as usual and use the quadratic formula to solve the second order equation for Q.

(d) Graph the market demand curve, the *MR* curve, and the three marginal cost curves from parts a, b and c. Notice that the monopolist's profit-making ability is constrained by (1) the demand curve (along with its associated *MR* curve) and (2) the cost structure underlying production.

Solution:

(a) We have that the demand is given by $Q = 70 - p$ or $p = 70 - Q$ thus the total revenue is $TR = p \cdot Q = (70 - Q)Q$, then the marginal revenue for the monopolist is $MR = 70 - 2Q$. We know that the monopolist profit maximization condition is $MR = MC$ and by the information we know that $MC = 6$ then we can set $70 - 2Q = 6$ and solving for the

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quantities we have that $Q = 32$, $P = 38$ and $\pi = (p - MC)Q = (38 - 6)32 = 1024$.

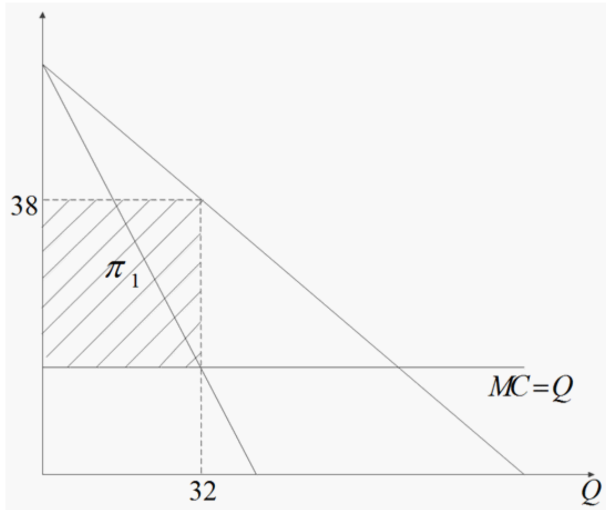

Figure 1

(b) If the total costs are described by $C(Q) = 0.25Q^2 - 5Q + 300$ then the marginal costs are $MC = 0.5Q - 5$. Thus, equalizing again $MR = MC$ we have that $70 - 2Q = 0.5Q - 5$. In this case $Q = 30$, $P = 40$ and $\pi = pQ - TC = (40 \cdot 30) - (0.25(30)^2 - 5(30) + 300) = 825$. As we can see, the change in the costs structure reduces the total production, increases the price and reduce the profits of the firm.

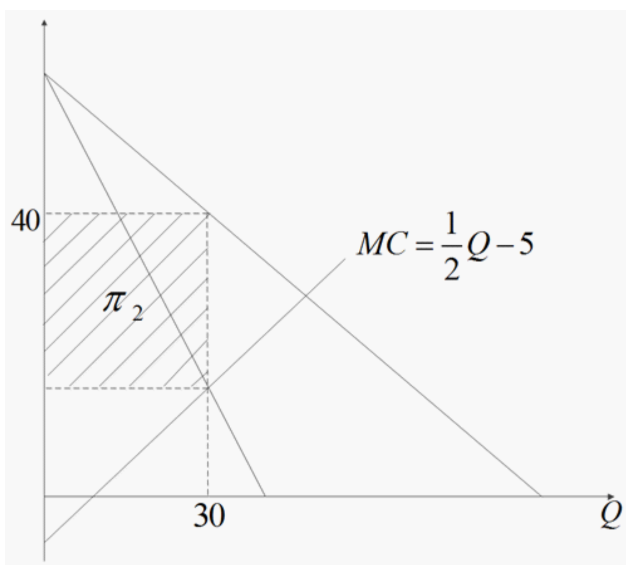

Figure 2

(c) If the total costs are described by $C(Q) = 0.0133Q^3 - 5Q + 250$ then the marginal costs are $MC = 0.0399Q^2 - 5$. Thus, equalizing again $MR = MC$ we have that $70 - 2Q =$

$0.0399Q^2 - 5$. In this case the positive solution of the quadratic equation is $Q = 25$, $P = 45$ and $\pi = pQ - TC = (45 \cdot 25) - (0.0133(25)^3 - 5(25) + 250) = 792.2$. The new change in the costs structure reduces the total production, increases the price and reduce the profits of the firm.

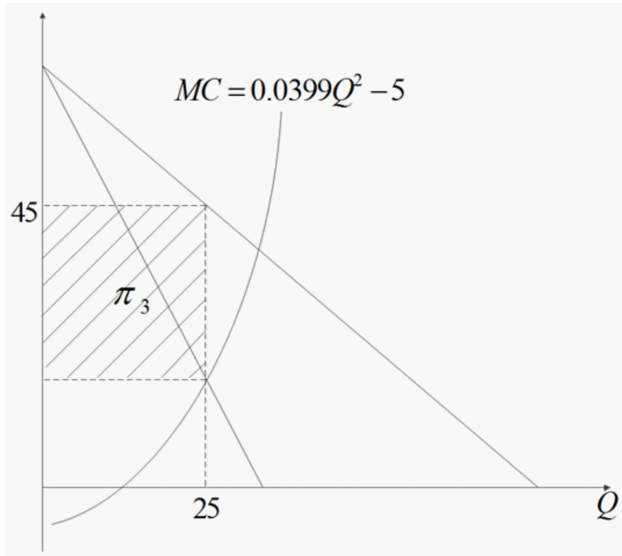

Figure 3

Exercise 2

1. [**Alternative PMP**] The monopolist can also be thought of as choosing the price and letting the market determine how much is sold. Write down the profit maximization problem and verify that $p\left[1 + \frac{1}{\varepsilon}\right] = c'(q)$ also holds when the monopolist chooses a profit-maximizing price rather than a profit-maximizing output.

   - The monopolist maximization problem is

     $$\max_{p} \quad pq(p) - c(q(p)).$$

     Differentiating, we have

     $$pq'(p) + q(p) - c'(q)\, q'(p) = 0.$$

     This can also be written as

     $$p + \frac{q(p)}{q'(p)} - c'(q) = 0,$$

     or

     $$p\left[1 + \frac{1}{\varepsilon}\right] = c'(q).$$

     which coincides with the inverse elasticity pricing rule (IEPR) we found in class.

Exercise 3

[**Monopoly subsidies**] Suppose a government wishes to combat the undesirable allocational effects of a monopoly through the use of a subsidy.

(a) Why would a lump-sum subsidy not achieve the government's goal?

(b) Use a graphical proof to show how a per-unit-of-output subsidy might achieve the government's goal.

(c) Suppose the government wants its subsidy to maximize the difference between the total value of the good to consumers and the good's total cost. Show that, in order to achieve this goal, the government should set:

$$\frac{t}{P} = -\frac{1}{e_{Q,\,P}},$$

where t is the per-unit subsidy and P is the competitive price. Explain your result intuitively.

Solution:

(a) The government wishes the monopoly to expand output toward $P = MC$. A lump-sum subsidy (T) will have no effect on the monopolist's profit maximizing choice, so this will not achieve the goal. If the monopoly maximizes $\pi = p \cdot Q - TC + T$ then the profit maximization condition is $MR = MC$.

(b) A subsidy per unit of output (t) will effectively shift the MC curve downward. If the monopoly maximizes $\pi = p \cdot Q - (TC - t \cdot Q)$ then the profit maximization condition is $MR = MC - t$, thus if the marginal cost curve shifts to the right (or downward if the monopoly has constant marginal costs), then the monopoly will produce more units at a lower price.

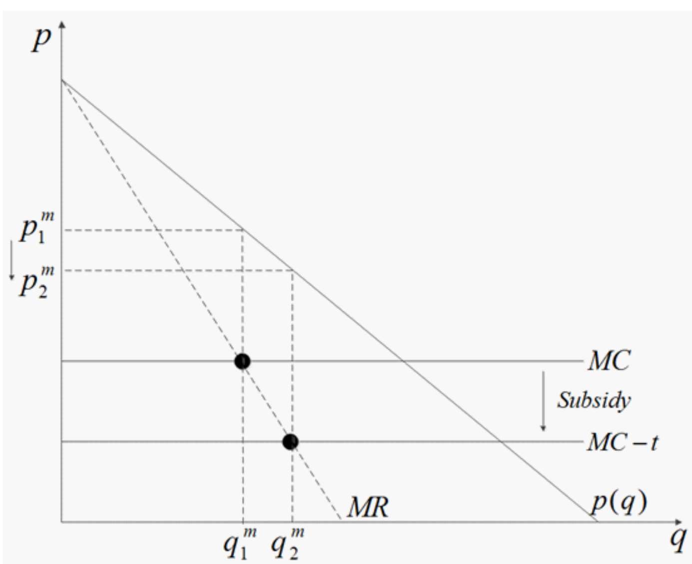

Figure 4

(c) A subsidy (t) must be chosen so that the monopoly chooses the socially optimal quantity, given t. Since the social optimality requires $P = MC$ and profit maximization requires that

$MR = MC - t$. On one hand, $MC - t = P - t$ since $P = MC$. On the other hand, note that marginal revenues are

$$MR = P + Q \cdot P'(Q) = P \cdot \left(1 + Q \cdot \frac{P'}{P}\right) = P\left(1 + \frac{1}{e}\right),$$

substituting this result into $MR = MC - t$. yields $P - t = P\left(1 + \frac{1}{e}\right)$. Rearranging, we find $1 - \frac{t}{P} = 1 + \frac{1}{e}$ which can be more compactly expressed as $\frac{t}{P} = -\frac{1}{e}$ as was to be shown. Intuitively, the monopoly creates a gap between price and marginal cost and the optimal subsidy is chosen to equal that gap expressed as a ratio to price.

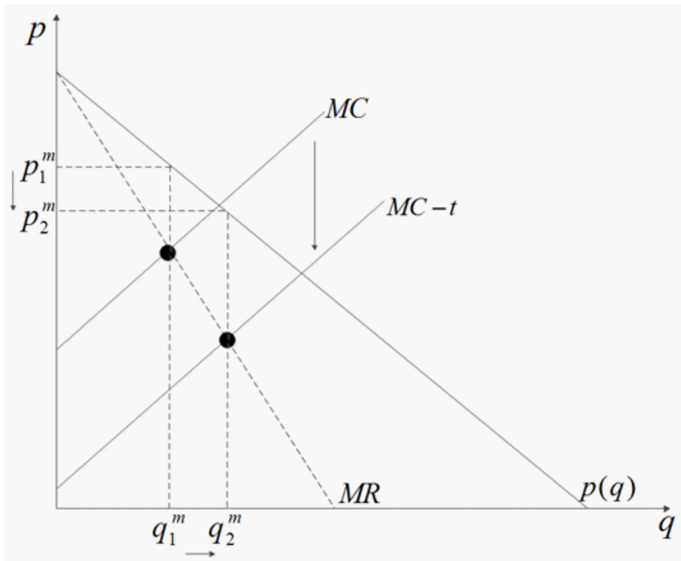

Figure 5

Exercise 4

[**Taxing a monopoly**] The taxation of monopoly can sometimes produce results different from those that arise in the competitive case. This problem looks at some of those cases. Most of these can be analyzed by using the inverse elasticity rule.

(a) Consider first an ad valorem tax on the price of a monopoly's good. This tax reduces the net price received by the monopoly from P to $P(1-t)$ where t is the proportional tax rate. Show that, with a linear demand curve and constant marginal cost, the imposition of such a tax causes price to rise by less than the full extent of the tax. That is, if p_B is the initial price and p_A is the final price, $p_A - p_B < t \cdot p_A$.

(b) Suppose that the demand curve in part (a) were a constant elasticity curve. Show that the price would now increase by precisely the full extent of the tax. [That is, if p_B is the initial price and p_A is the final price, $p_A - p_B = t \cdot p_A$, or $\frac{p_A - p_B}{p_A} = t$.] Explain the difference between your results in part (a) and (b).

(c) Describe a case where the imposition of an ad valorem tax on a monopoly would cause the price to rise by more than the tax [That is, $p_A - p_B > t \cdot p_A$].

(d) A specific tax is a fixed amount per unit of output. If the tax rate is τ per unit, total tax collections are τQ. Show that the imposition of a specific tax on a monopoly will reduce output more (and increase price more) than will the imposition of an ad valorem tax that collects the same tax revenue.

Solution:

(a) Recall that the Inverse Elasticity Rule is $P = \frac{MC}{1+\frac{1}{e}}$ when the monopoly is subject to an ad valorem tax of t, this becomes $P(1-t) = \frac{MC}{1+\frac{1}{e}}$, or $P = \frac{MC}{(1-t)} \cdot \frac{1}{1+\frac{1}{e}}$.

With linear demand, e falls (becomes more elastic) as prices rises[2] Hence, the increase in the price of the good, $P_{aftertax} - P_{pretax}$ is smaller than the extent of the tax, $t \cdot P_{aftertax}$, i.e., $P_{aftertax} - P_{pretax} < t \cdot P_{aftertax}$, or alternatively, $(1-t)P_{aftertax} < P_{pretax}$ if and only if

$$P_{aftertax} = \frac{MC}{(1-t)} \cdot \frac{1}{1+\frac{1}{e_{aftertax}}} < \frac{MC}{(1-t)} \cdot \frac{1}{1+\frac{1}{e_{pretax}}} = \frac{P_{pretax}}{(1-t)}, \text{ that is}$$
$$\frac{1}{1+\frac{1}{e_{aftertax}}} < \frac{1}{1+\frac{1}{e_{pretax}}},$$

or if $e_{aftertax} < e_{pretax}$, which is true since, for a constant marginal cost, the introduction of the tax forces the monopolist to charge a higher price (located at a point of the linear demand curve with a more negative elasticity).

(b) With a constant elasticity demand $q(p) = Ap^e$, we have that $e_{aftertax} = e_{pretax} = e$. Thus the inequality in part (a) becomes an equality so $P_{aftertax} = \frac{P_{pretax}}{(1-t)}$, or $P_{aftertax} - P_{pretax} = t \cdot P_{aftertax}$.

(c) For the price to raise more than the extent of the tax, we need $P_{aftertax} - P_{pretax} > t \cdot P_{aftertax}$, or alternatively that $(1-t)P_{aftertax} > P_{pretax}$. This occurs when

$$\frac{MC}{(1-t)} \cdot \frac{1}{1+\frac{1}{e_{aftertax}}} > \frac{MC}{(1-t)} \cdot \frac{1}{1+\frac{1}{e_{pretax}}}$$

A case in which this inequality holds is that in which the monopolist: (1) faces a constant elasticity demand curve $q(p) = Ap^e$ as in part (b) so that $e_{aftertax} = e_{pretax} = e$; and (2) the monopoly exhibits a positively sloped marginal cost curve, so $MC_{aftertax} > MC_{pretax}$. In this setting, the above inequality $\frac{MC}{(1-t)} \cdot \frac{1}{1+\frac{1}{e_{aftertax}}} > \frac{MC}{(1-t)} \cdot \frac{1}{1+\frac{1}{e_{pretax}}}$ becomes

$$P_{aftertax} = \frac{MC_{aftertax}}{(1-t)} \cdot \frac{1}{1+\frac{1}{e}} > \frac{MC_{pretax}}{(1-t)} \cdot \frac{1}{1+\frac{1}{e}} = \frac{P_{pretax}}{(1-t)}$$

which holds, given that $MC_{aftertax} > MC_{pretax}$.

(d) The key part of this question is the requirement of equal tax revenues. That is $tP_aQ_a = \tau Q_s$ where the subscripts refer to the monopoly's choices under the two tax regimes, and subscript a (s) denotes an ad-valorem tax (specific tax, respectively). Suppose that the tax rates were chosen so as to raise the same revenue for a given output level, Q. For

[2] Recall the standard figure in intermediate micro textbooks whereby the linear demand curve is described in terms of its price elasticity of demand, satisfying $\varepsilon = -\infty$ at the vertical intercept, $\varepsilon = -1$ at the midpoint of the demand line, and $\varepsilon = 0$ at the horizontal intercept.

compactness, let $\tau = tP_a$, thus implying that $\tau = tP_a > tMR_a$, given that $P_a > MR_a$, i.e., the demand curve lies above the marginal revenue curve regardless of the demand function we are considering.
Generally, under an ad valorem tax, total revenues are reduced to $R = (1-t)PQ$, entailing a marginal revenue of

$$MR_a = (1-t)MR = MR - tMR,$$

whereas under a specific tax, total revenues are $R = PQ - \tau Q$, which entail a marginal revenue of

$$MR_s = MR - \tau$$

Therefore, $MR_s < MR_a$, which implies that, for a given output Q, the specific tax reduces marginal revenue by more than does the ad valorem tax (as long as both taxes raise the same total revenue). With a constant (or upward sloping) marginal cost curve, less would be produced under the specific tax, i.e., $Q_s < Q_a$ (see figure below), thereby dictating an even higher tax rate. In all, a lower output would be produced, at a higher price than under the ad valorem tax, i.e., $Q_s < Q_a$ and $P_s > P_a$. Under perfect competition, the two equal-revenue taxes would have equivalent effects.

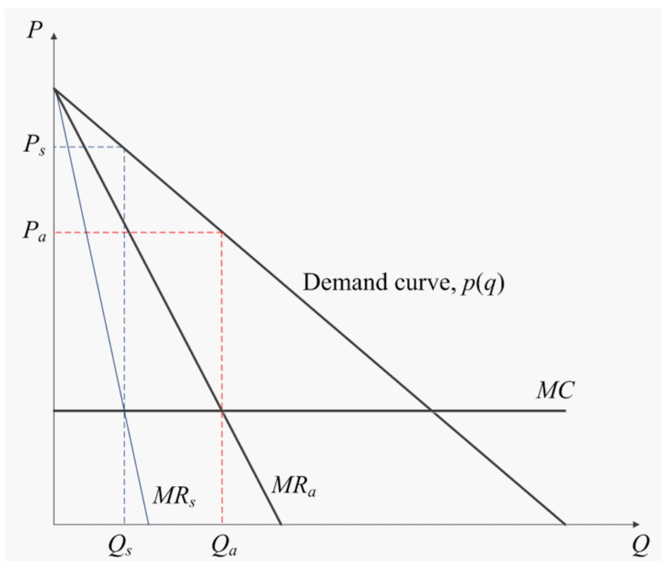

Figure 6

Exercise 5

[Pricing with discontinuous demand] Consider the market for the G-Jeans (the latest fashion among people in their late thirties). G-Jeans are sold by a single firm that carries the patent for the design. On the demand side, there are $n^H = 200$ high-income consumers who are willing to pay a maximum amount of $V^H = \$20$ for a pair of G-Jeans, and $n^L = 300$ low-income consumers who are willing to pay a maximum amount of $V^L = \$10$ for a pair of G-Jeans. Each consumer chooses whether to buy one pair of jeans or not to buy at all.

(a) Draw the market aggregate-demand curve facing the monopoly.

- The aggregate demand curve should be drawn according to the following formula:

$$Q(p) = \begin{cases} 0 & \text{if } \ p > \$20 \\ 200 & \text{if } \ \$10 < p \leq \$20 \\ 200 + 300 & \text{if } \ p \leq \$10. \end{cases}$$

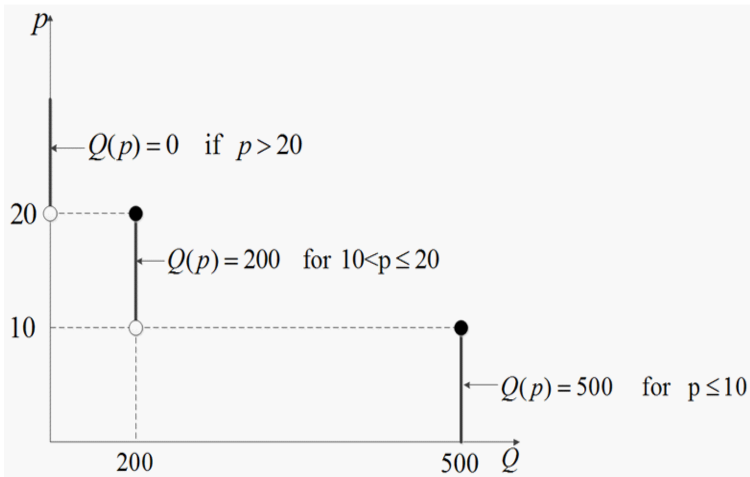

Figure 7

(b) The monopoly can produce each unit at a cost of $c = \$5$. Suppose that the G-Jeans monopoly cannot price discriminate and is therefore constrained to set a uniform market price. Find the profit-maximizing price set by G-Jeans, and the profit earned by this monopoly.

- Setting a high price, $p = \$20$ generates $Q = 200$ consumers and a profit of $\pi_H = (20 - 5)\,200 = \$3000$.

  - Setting a low price, $p = \$10$ generates $Q = 200 + 300$ consumers and a profit of $\pi_H = (10 - 5)\,500 = \$2500 < \$3000$. Hence, $p = \$20$ is the profit-maximizing price. Type L consumers will not buy under these prices.

(c) Compute the profit level made by this monopoly assuming now that this monopoly can price discriminate between the two consumer populations. Does the monopoly benefit from price discrimination. Prove your result!

- The monopoly will change $p = \$20$ in market H and $p = \$10$ in market L. Hence, total profit is given by

$$\Pi = \pi_H + \pi_L = (20 - 5)\,200 + (10 - 5)\,300 = 3000 + 1500 = \$4500 > \$3000.$$

  Clearly, the ability to price discriminate cannot reduce the monopoly profit since even with this ability, the monopoly can always set equal prices in both markets. The fact that the monopoly chooses different prices implies that profit can only increase beyond the profit earned when the monopoly is unable to price discriminate.

Exercise 6

[**Third-degree price discrimination**] The demand function for concert tickets to be played by the Pittsburgh symphony orchestra varies between nonstudents (N) and students (S). Formally, the two demand functions of the two consumer groups are given by

$$q_N = 240\frac{1}{p_N^2} \quad \text{and} \quad q_S = 540\frac{1}{p_S^3}.$$

Assume that the orchestra's total cost function is $C(Q) = 2Q$ where $Q = q_N + q_S$ is to total number of tickets sold. Compute the concert ticket prices set by this monopoly orchestra, and the resulting ticket sales, assuming that the orchestra can price discriminate between the two consumer groups.

- The demand price elasticity is -2 in the nonstudents' market, and -3 in the students' market. In the nonstudents' market, the monopoly sets p_N to solve

$$p_N\left[1 + \frac{1}{-2}\right] = \$2 \text{ yielding } p_N = \$4 \text{ and hence } q_N = \frac{240}{4^2} = 15.$$

  In the students' market, the monopoly sets p_S to solve

$$p_S\left[1 + \frac{1}{-3}\right] = \$2 \text{ yielding } p_S = \$3 \text{ and hence } q_S = \frac{540}{3^3} = 20.$$

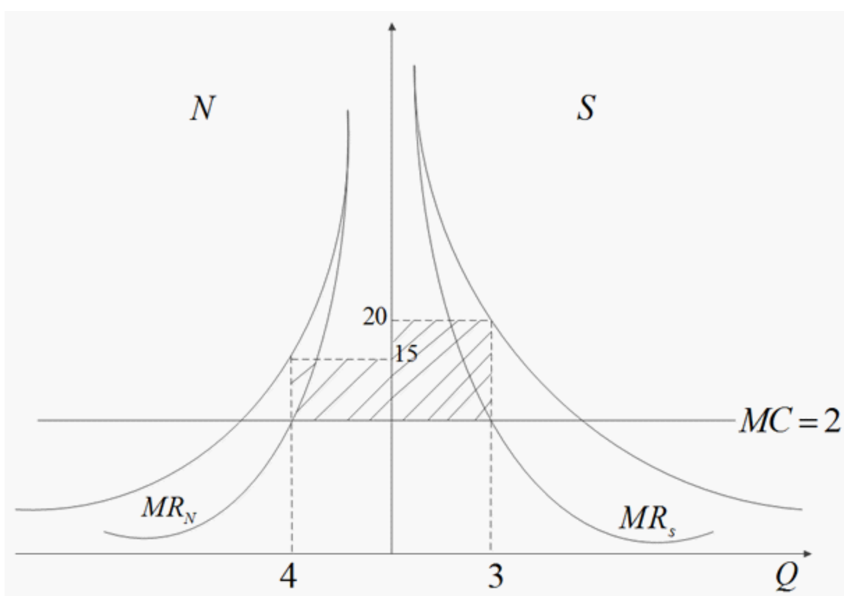

Figure 8

Exercise 7

1. **[Comparative statics]** Suppose that the inverse demand curve facing a monopolist is given by $p(q,t)$, where t is an exogenous parameter that shifts the demand curve, e.g., a new fad making that product attractive for more customers. For simplicity, assume that the monopolist has a technology that exhibits constant marginal costs.

   (a) Derive an expression showing how output responds, q, to a change in t.

   - The monopolist's profit maximization problem becomes

     $$\max_{y} \quad p(q,t)\,q - cq.$$

     where the monopolist's choice variable is now the price he charges, p. The first-order condition for this problem is

     $$\Phi(q,t) = p(q,t) + \frac{\partial p(q,t)}{\partial q} q - c = 0.$$

     Using the implicit function therorem, we obtain

     $$\frac{dq}{dt} = -\frac{\frac{\partial \Phi}{\partial t}}{\frac{\partial \Phi}{\partial q}} = -\frac{p_t + p_{qt} q}{2p_q + p_{qq} q}$$

   (b) How does this expression simplify if the inverse demand function takes the special form $p(y,t) = a(q) + b(t)$?

   - For the special case $p(q,t) = a(q) + b(t)$,

     $$\frac{dq}{dt} = -\frac{p_t' + p_{qt}'' q}{2p_q' + p_{qq}'' q} = -\frac{\frac{\partial b}{\partial t}}{2\frac{\partial a}{\partial q} + \frac{\partial^2 a}{\partial q^2} q}$$

     since in this case $\frac{\partial\left(\frac{\partial a}{\partial q}\right)}{\partial t} = 0$.