EconS 501 - Microeconomic Theory I[1]
Recitation #8 - Choice under Uncertainty-I

Exercise 1

1. **Exercise 6.B.2, MWG:** Show that if the preference relation $\succsim$ on $\mathcal{L}$ is represented by a utility function $U(\cdot)$ that has the expected utility form, then $\succsim$ satisfies the independence axiom.

   - Assume that the preference relation $\succsim$ is represented by an $v.N-M$ expected utility function $U(L)=\sum_n u_n p_n$ for every $L=(p_1,...,p_N)\in\mathcal{L}$. Let

     $$L=(p_1,...,p_N)\in\mathcal{L},\ L'=\left(p_1^{'},...,p_N^{'}\right)\in\mathcal{L},\ L''=\left(p_1^{''},...,p_N^{''}\right)\in\mathcal{L},$$

     and $\alpha\in(0,1)$. Then $L\succsim L'$ if and only if $\sum_n u_n p_n \geq \sum_n u_n p_n^{'}$. This inequality is equivalent to

     $$\alpha\left(\sum_n u_n p_n\right)+(1-\alpha)\left(\sum_n u_n p_n^{''}\right)\geq\alpha\left(\sum_n u_n p_n^{'}\right)+(1-\alpha)\left(\sum_n u_n p_n^{''}\right).$$

     where we just added the same number, i.e., the utility of lottery L'', to both sides of the inequality. This latter inequality holds if and only if

     $$\alpha L+(1-\alpha)L''\succsim\alpha L'+(1-\alpha)L''.$$

     Hence $L\succsim L'$ if and only if

     $$\alpha L+(1-\alpha)L''\succsim\alpha L'+(1-\alpha)L''.$$

     Thus the independence axiom holds.

Exercise 2

2. **Exercise 6.B.5, MWG:** The purpose of this exercise is to show that the Allais paradox is compatible with a weaker version of the independence axiom. We consider the following axiom, known as the *betweenness axiom* [see Dekel (1986)]:

   $$\text{For all } L,\ L' \text{ and } \alpha\in(0,1),\ \text{if } L\sim L', \text{ then } \alpha L+(1-\alpha)L'\sim L.$$

   Suppose that there are three possible outcomes.

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a. Show that a preference relation on lotteries satisfying the independence axiom also satisfies the betweenness axiom.

- **Answer:** Following the independence axiom we can state that

$$\text{if } \; L \sim L' \; \text{ then } \; \underbrace{(1-\alpha)L + \alpha L}_{L} \sim \alpha L + (1-\alpha)L'$$

Thus $L \sim \alpha L + (1-\alpha)L'$. This means that if the preference relation satisfies the independence axiom it then also satisfies the betweenness axiom.

b. Using a simplex representation for lotteries similar to the one in Figure 6.B.1 (page 169 in MWG), show that if the continuity and betweenness axioms are satisfied, then the indifference curves of a preference relation on lotteries are straight lines. Conversely, show that if the indifference curves are straight lines, then the betweenness axiom is satisfied. Do these straight lines need to be parallel?

- **Answer:** Indifference courves are straight lines if for every pair of lotteries L, L', we have that $L \sim L'$ implies $\alpha L + (1-\alpha)L' \sim L$ for all $\alpha \in (0,1)$. That is, if decision maker is indiferent between the compond lottery $\alpha L + (1-\alpha)L'$ (the linear combination of two simple lotteries) and either of the simple lotteries L or L' that generated such compound lottery.

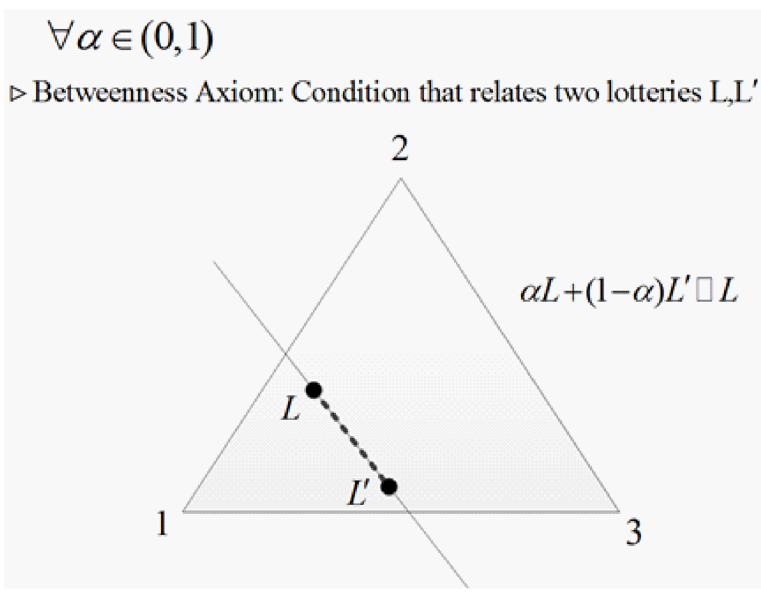

Figure 1. The betweeness axiom

The independence axiom guarantees that indifference curves over lotteries must be not only straight lines but also parallel; as depicted in figure 2.

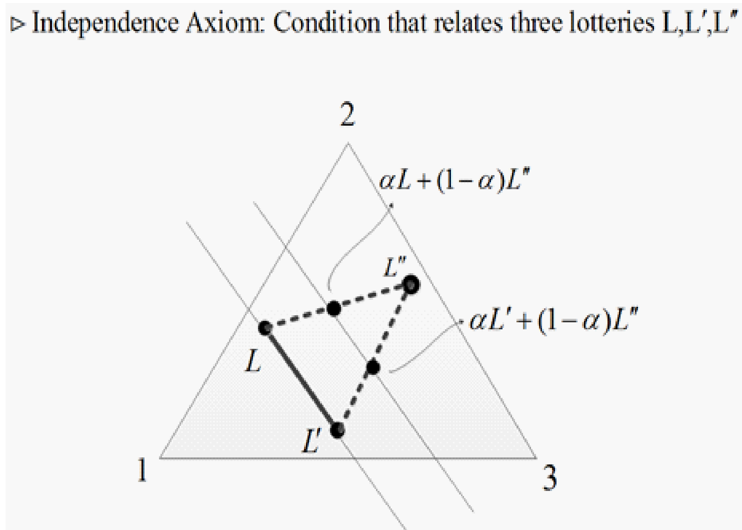

Figure 2. The independence axiom.

When continuity and betweenness axioms are satisfied, then $L \sim L' \Rightarrow \alpha L + (1-\alpha)L' \sim L$ or L' for all $\alpha \in (0,1)$. That is any linear combination is indiferent, which means indifference courves are linear or straight lines.

Also when indifference courves are straight lines, any linear combination of the indiffernce lotteies is is also indifferent. That is $L \sim L' \Rightarrow \alpha L + (1-\alpha)L' \sim L$ or L', then continuity and betweenness axioms are satisfied. The independence axiom guarantees that indifference curves over lotteries must be not only straight lines but also parallel.

That is, if independence axiom holds, $L \sim L' \Rightarrow \alpha L + (1-\alpha)L'' \sim \alpha L' + (1-\alpha)L''$ for all $\alpha \in (0,1)$, set $L^* = \alpha L + (1-\alpha)L''$, $L^{**} = \alpha L' + (1-\alpha)L''$, then $L^* \sim L^{**}$. For each $\alpha \in (0,1)$, we have one pair of L^* and L^{**}. Thus, the linear combinations of L^* and L^{**} for different α are on parallel lines.

c. Using (b), show that the betweenness axiom is weaker (less restrictive) than the independence axiom.

- **Answer:** Any preference represented by straight, but not parallel indifference curves, satisfies the betweenness axiom but does not satisfy the independence axiom. Hence the betweenness axiom is weaker than the independence axiom. In other words, the IA $\Longrightarrow$ BA, but IA$\not\Leftarrow$BA. (See figure 3, illustrating an example of indifference curves that satisfy the BA but do not satisfy the IA).

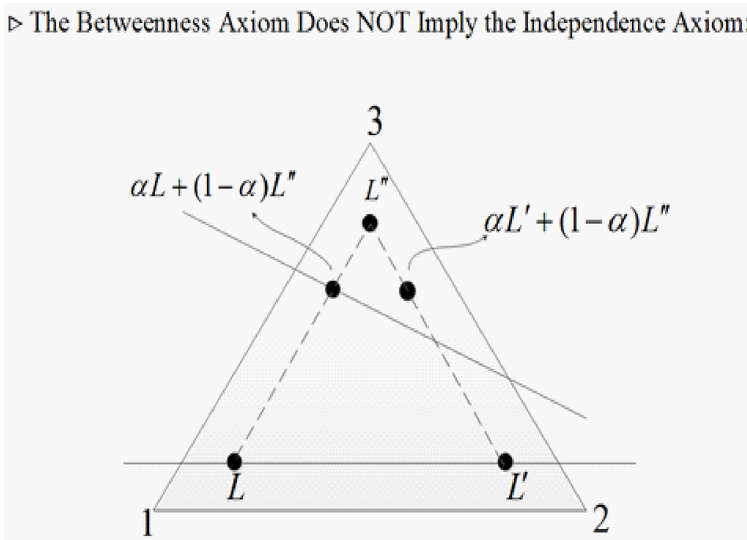

Figure 3. The betweeness and independence axioms.

Exercise 3

3. A security agency with vNM utility function u evaluates two disaster plans for the evacuation of an area prone to flooding. The probability of flooding is 1%. There are four possible outcomes:

$$\begin{cases} a_1: & \text{no evacuation, no flooding,} \\ a_2: & \text{no evacuation, but flooding,} \\ a_3: & \text{evacuation, no flooding,} \\ a_4: & \text{evacuation, flooding.} \end{cases}$$

The agency is indifferent between the sure outcome a_3 and the lottery of a_1 with probability $p \in (0,1)$ and a_2 with probability $1-p$ and between the sure outcome a_4 and the lottery of a_1 with probability $q \in (0,1)$ and a_2 with probability $1-q$. Further, $u(a_1) = 1$ and $u(a_2) = 0$. Moreover,

$$\begin{aligned} a_3 &\sim (a_1, a_2; p, 1-p) \\ a_4 &\sim (a_1, a_2; q, 1-q) \\ u(a_1) &= 1, \quad u(a_2) = 0 \end{aligned}$$

(a) Express $u(a_3)$ and $u(a_4)$ in terms of p and q.

- **Answer:** Given $\succsim$ on $\mathcal{L}$ can be represented by a utility function $u(\cdot)$

$$\begin{aligned} u(a_3) &= pu(a_1) + (1-p)u(a_2) = p \\ u(a_4) &= qu(a_1) + (1-q)u(a_2) = q \end{aligned}$$

The two disaster plans are summarized as follows:

- *Plan 1:* results in an evacuation in 90% of the cases where a flooding does occur and in 10% of the cases where no flooding occurs.
- *Plan 2:* results in an evacuation in 95% of the cases where a flooding does occur and in 15% of the cases where no flooding occurs.

b. For each of these two plans, compute the probability distribution over the four outcomes $\{a_1, a_2, a_3, a_4\}$.

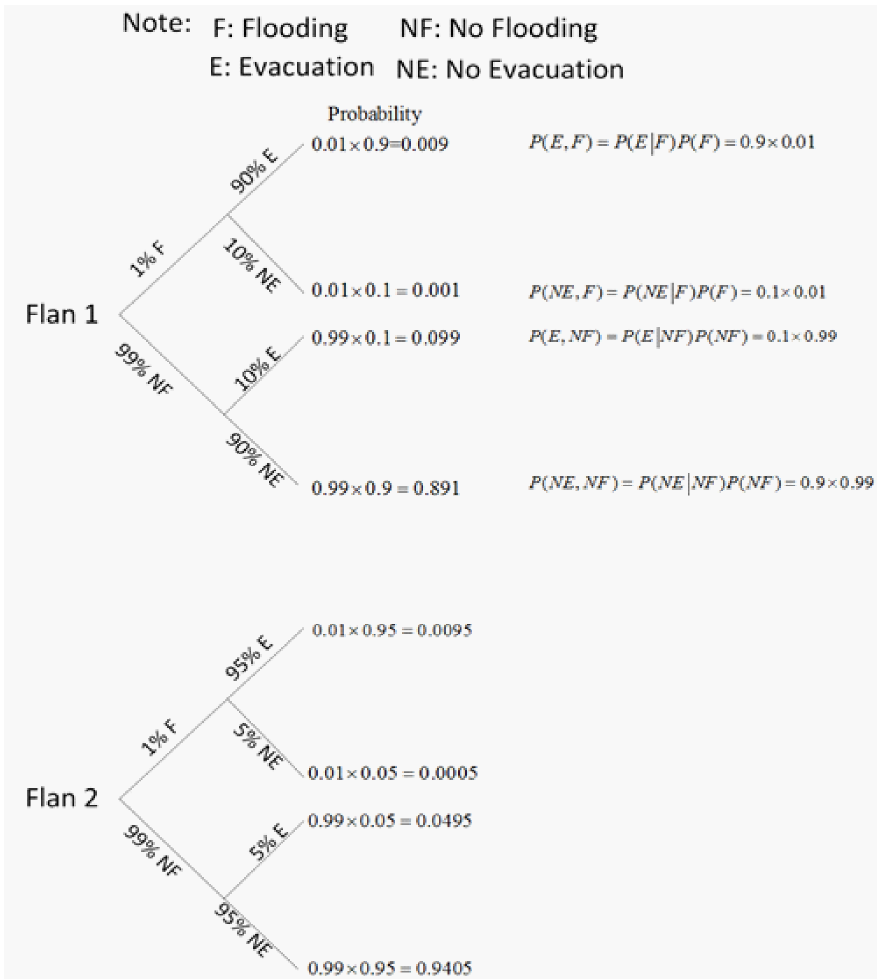

Figure 5

c. Compute the expected utility of each of the two plans. When is plan 1 strictly preferred over plan 2?

- **Answer:**

$$\begin{aligned} u(a_1) &= 1 \\ u(a_2) &= 0 \\ u(a_3) &= p \\ u(a_4) &= q \end{aligned}$$

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$$\begin{aligned} u\left(Plan_1\right) &= 0.891 \cdot u\left(a_1\right) + 0.001 \cdot u\left(a_2\right) + 0.099 \cdot u\left(a_3\right) + 0.009 \cdot u\left(a_4\right) \\ &= 0.891 + 0.099p + 0.009q \\ u\left(Plan_2\right) &= 0.8415 \cdot u\left(a_1\right) + 0.0005 \cdot u\left(a_2\right) + 0.1485 \cdot u\left(a_3\right) + 0.00954 \cdot u\left(a_4\right) \\ &= 0.8415 + 0.1485p + 0.0095q \end{aligned}$$

Hence, Plan 1 is strictly preferred to Plan 2 if and only if

$$u\left(Plan_1\right) > u\left(Plan_2\right)$$

$$\begin{aligned} &\Longleftrightarrow 0.891 + 0.099p + 0.009q > 0.8415 + 0.1485p + 0.0095q \\ &\Longleftrightarrow 0.0495p + 0.0005q < 0.0495 \\ &\Longleftrightarrow q < 99\left(1 - p\right) \end{aligned}$$

But given that $q \in (0, 1)$, this condition can **always** be satisfied when,

$$1 < 99\left(1 - p\right) \Longleftrightarrow \frac{98}{99} > p$$

i.e., for almost all possible values of p, Plan 1 will always be strictly preferred to Plan 2.