EconS 501 – Micro Theory I[1]

Recitation #12 – Imperfect Competition

Exercise 15.3 (NS). [ON YOUR OWN] This exercise analyzes Cournot competition when firms have different marginal costs. This departure from identical firms allows the student to shift around firm's best-responses independently on a diagram.

Let c_i be the constant marginal and average cost for firm *i* (so that firms may have different marginal costs). Suppose demand is given by $P = 1 - Q$.

a. Calculate the Nash equilibrium quantities assuming there are two firms in a Cournot market. Also compute market output, market price, firm profits, industry profits, consumer surplus, and total welfare.

   ANSWER: Let us find the output level that firm 1 selects when maximizing its profits in this Cournot duopoly

$$\max_{q_1} q_1(1 - q_1 - q_2) - c_1 q_1$$

$$\frac{\partial}{\partial q_1} : 1 - 2q_1 - q_2 = c_1$$

$$q_1 = \frac{1 - q_2 - c_1}{2} \quad \text{best response for consumer 1}$$

   Likewise

$$q_2 = \frac{1 - q_1 - c_2}{2} \quad \text{best response for consumer 2}$$

   Solving simultaneously for q_1 and q_2,

$$q_1 = \frac{1 - \frac{1 - q_1 - c_2}{2} - c_1}{2}$$

$$\frac{4q_1}{2} - \frac{q_1}{2} = 1 - \frac{1}{2} + \frac{c_2}{2} + -c_1$$

   which yields an output for firm 1 of

$$q_1^c = \frac{1 - 2c_1 + c_2}{3}$$

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And similarly, an output for firm 2 of $q_2^c = \frac{1-2c_2+c_1}{3}$.

Further, aggregate output is

$$Q^c = q_1^c + q_2^c = \frac{1-2c_2+c_1}{3} + \frac{1-2c_1+c_2}{3} = \frac{(2-c_1-c_2)}{3},$$

which implies that equilibrium price is

$$P^c = 1 - Q^c = \frac{(1+c_1+c_2)}{3},$$

and that firm 1's profits are

$$\pi_1^c = q_1 p - c_1 q_1 = \frac{(1-2c_1+c_2)^2}{9},$$

and those of firm 2 are

$$\pi_2^c = q_2 p - c_2 q_2 = \frac{(1-2c_2+c_1)^2}{9} \;,$$

As a consequence, aggregate profits are

$$\Pi^c = \pi_1^c + \pi_2^c = \frac{(1-2c_1+c_2)^2 + (1-2c_2+c_1)^2}{9}$$

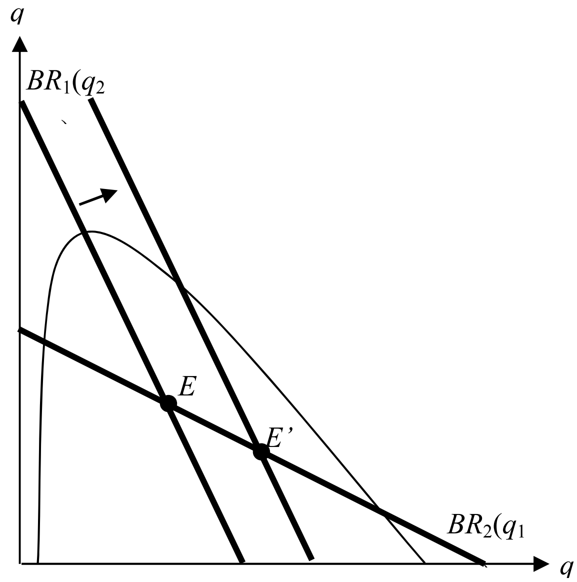
and consumer surplus is

$$CS^c = \frac{1}{2} \cdot Q \cdot (1-p) = \frac{(2-c_1-c_2)^2}{18},$$

and aggregate welfare is thus the sum of aggregate profits and consumer surplus, $W^c = \Pi^c + CS^c$.

b. Represent the Nash equilibrium on a best-response function diagram. Show how a reduction in firm 1's cost would change the equilibrium. Draw a representative isoprofit for firm 1.

ANSWER: Point E in following figure represents the Nash equilibrium. The curved line represents firm 1's isoprofit.



The reduction in firm 1's marginal cost shifts its best response outwards ($q_1 = \frac{1 - q_2 - c_1}{2}$) and shifts the equilibrium from *E* to *E'*. Firm 1 will produce more for any given q_2.

Exercise 15.7 (NS). This exercise analyzes the Stackelberg game both with and without the possibility of entry-deterring investment.

Assume as in Problem 15.1 of NS that two firms with no production costs, facing a direct demand function $Q = 150 - P$, choose quantities q_1 and q_2. [In Problem 15.1, firms were assumed to compete a la Cournot, simultaneously selecting quantities. In that context, you can easily show that every firm *i*'s best response function is $q_i = 75 - \frac{1}{2} q_j$.]

a. Compute the subgame-perfect equilibrium of the Stackelberg version of the game in which firm 1 chooses q_1 first and then firm 2 chooses q_2.

   ANSWER: We solve the game using backward induction starting with firm 2's action. Firm 2 moves second and best responds to firm 1's choice. We saw from Problem 15.1 (b) that firm 2's best-response function is $q_2 = 75 - \frac{q_1}{2}$. We substitute this back into firm 1's profit function so that firm 1 is making its optimal choice given what it expects firm 2 to do

$$\pi_1 = q_1[150-(q_1+q_2)] = q_1\left[150 - q_1 - \left(75 - \frac{q_1}{2}\right)\right]$$

$$\frac{\partial}{\partial q_1}: 150 - 2q_1 - 75 = -q_1$$

$$q_1 = 75$$

Taking the first-order condition with respect to q_1 and solving yields $q_1^* = 75$. Substituting this back into firm 2's best-response function yields

$$q_2 = 75 - \tfrac{75}{2}$$

$$q_2^* = 37.5$$

b. Now add an entry stage after firm 1 chooses q_1. In this stage, firm 2 decides whether or not to enter. If it enters it must sink cost K_2, after which it is allowed to choose q_2. Compute the threshold value of K_2 above which firm 1 prefers to deter firm 2's entry.

ANSWER: If firm 1 accommodates 2's entry, the outcome in part (a) arises. P=150 – Q. P = 37.5. There are no production costs, thus profit for firm 1= P*Q=37.5*75= 2,812.5.

When firm 1 produces $\bar{q}_1$ if firm 2 best responds to $\bar{q}_1$, and enters it will generate profit $p \cdot q_2 - K_2 = (150-\bar{q}_1)^2/4 - K_2$. In order to deter entry this profit must be less than or equal to 0. Setting firm 2's profit function equal to zero

$$\frac{(150-\bar{q}_1)^2}{4} - K_2 = 0$$

$$\bar{q}_1 = 150 - 2\sqrt{K_2}$$

The threshold value of $\bar{q}_1 = 150 - 2\sqrt{K_2}$. Firm 1's profit from operating alone in the market and producing this output is *Q**P*= $(150-2\sqrt{K_2})(2\sqrt{K_2})$, which exceeds 2,812.5, the profit in part (a), if $K_2 \geq 120.6$. (as can be shown by graphing both sides of the inequality)

Exercise 15.9 (NS). This exercise examines the "*Herfindahl index*" of market concentration. Many economists subscribe to the conventional wisdom that increases in concentration are bad for social welfare. This problem leads students through a series of calculations showing that that the relationship between welfare and concentration is not this straightforward.

One way of measuring market concentration is through the use of the Herfindahl index, which is defined as:

$$H = \sum_{i=1}^{n} s_i^2 \qquad \text{where} \quad s_i = \frac{q_i}{Q}$$

Where s_i is firm *i*'s market share. The higher is *H*, the more concentrated the industry is said to be. Intuitively, more concentrated markets are thought to be less competitive because dominant firms in concentrated markets face little competitive pressure. We will assess the validity of this intuition using several models.

a. Recall from class the equilibrium output levels in the *n*-firm Cournot game. If you don't remember it, don't worry, here it is again for you to practice. Find equilibrium output level for every firm *i*, the resulting aggregate output, and the market price. In addition, evaluate consumer surplus, industry profit, and total welfare in this equilibrium. Once you are done, compute the Herfindahl index for this equilibrium.

ANSWER: Firm *i*'s profit is $q_i(a - bq_i - bQ_{-i} - c)$ with associated first-order condition $a - 2b - bQ_{-i} - c = 0$.
This is the same for every *n* firm so we may impose symmetry [$Q_{-i}^* = (n-1)q_i^*$]. Plugging in the symmetry condition in the above first-order condition, yields

$$a - 2b - b(n-1)q_i^* - c = 0$$

$$q_i^* = \frac{(a-c)}{(n+1)b}$$

Further, aggregate output is

$$Q^* = \frac{n(a-c)}{(n+1)b},$$

Thus implying that equilibrium price is

$$P^* = \frac{(a+nc)}{(n+1)},$$

And that aggregate (industry) profits are

$$\Pi^* = n\pi_i^* = \frac{n}{b}\left[\frac{(a-c)}{(n+1)}\right]^2,$$

where individual profits are just 1/*n* share of aggregate profits since firms are symmetric. As a consequence, consumer surplus is

$$CS^* = \frac{n^2}{b}\left[\frac{(a-c)}{(n+1)}\right]^2$$

while overall social welfare (i.e., the sum of aggregate profits plus consumer surplus) is

$$W^* = \frac{n}{(n+1)}\left[\frac{(a-c)^2}{b}\right].$$

Because firms are symmetric, $s_i = 1/n$, thus we can solve for the Herfindahl index

$$H = n(\tfrac{1}{n})^2 = \tfrac{1}{n}.$$

b. Suppose two of the *n* firms merge, leaving the market with *n*-1 firms. Recalculate the Nash equilibrium and the rest of the items requested in part (a). How does the merger affect price, output, profit, and total welfare. Compute the Herfindahl index for this equilibrium.

ANSWER: We can obtain a rough idea of the effect of merger by seeing how the variables in part (a) change with a reduction in n. Per-firm output, price, industry profit, and the Herfindahl index increase with a reduction in *n*, caused by the merger. Total output, consumer surplus, and welfare decrease with a reduction in *n*, caused by the merger.

c. Put the model used in parts (a) and (b) aside and turn to a different setup: that of Problem 15.3, where Cournot duopolists face different marginal costs. Use your answer to Problem 15.3(a) to compute equilibrium firm outputs, market output, price, consumer surplus, industry profit, and total welfare, substituting the particular cost parameters $c_1 = c_2 = \frac{1}{4}$. Also compute the Herfindahl index.

ANSWER: Substituting $c_1 = c_2 = 1/4$ into the answers for 15.3, we have $q_i^* = 1/4$, $Q^* = 1/2$, $P^* = 1/2$, $\Pi^* = 1/8$, $CS^* = 1/8$, and $W^* = 1/4$. Also, $H = 1/2$.

d. Repeat your calculations in part (c) while assuming that firm 1's marginal cost c_1 falls to 0 but c_2 stays at $\frac{1}{4}$. How does the merger affect price, output, profit, consumer surplus, total welfare, and the Herfindahl index.

ANSWER: Substituting $c_1 = 0$ and $c_2 = 1/4$ into the answers for 15.3, we have $q_1^* = 5/12$, $q_1^* = 2/12$, $Q^* = 7/12$, $P^* = 5/12$, $\Pi^* = 29/144$, $CS^* = 49/288$, and $W^* = 107/288$. Also, $H = 29/49$.

e. Given your results from parts (a)-(d), can we draw any general conclusions about the relationship between market concentration on the one hand and price, profit, or total welfare on the other?

   ANSWER: Comparing part (a) with (b) suggests that increases in the Herfindahl index are associated with lower welfare. The opposite is evidenced in the comparison of part (c) to (d): welfare and the Herfindahl increase together. General conclusions are thus hard to reach.

Exercise 15.10 (NS). [ON YOUR OWN] This exercise extends the *Inverse Elasticity Pricing Rule* (IEPR) from a market structure with only one firm (monopoly) to market structures with more than one firm. It derives an alternatives form of the IEPR we know under monopoly that we can apply into a Cournot model of quantity competition.

a. Use the first-order condition (Equation 15.2) for a Cournot firm to show that the usual inverse elasticity rule form Chapter 11 holds under Cournot competition (where the elasticity is associated with an individual firm's residual demand, the demand left after all rivals sell their output on the market). Manipulate Equation 15.2 in a different way to obtain an equivalent version of the inverse elasticity rule:

$$\frac{P-MC}{P}=-\frac{s_i}{e_{Q,P}} \quad \text{where} \quad s_i=\frac{q_i}{Q}$$

where s_i is firm i's market share and $e_{Q,P}$ is the elasticity of market demand. Compare this version of the inverse elasticity rule to that for a monopolist from the previous chapter.

ANSWER: Equation 15.2 can be rearranged as follows:

Equation 15.2

$$\frac{\partial \pi_i}{\partial q_i}=P(Q)+P'(Q)q_i-C_i'(q_i)=0$$

$$\frac{P-C'}{P}=\frac{-P'q_i}{P} \qquad ,$$

$$\frac{P-C'}{P}=\frac{-P'q_i}{P}=\frac{-dP/dq_i\cdot q_i}{P}=\frac{1}{|\varepsilon_{q_i,P}|}$$

where $\varepsilon_{q_i,P}$ is the elasticity of demand with respect to firm i's output. The second equality uses the fact that $P'=dP/dQ=dP/dq_i$. Multiplying numerator and denominator by Q, we can also rearrange Equation 15.2 as

$$\frac{P - C'}{P} = \frac{-P' q_i}{P} = \frac{-dP/dq_i \cdot q_i}{P} = \left(\frac{-dP/dQ \cdot Q}{P} \right) \left(\frac{q_i}{Q} \right) = \frac{s_i}{|\varepsilon_{Q,P}|}$$

MWG 12.C.10. Consider a *J*-firm Cournot model in which firms' costs differ. Let $c_j(q_j) = \alpha_j \tilde{c}(q_j)$ denote firm *j*'s cost function, and assume that $\tilde{c}(\cdot)$ is strictly increasing and convex. Assume that $\alpha_1 > \ldots > \alpha_j$.

(a) Show that if more than one firm is making positive sales in a Nash equilibrium of this model, then we cannot have productive efficiency; that is, the equilibrium aggregate output Q^* is produced inefficiently.

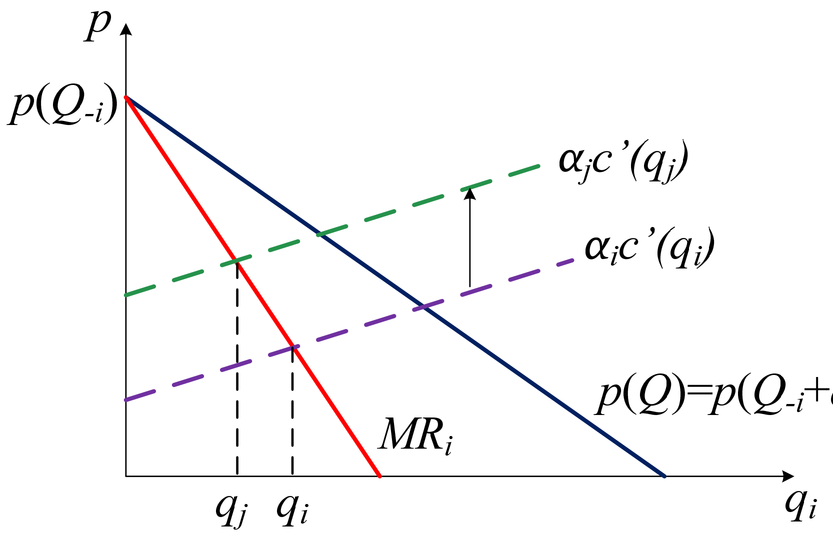
ANSWER: Each firm *i* chooses its output $q_i \geq 0$ to maximize its profits

$$\pi = p(Q_{-i} + q_i)q_i - \alpha_i \tilde{c}(q_i)$$

Taking first-order conditions with respect to q_i, and assuming an interior solution, yields

$$p(Q) + p'(Q)q_i = \alpha_i \tilde{c}(q_i)$$

The left-hand side represents the marginal revenue that, for a given Q_{-i} output produced by all other firms, firm *i* obtains when it increases its individual production q_i. Since $p'(Q)<0$, the marginal revenue lies below the demand curve $p(Q)$; as illustrated in the figure below. The right-hand side, in contrast, represents the marginal cost of firm *i*, which graphically is increasing in q_i given convexity in costs. You can visually notice that a given increase in α, for instance if firm *j* has $\alpha_j > \alpha_i$ then the marginal cost curve shifts upwards, thus crossing the marginal revenue of firm *j* to the left-hand side, i.e., $q_j < q_i$.



Imagine the case where firm *i* differs from firm *j* we may calculate the difference in marginal cost between the two firms as

$$\alpha_j \tilde{c}'(q_j) - \alpha_i \tilde{c}'(q_i)$$

From our first-order conditions we know that after canceling out the $p(Q)$ terms this is equivalent to

$$-p'(Q)(q_i - q_j)$$

Imagine the particular case where $\alpha_j > \alpha_i$ so that also $q_i > q_j$. Because $p'(Q) < 0$ the term $-p'(Q)(q_i - q_j)$ is positive. This implies that $\alpha_j \tilde{c}'(q_j) > \alpha_i \tilde{c}'(q_i)$. Thus the marginal cost for firm *j* is greater in this case, implying that marginal costs across firms are not necessarily equalized.

Likewise aggregate output is not necessarily produced efficiently. Indeed, for aggregate output *Q* to be efficient, we would need that the sum of marginal costs coincides with the marginal revenue curve described above. In the setting in which firms independently choose their output levels, every firm *i*'s output is increased until the point in which the marginal revenue is equal to firm *i*'s marginal costs. If firms' marginal costs were constant and symmetric, then their aggregate output would be efficient. However, given that firms' marginal costs are increasing (convexity), aggregate output is not necessarily efficient.

(b) If so, what is the correct measure of welfare loss relative to a fully efficient (competitive) outcome? [*Hint*: Reconsider the discussion in Section 10.E]

ANSWER: The correct measure of welfare loss relative to a fully efficient outcome in this case is equal to the loss of consumer surplus due to non-competitive pricing plus the higher production cost due to productive inefficiency.

(c) Provide an example in which welfare decreases when a firm becomes more productive (i.e., when α_j falls for some *j*). [*Hint*: Consider an improvement in cost for firm 1 in the model of Exercise 12.C.9.] Why can this happen?

ANSWER: Let's use our results from Exercise 12.C.9 in order to provide an example in which welfare decreases when a firm becomes more productive. From 12.C.9 the Cournot equilibrium output and price levels are

$$q_1 = \frac{a - 2c_1 + c_2}{3b}$$

$$q_2 = \frac{a - 2c_2 + c_1}{3b}$$

$$p = \frac{a + c_1 + c_2}{3}$$

Total profits of the two firms can now be computed as

$$(p - c_1)q_1 + (p - c_2)q_2 = (2a^2 + 5c_1^2 + 5c_2^2 - 2a(c_1 + c_2) - 8c_1c_2)/9b$$

Consumer surplus can be computed as

$$\int_0^{q_1+q_2} p(q)dq = (aq - bq^2/2)\Big|_0^{q_1+q_2} = (a - c_1 - c_2)(5a + c_1 + c_2)/18b$$

Adding up total profits and consumer surplus, and differentiating with respect to c_1, we obtain

$$\frac{\partial Surplus}{\partial c_1} = \frac{9c_1 - 9c_2 - 4a}{9b}$$

This derivative is positive when $c_1 > c_2 + (\frac{4}{9})a$. This will occur when firm 1's costs are much greater than firm 2's costs. In this case a decrease in c_1 reduces social welfare. The reason is that when c_1 slightly falls, firm 1 steals more business from firm 2, which raises production inefficiency. When c_1 is substantially larger than c_2, this effect actually dominates the increase in consumer surplus due to a lower price.

MWG 12.C.12. Consider two strictly concave and differentiable profit functions $\pi_j(q_i, q_j),\ i,j \in \{1,2\}$, defined on $q_j \in [0, q]$.

(a) Give sufficient conditions for the best-response functions $b_j(q_j)$ to be increasing or decreasing.

ANSWER: Assume that $\pi_{11}^i(q_i, q_j) < 0$ for $i = 1,2$. Where the subscript *ii* means differentiate twice with respect to the first element.

Every firm *i* will maximize profit by finding its best response function $b_i(q_j)$. We can evaluate the sign of firm *i*'s best response function (to check if it increases or decreases with its rival's output q_j) by applying the implicit function theorem, as follows

$$\frac{\partial}{\partial q_j} : \frac{\partial b_i(q_j)}{\partial q_j} = -\frac{\pi_{11}^i(b_i(q_j), q_j)}{\pi_{12}^i(b_i(q_j), q_j)}$$

Therefore, since the numerator satisfies $\pi_{11}^i(q_i, q_j) < 0$ for $i = 1,2$ by definition, the sign of $\frac{\partial b_i(q_j)}{\partial q_j}$ is the same as the sign of $\pi_{12}^i(b_i(q_j), q_j)$. In other words, firm *i*'s best-response function is increasing (decreasing) when the cross-derivative π_{12}^i is positive (negative, respectively).

(b) Specialize to the Cournot model. Argue that a decreasing (downward-sloping) best-response function is the "normal" case.

ANSWER: In the Cournot model

$$\pi^1(q_i, q_j) = p(q_i + q_j)q_i - c(q_i)$$

Then differentiating further with respect to q_j

$$\pi^i_{12}(q_i, q_j) = p''(q_i + q_j)q_i + p'(q_i + q_j)$$

which is negative if $p(\cdot)$ is downward sloping and not to convex. This seems likely for the inverse demand function, thus the "normal" slope of the best response functions in the Cournot model is negative.

MWG 12.D.1. Consider an infinitely repeated Bertrand duopoly with discount factor $\delta < 1$. Determine the conditions under which strategies of the form in (12.D.1) sustain the monopoly price in each of the following cases:

From page 401 (12.D.1 in MWG):

$$P_{jt}(H_{t-1}) = \begin{cases} p^m & \text{if all elements of } H_{t-1} \text{ equal } (p^m, p^m) \text{ or } t = 1 \\ c & \text{otherwise} \end{cases}$$

(a) Market demand in period t is $x_t(p) = \gamma^t x(p)$ where $\gamma > 0$ is the rate of growth of demand across periods.

ANSWER: Monopoly profit in period t is

$$\max_p \gamma^t x(p)(p-c) = \gamma^t \max_p x(p)(p-c) = \gamma^t \pi^m$$

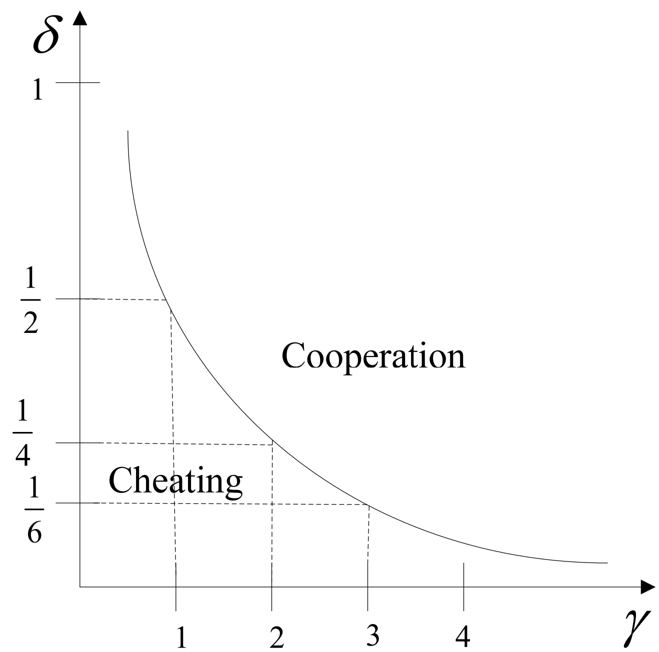
If a firm deviates at period $t = \tau$, it can obtain $\gamma^\tau \pi^m$ in that period, and it will get zero forever after. If it does not deviate, its payoff is

$$\gamma^\tau \sum_{t=0}^{\infty} (\gamma\delta)^t \frac{\pi^m}{2} = \frac{1}{(1-\gamma\delta)} \gamma^\tau \frac{\pi^m}{2}$$

Monopoly price can be sustained when deviation from the strategy is not profitable. Deviation is not profitable if and only if

$$\frac{1}{(1-\gamma\delta)} \gamma^\tau \frac{\pi^m}{2} \geq \gamma^\tau \pi^m \quad \text{or} \quad \delta \geq \frac{1}{2\gamma}$$

Hence, the minimal discount factor supporting cooperation decreases in the rate of growth of demand, i.e., cooperation can be sustained under a larger set of discount factors as demand grows faster across periods. [See figure below].



(b) At the end of each period, the market continues to exist with probability $\gamma \in [0,1]$.

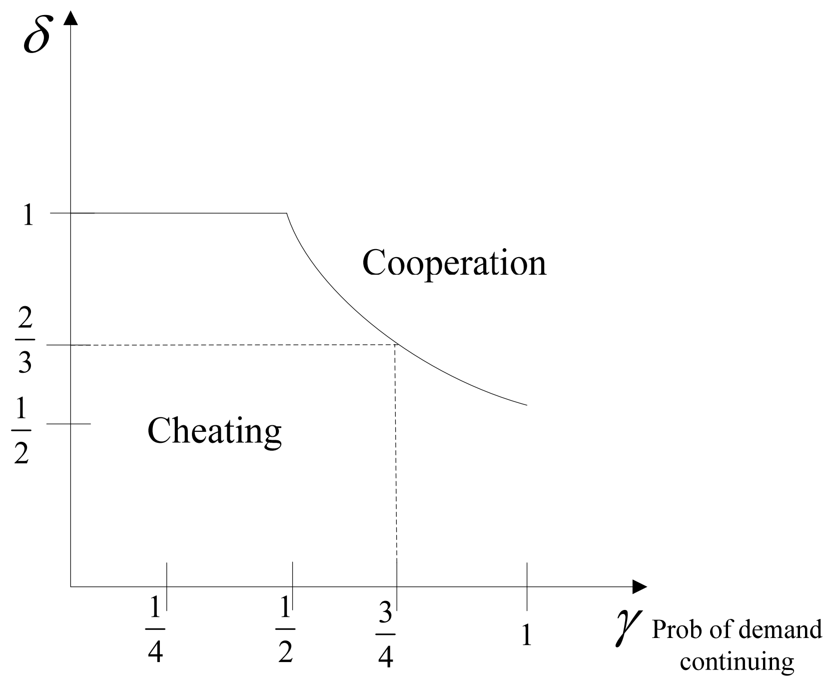
ANSWER: If a firm deviates, it can obtain π^m in that period, and it will get zero forever after. If it does not deviate, its payoff is

$$\sum_{t=0}^{\infty} (\gamma\delta)^t \frac{\pi^m}{2} = \frac{1}{(1-\gamma\delta)} \frac{\pi^m}{2}$$

Therefore, deviation is not profitable if and only if

$$\frac{1}{(1-\gamma\delta)} \frac{\pi^m}{2} \geq \pi^m \quad \text{or} \quad \delta \geq \frac{1}{2\gamma}$$

Thus, cooperation cannot be sustained under any discount factor (between zero and one) when the probability that demand continues existing is relatively low, but can be sustained when the probability that demand continues existing is sufficiently high (and decreases as this probability gets closer to 100%). [See figure, where δ and γ are both smaller than 1.]



(c) It takes K periods to detect and respond to a deviation from the collusive agreement.

ANSWER: If a firm deviates, it can obtain

$$\sum_{t=0}^{K-1} \delta^t \pi^m = \frac{(1-\delta^K)}{(1-\delta)} \pi^m$$

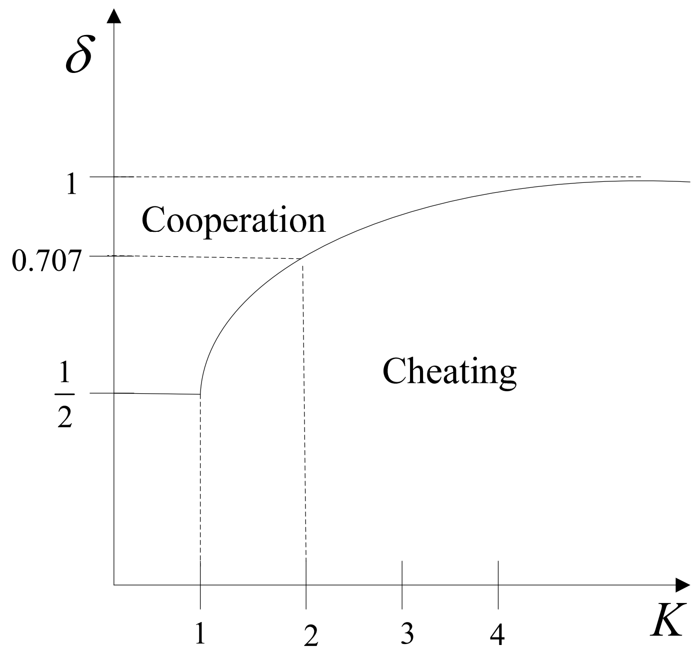
In the next K periods, and it will get zero forever after. If it does not deviate, its payoff is

$$\sum_{t=0}^{\infty} \delta^t \frac{\pi^m}{2} = \frac{1}{(1-\delta)} \frac{\pi^m}{2}$$

Therefore, deviation is not profitable if and only if

$$\frac{1}{(1-\delta)} \frac{\pi^m}{2} \geq \frac{(1-\delta^K)}{(1-\delta)} \pi^m \quad \text{or} \quad \delta \geq \left(\frac{1}{2}\right)^{\frac{1}{K}}$$

Hence, the more periods of time K that a cheating firm remains undetected by its colluding partners, the more attractive cheating becomes. Cooperation therefore can only be sustained under more restrictive sets of parameter values. [See figure].



δ
1
Cooperation
0.707
$\frac{1}{2}$
Cheating
1
2
3
4
K