EconS 301

Quiz #3 – *Answer key*

Exercise #1. Consider a firm with Cobb-Douglas production function

$$q = 4L^{1/3}K^{2/3}$$

a) Find the marginal rate of technical substitution of this firm, *MRTS*.

$$MP_L = \frac{1}{3}4L^{\frac{-2}{3}}K^{\frac{1}{3}} = \frac{4K^{\frac{2}{3}}}{3L^{\frac{2}{3}}}$$

$$MP_K = \frac{2}{3}4L^{\frac{1}{3}}K^{\frac{-1}{3}} = \frac{8L^{\frac{1}{3}}}{3K^{\frac{1}{3}}}$$

$$MRTS = \frac{\frac{4K^{\frac{2}{3}}}{3L^{\frac{2}{3}}}}{\frac{8L^{\frac{1}{3}}}{3K^{\frac{1}{3}}}} = \frac{4K^{\frac{2}{3}}}{3L^{\frac{2}{3}}} * \frac{3K^{\frac{1}{3}}}{8L^{\frac{1}{3}}} = \frac{K}{2L}$$

b) Solve the firm's cost-minimization problem (CMP) for *any* output level q units, and assuming that input prices are $w = \$4$ and $r = \$3$ for labor and capital, respectively. Which are the cost-minimizing amount of labor and capital that the firm hires?

- [*Hint*: Set the $MRTS$ found in part (a) equal to the input price ratio. Then, insert your result into the firm's production function. Last, solve for either L to find the cost-minimizing amount of labor, or for K to find the cost-minimizing amount of capital. Note that the expressions you find should be a function of the output that the firm seeks to produce, q.]

First, we take our MRTS and set it equal to our price ratio,

$$MRTS = \frac{w}{r}$$

$$\frac{K}{2L} = \frac{4}{3}$$

Solving for K gives us,

$$3K = 8L$$

$$K = \frac{8}{3}L$$

We then plug this into our production function and solve for q.

$$q = 4L^{1/3}K^{2/3}$$

$$q = 4L^{\frac{1}{3}}\left(\frac{8}{3}L\right)^{\frac{2}{3}}$$

Solving for L gives us,

$$L^{\frac{1}{3}}L^{\frac{2}{3}} = q\ (4^{-1})\left(8^{-\frac{2}{3}}\right)\left(3^{\frac{2}{3}}\right)$$

And, after rearranging, we obtain

$$L = \frac{q}{7.69} = 0.13q$$

Plugging this back into the K function we found earlier gives us

$$K = \frac{8}{3}L$$
$$K = \frac{8}{3}(0.13q)$$
$$K = 0.347q$$

c) What is the firm's total cost from hiring the optimal units of labor and capital you found in part (b)? The total cost $TC(q)$ that you find should also be a function of output q.

In this setting, our total cost can be defined as

$$TC = wL + rK$$
$$TC = 4(0.13q) + 3(0.347q)$$
$$TC = 0.52q + 1.04q = 1.56q$$

d) Use the total cost $TC(q)$ found in part (c) to find the marginal cost, $MC(q)$, and the average cost, $AC(q)$.

$$MC = \frac{\partial TC}{\partial q} = 1.56$$

$$AC = \frac{TC}{q} = \frac{1.56q}{q} = 1.56$$

e) If the firm operates in a perfectly competitive market, facing a given price of p for each unit of output, which is its "shut-down" price?

- [*Hint*: This "shut-down" price occurs at the minimum of the $AC(q)$ curve or, alternatively, at the point where the $AC(q)$ crosses the $MC(q)$ curve.]

In this setting, our "shut-down" price is anytime that $p < AC$. Since we know from above that our $MC = AC = 1.56$, our "shut-down" price would be anytime that $p < 1.56$.

f) Use the "shut-down" price of part (e) to identify the firm's supply curve. (This is a long-run supply curve, since there are no fixed costs.)

When $p <\ 1.56$, we have q = 0. When $p \geq 1.56$, then our q is determined from $p = mc$.