EconS 301

Quiz #2 – *Answer key*

Exercise #1. Consider a consumer with Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$$

Assume that the consumer faces a price of \$1 for good 2, and a total income of \$150. The price of good 1 decreases from \$3 to \$2. We next analyze the substitution and income effect of this price change.

a) Find the optimal consumption bundle at the *initial* price of \$3. Label it bundle *A*.

ANSWER: The marginal rate of substitution in this case is

$$MRS_{1,2} = \frac{MU_1}{MU_2} = \frac{\frac{1}{3} x_1^{\frac{1}{3}-1} x_2^{2/3}}{\frac{2}{3} x_1^{1/3} x_2^{\frac{2}{3}-1}} = \frac{x_1^{-\frac{2}{3}} x_2^{2/3}}{2x_1^{1/3} x_2^{-\frac{1}{3}}} = \frac{x_2^{2/3} x_2^{1/3}}{2x_1^{1/3} x_1^{2/3}} = \frac{x_2}{2x_1}$$

Hence, the tangency condition, $MRS = \frac{p_1}{p_2}$, is

$$\frac{x_2}{2x_1} = \frac{3}{1}$$

since the price of good 1 is \$3, and that of good 2 is \$1. Rearranging, we obtain

$$x_2 = 6x_1$$

Plugging this result into the budget line, $p_1 x_1 + p_2 x_2 = I$, we find

$$3x_1 + 1 \cdot \underbrace{(6x_1)}_{x_2} = 150$$

or $9x_1 = 150$, which simplifies to $x_1 = \frac{150}{9} = 16.6$ units. Last, we can use the tangency condition to find the optimal amount of good 2,

$$x_2 = 6x_1 = 6 \cdot 16.6 = 100 \text{ units.}$$

Hence, bundle $A = (16.6, 100)$.

b) Find the optimal consumption bundle at the *final* price of \$2. Label it bundle *C*.

ANSWER: In this case, the tangency condition is

$$\frac{x_2}{2x_1} = \frac{2}{1}$$

since $p_1 = \$2$ and $p_2 = \$1$. Rearranging, we obtain $x_2 = 4x_1$. Plugging this result in the budget line, we obtain

$$2x_1 + 1 \cdot \underbrace{(4x_1)}_{x_2} = 150$$

or $6x_1 = 150$, which yields

$$x_1 = \frac{150}{6} = 25 \text{ units}$$

And using the tangency condition again, we find the optimal amount of good 2

$$x_2 = 4x_1 = 4 \cdot 25 = 100 \text{ units}$$

Hence, bundle *C* is $C = (25{,}100)$

c) What is the total effect of the price change?

ANSWER: Total effect on good 1 is

$$TE = 25 - 16.6 = 8.3 \text{ units}$$

d) We next seek to break down the total effect you found in part (c) into the substitution and income effects. In order to do that, let us start by finding the decomposition bundle. Label it bundle *B*.
[*Hint*: Recall that the decomposition bundle must satisfy two conditions: (1) it must generate the same utility level as the initial bundle *A*; and (2) we must have that the slope of the consumer's indifference curve, *MRS*, coincides with the new price ratio.]

ANSWER: At bundle *B* the consumer receives the same utility level as in bundle *A*. Hence,

$$x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} = (16.6)^{\frac{1}{3}}(100)^{\frac{2}{3}} =$$
$$= 2.55 \times 21.54 = 55.02$$

In addition, at bundle *B*, we have that the indifference curve is tangent to the budget line at the new prices, that is,

$$MRS_{1,2} = \frac{2}{1} \rightarrow \frac{x_2}{2x_1} = \frac{2}{1}$$

Rearranging, $x_2 = 4x_1$. Plugging this result into the utility constraint found above, $x_1^{\frac{1}{3}} x_2^{\frac{2}{3}} = 55.02$, we obtain that

$$x_1^{\frac{1}{3}} \underbrace{(4x_1)}_{x_2}^{\frac{2}{3}} = 55.02$$

Rearranging,

$$\underbrace{4^{\frac{2}{3}}}_{2.51} \underbrace{x_1^{\frac{1}{3}} x_1^{\frac{2}{3}}}_{x_1} = 55.02$$

or $x_1 = \frac{55.02}{2.51} = 21.84$ units.

e) Write the amount of good 1 that this individual consumes on bundles *A*, *B* and *C*. What is the increase in consumption of good 1 due to the substitution effect? What is due to the income effect?

ANSWER: Summarizing,

$$SE = x_B - x_A = 21.84 - 16.6 = 5.174$$
$$IE = x_C - x_B = 25 - 21.84 = 3.16$$

f) Using the sign of the income effect, what can you say about good 1? Is it a normal, or an inferior good?

ANSWER: The good is normal since the income effect is positive, $IE = 3.16 > 0$.