EconS 301

Quiz #1 – Utility maximization problems

Answer Key

Consider an individual with Cobb-Douglas utility function $u(x, y) = \sqrt{xy}$. Assume that his income is I=\$120, the price of good x is $p_x = \$4$, and the price of good y is p_y = \$10.

a) Find the marginal utility of x, MU_x, and the marginal utility of y, MU_y.

***ANSWER**:* The utility function $u(x, y) = \sqrt{xy}$ can be expressed as $u(x, y) = x^{1/2}y^{1/2}$.

$$MU_x = \frac{1}{2}x^{-1/2}y^{1/2} = \frac{1}{2}\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}$$

$$MU_y = \frac{1}{2}x^{1/2}y^{-1/2} = \frac{1}{2}\frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}$$

b) Given your results in part (a), does the "more is better" assumption hold on the above utility function?

***ANSWER**:* Yes, an increase in *x* increases the individual's utility since $MU_x > 0$ for all combinations of good *x* and *y*.

Yes, an increase in *y* increases the individual's utility since $MU_y > 0$ for all combinations of good *x* and *y*.

c) Using the marginal utilities you found in part (a), find the marginal rate of substitution of this consumer (*MRS*).

***ANSWER**:*

$$MRS_{x,y} = \frac{MUx}{MUy} = \frac{\frac{1}{2}\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}}{\frac{1}{2}\frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}} = \frac{y^{\frac{1}{2}-\left(-\frac{1}{2}\right)}}{x^{\frac{1}{2}-\left(-\frac{1}{2}\right)}} = \frac{y}{x}$$

d) Using the *MRS* you found in part (c), what is the optimal consumption bundle? [*Hint*: Use the tangency condition. Then, insert your result into the budget line. This should give you the optimal amount of good *x* and the optimal amount of good *y*.]

ANSWER*:* Using the tangency condition, we obtain:

$$\text{MRS} = \frac{y}{x} = \frac{4}{10} = \frac{Py}{Px} \text{ , which simplifies to } \quad y = \frac{4}{10}x$$

Plugging this result into the budget line, 4x + 10y = 120, yields

$$4x + 10\left(\frac{4}{10}x\right) = 120$$

which simplifies to,

$$4x + 4x = 120 \text{ or } 8x = 120$$

Solving for *x*, we obtain an optimal amount of good *x*, $x = \frac{120}{8} = 15$ units.

Finally, using the tangency condition again, $y = \left(\frac{4}{10}\right)x = \left(\frac{4}{10}\right)15 = 6$ units .

e) Consider now that the price of good *x* is left undefined at p_x (that is, it can take any number). Repeat your analysis in part (d), finding the optimal consumption of good *x* and of good *y*. [*Hint*: The expression that you obtain will be a function of price p_x]

ANSWER*:* Using the tangency condition,

$$\text{MRS} = \frac{y}{x} = \frac{4}{10} = \frac{Py}{Px} \text{ , which simplifies to } \ y = \frac{Px}{10}x$$

Plugging this result into the budget line, $p_x x + 10y = 120$

$$p_x x + 10\left(\frac{Px}{10}x\right) = 120,$$

which yields to

$$p_x x + p_x x = 120, \quad \text{or} \quad p_x x = 60$$

Solving for *x*, we obtain the demand for good *x*,

$$x = \frac{60}{Px},$$

As a remark, the demand for x is then decreasing in its own price (which shows up in the denominator). Finally, using the tangency condition again, we obtain the number of units of good *y*,

$$y = \frac{Px}{10}\left(\frac{60}{Px}\right) = \frac{60}{10} = 6 \ \text{units.}$$