Extensions-I

- Consider a model in which:
    - The difficulty of the task, θ, is unobservable; and
    - The effort that the manager exerts, e, is also unobservable.
- However, assume that the relationship between effort and profits is deterministic, given by function $\pi(e)$,...
    - rather than stochastic (as we described at the beginning of this week).

Extensions-I

- In this setting, contract pairs (w_H, e_H) and (w_L, e_L) can be designed in the same fashion as in the last model we considered (in which θ was the only piece of information the principal could not observe).
  - Intuitively, the observation of profits allows the principal to perfectly infer effort, even if effort was not directly observable.
- In particular:
  - The principal offers contract pairs (w_H, e_H) and (w_L, e_L) to the agent (same pairs as those found in the hidden information model we just described).
  - Then, the agent privately observes the realization of θ.
  - The agent then chooses one of the two contract pairs anticipating that, when profits $\pi(e_H) = \pi_H$ are observed by the principal, he pays w_H; whereas when profits $\pi(e_L) = \pi_L$ are observed, he pays w_L.

Extensions-I

- We can think about this contract as a **direct mechanism** in which:
  - For a given announcement $\widehat{\theta}$ from the agent to the principal, the principal offers a wage-profit pair $\left(w\left(\widehat{\theta}\right), \pi\left(\widehat{\theta}\right)\right)$.
- From the above analysis of hidden information models (specifically, from the I.C. constraints), the agent has incentives to truthfully report $\widehat{\theta} = \theta_L$ when he observes that the realization of parameter θ is θ_L, and similarly when he observes that its realization is θ_H.

Extensions-I

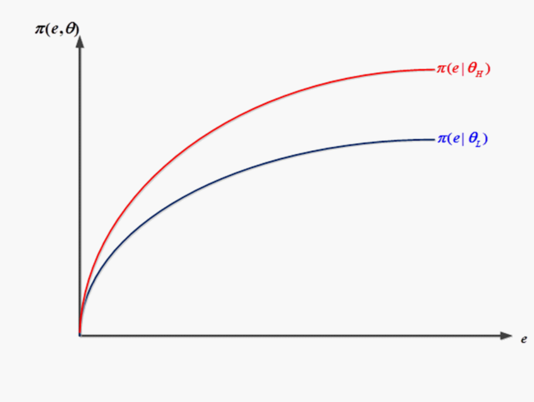
- Indeed, note that for any required profit π, the effort $\widetilde{e}$ necessary to achieve such profit is that solving $\pi(\widetilde{e}) = \pi$.
- Solving for e, we obtain the effort function $\widetilde{e}(\pi)$.
- We can then relabel the manager's effort function $g(\widetilde{e}(\pi), \theta) = \widetilde{g}(\pi, \theta)$
- But then, the model is exactly equivalent to the hidden information model we solved above, where:
  - The observable variable is the effort $\widetilde{e}(\pi) = \pi$, and
  - The unobservable variable is the disutility of effort $\widetilde{g}(\pi, \theta)$.

Extensions-II

- In the hidden information model we solved the principal could not observe the realization of θ, and thus didn't know the agent's disutility of effort, $g(e, \theta)$.
- What if, instead, the principal cannot observe the relationship between effort and profits, i.e., the marginal productivity of effort?
  - Now the disutility of effort is perfectly known, $g(e)$.
  - However, the profit function is $\pi(e, \theta)$, where...
  - $\pi_e(e, \theta) > 0$, $\pi_{ee}(e, \theta) < 0$, $\pi_\theta(e, \theta) > 0$, and $\pi_{e\theta}(e, \theta) > 0$
  - That is, effort increases profits at a decreasing rate; profits increase in the realization of parameter θ, and marginal profits also increase in this realization.

Extensions-II

- Profit function $\pi(e, \theta)$



Extensions-II

- Similarly as in the first extension, we can think of direct mechanisms specifying:
    - For a given announcement $\widehat{\theta}$ from the agent to the principal, the principal offers a wage-profit pair $\left(w\left(\widehat{\theta}\right), \pi\left(\widehat{\theta}\right)\right)$.
- In this context, the effort $\widetilde{e}$ necessary to achieve any profit level π is that solving $\pi(\widetilde{e}, \theta) = \pi$.
- Solving for e, we obtain the effort function $\widetilde{e}(\pi, \theta)$.
- We can then relabel the manager's effort function as $g(\widetilde{e}(\pi, \theta)) = \widetilde{g}(\pi, \theta)$
- But then, the model is exactly equivalent to the hidden information model we solved above, where:
    - The observable variable is the effort $\widetilde{e}(\pi, , \theta) = \pi$, and
    - The unobservable variable is the disutility of effort $\widetilde{g}(\pi, \theta)$.

Moral Hazard with multiple signals

- Consider a setting in which the principal, still not observing effort e, observes profits π and a signal s, e.g., a middle management report about the manager.
  - Such a signal provides no intrinsic economic value (i.e., s does not affect profits), but provides information about the effort e.
- How shoud l the principal use this information?

$$\frac{1}{v'(w)} = \gamma + \mu \left[1 - \frac{f(\pi, s | e_L)}{f(\pi, s | e_H)} \right]$$

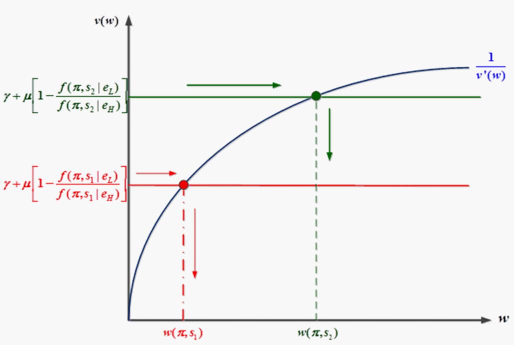
Moral Hazard with multiple signals

- Variations in s affect wages only if

$$f\left(\pi, s|e\right) \neq f\left(\pi|e\right)$$

  i.e., π is *not* a sufficient statistic of e. Intuitively, (π, s) contains more information about e than π alone.
- Figure

Moral Hazard with multiple signals



- Hence, salary is increasing in signal (e.g., middle management report) s, i.e., $w(\pi, s_2) > w(\pi, s_1)$, if only if $\frac{f(\pi,s_1|e_L)}{f(\pi,s_1|e_H)} > \frac{f(\pi,s_2|e_L)}{f(\pi,s_2|e_H)} \leftarrow$ the likelihood ratio decreases in the report s.

Moral Hazard with multiple signals

- **Sufficient statistic theorem** (Holstrom, 1979):
  - The principal conditions the agent's wage on a sufficient statistic for all the signals he receives, e.g., $w(\pi, s)$.

Moral Hazard with several agents

- Consider now a setting in which the principal deals with N agents, and can offer a different salary w_i to each of them (e.g., WSU).
- If the profits that employee i brings to the firm, π_i, are a function of his effort, e_i, and that of other employees, e_{-i}, then

$$f(\pi_i, \pi_{-i} | e_i) \neq f(\pi_i | e_i)$$

- Therefore, the principal uses π_{-i} as an additional signal, and the previous "sufficient statistic theorem" applies, i.e., $w_i(\pi_i, \pi_{-i})$.

Moral Hazard with several agents

- A similar argument applies if π_i depends on my own effort as agent i, e_i, on an idiosyncratic noise only affecting π_i, ε_i, and on a common noise affecting all employees, ε, e.g.,

$$\pi_i = ae_i + b\varepsilon_i + c\varepsilon.$$

  - Typical example: sellers of the same product to different clients.
- Then, by the sufficient statistic theorem, the principal offers a salary $w_i(\pi_i, \pi_{-i})$ to agent i.

Moral Hazard with several agents

- If there was no common noise, the principal could separately treat his relationship with each agent as a standard principal-agent model, offering $w_i(\pi_i)$ to agent i and inducing effort e_i for every agent $i \in N$.
- However, the presence of a common noise affecting all agents, leads the principal to design a compensation $w_i\left(\pi_i, \pi_{-i}\right)$ which depends on both π_i and π_{-i}.
  - Importantly, the salary induces competition, i.e., $w_i\left(\pi_i, \pi_{-i}\right)$ increases in my own outcomes π_i but decreases in the other agents', π_{-i}.
  - This incentive structure helps the principal extract better information about the common noise; see Holsmtrom (1982).
- More references in Chapter 8 of Bolton and Dewatripont's *Contract Theory* textbook.

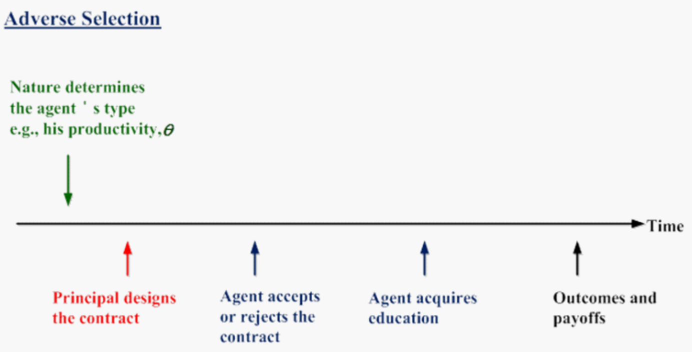
Future extensions

- Moral hazard with multiple tasks.
  - Tradeoff so the agent dedicates enough time to each of them, which induces less powered incentives.
  - Section 6.2 in Bolton and Dewatripont's *Contract Theory*.
- Repeated moral hazard.
  - Sections 10.1 and 10.2 in Bolton and Dewatripont.
- Empirical models on moral hazard and adverse selection.
  - Chapter 8 in Bernard Salanie's *The Economics of Contracts*.

Summary

- **Adverse selection:**
  - The employer does not know which type of employee he is hiring.
  - That is, the employer doesn't observe the productivity of the employee, θ.
    - (Importantly, in adverse selection models the employer doesn't observe the realization of a random variable which is observed by the agent before the contractual relationship starts.)
  - In a Spence's labor market signaling game, the employee acts first, using education as a signal of his type.
  - In a screening game, the employer acts first, offering a menu of contracts (w_H, t_H) for the high productivity, and (w_L, t_L) for the low productivity worker, where t_i denotes the difficulty of the task.

Summary

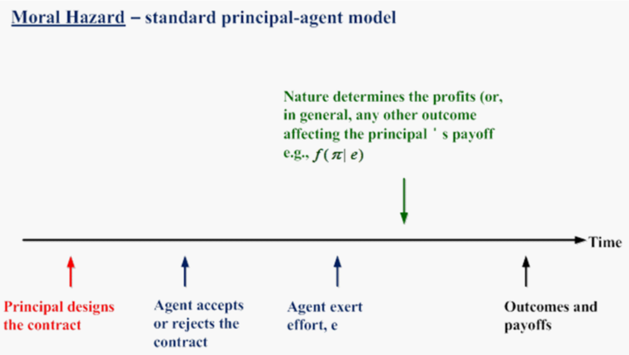


Adverse Selection
Nature determines the agent ' s type e.g., his productivity, θ
Time
Principal designs the contract
Agent accepts or rejects the contract
Agent acquires education
Outcomes and payoffs

Summary

- **Moral hazard:**
  - The employer knows which type of employee he is hiring, but cannot observe the effort that the employee will exert once he is on the job, e.
  - However, the employer observes profits π (and potentially more signals, s) as an imperfect indication of the effort the worker exerted.
    - Hence, the source of uncertainty for the employer is post-contractual, as opposed to pre-contractual in adverse selection models.
  - *Standard moral hazard:* we determine salaries first. Then find which effort level is optimal for the principal given those salaries.

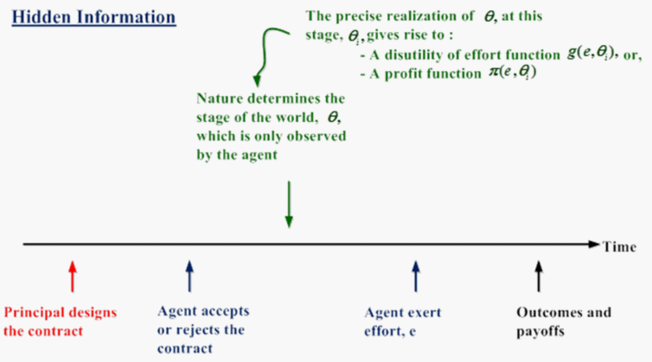
Summary


Moral Hazard – standard principal-agent model
Nature determines the profits (or, in general, any other outcome affecting the principal ' s payoff e.g., f(π| e)
Time
Principal designs the contract
Agent accepts or rejects the contract
Agent exert effort, e
Outcomes and payoffs

Summary

- **Moral hazard:**
  - *Hidden information:* the employer observes effort, but the employee gets to observe how difficult the task is (disutility of effort, $g(e, \theta)$) which the employer cannot observe.
    - Hence, the employer is still uninformed about a post-contractual element whose realization only the employee observes.
  - *Alternative of hidden info.*: the employee observes how profitable each unit of effort is, $\pi(e, \theta)$, but the employer doesn't.
    - He is still uncertain about a post–contractual element whose realization only the employee observes.

Summary



Hidden Information
The precise realization of θ, at this stage, θ_i, gives rise to :
- A disutility of effort function $g(e,\theta_i)$, or,
- A profit function $\pi(e,\theta_i)$
Nature determines the stage of the world, θ, which is only observed by the agent
Time
Principal designs the contract
Agent accepts or rejects the contract
Agent exert effort, e
Outcomes and payoffs