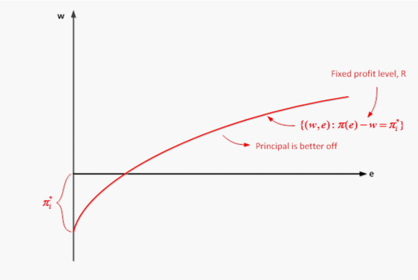
Hidden information

- Section 14.C. in MWG
- We still consider a setting with information asymmetries between the principal and agent.
- However, the effort is now perfectly observable.
- What is unobservable? An element arising after the contract is signed.
    - We will consider that the unobservable element is the disutility that the manager experiences from effort $g(e)$, e.g., high or low, which only the manager observes.
    - Alternative: the manager observes the profitability of different effort levels.

Hidden information

- **Principal's payoff:** $\pi(e) - w$,
  - where profits $\pi(e)$ are increasing in effort, but at a decreasing rate, i.e., $\pi'(e) > 0$ and $\pi''(e) < 0$,
  - No profits arise if the agent exert no effort, $\pi(0) = 0$.
  - The principal is hence risk neutral.

Indifference curve for the principal (isoprofit curve)

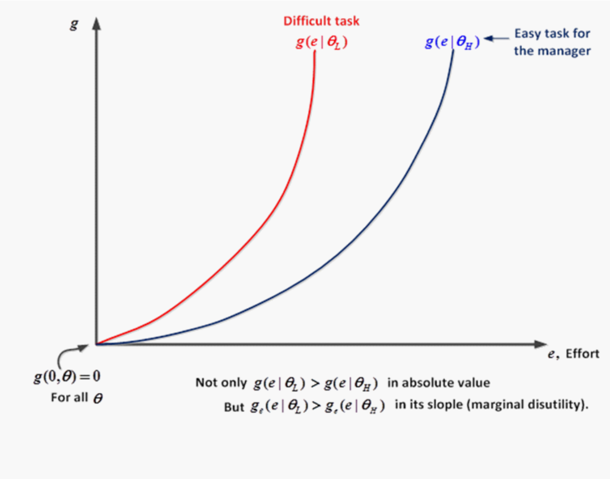


- The principal is better off the lower the salary the higher the effort shifts the principal's indifference curve southeast.
- An increase in effort keeps the principal's utility unaffected only if it is accompanied by an increase in his salary expenses (otherwise, he would be better off).

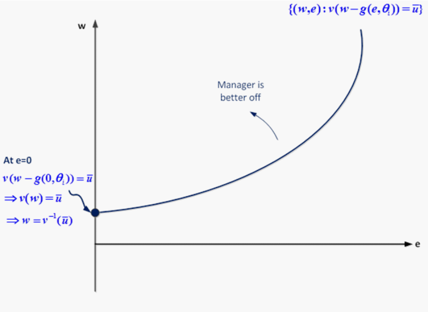
Hidden information

- **Manager's payoff:** $u(w, e, \theta) = v(w - g(e, \theta))$, where
  - $g(e, \theta)$ measures the disutility from effort, and $g(0, \theta) = 0$.
  - $g_e(e, \theta) \geq 0$, $g_e(0, \theta) = 0$, and $g_{ee}(e, \theta) > 0$, i.e., the cost of effort is increasing and convex in effort.
  - $g_\theta(e, \theta) < 0$, i.e., the cost of effort is lower for high states of nature $\theta_H > \theta_L$.
  - $g_{e\theta}(e, \theta) \leq 0$, i.e., the marginal cost of effort is lower for high states of nature $\theta_H > \theta_L$.
  - In addition, $v' > 0$ and $v'' < 0$, thus implying that the agent is risk averse.
  - For simplicity, we consider two types $\Theta = \{\theta_H, \theta_L\}$ with associated prob. p and $1 - p$.

Disutility of effort



Indifference curve for the agent



- The manager is better off as salary increases as effort decreases shifts of his indifference curve towards the northwest
- If effort is increased, the manager's utility is only unaffected if his salary increases.

Hidden information - Observable types

- In this setting, a contract can specify a salary as a function of both effort and θ (since both are observable).
  - That is, a wage-effort pair (w_H, e_H) for state θ_H, and similarly (w_L, e_L) for state θ_L.
- In particular, the principal chooses these two pairs to solve

$$\max \quad p\left[\pi\left(e_H\right) - w_H\right] + (1-p)\left[\pi\left(e_L\right) - w_L\right]$$

$$\text{subject to} \quad pv\left(w_H - g(e_H, \theta_H)\right) + (1-p)v\left(w_L - g(e_L, \theta_L)\right) \geq \bar{u} \quad \text{(P.C.)}$$

Hidden information - Observable types

- Taking FOCs, we find

$$
\begin{array}{cc}
\partial w_H & -p+\gamma p v'\left(w_H^*-g(e_H^*,\theta_H)\right)=0 \\
\partial w_L & -(1-p)+\gamma(1-p)v'\left(w_L^*-g(e_L^*,\theta_L)\right)=0 \\
\partial e_H & p\pi'\left(e_H^*\right)-\gamma p v'\left(w_H^*-g(e_H^*,\theta_H)\right)g_e(e_H^*,\theta_H)=0 \\
\partial e_L & \begin{array}{c}(1-p)\,\pi'\left(e_L^*\right) \\ -\gamma(1-p)v'\left(w_L^*-g(e_L^*,\theta_L)\right)g_e(e_L^*,\theta_L)=0\end{array}
\end{array}
$$

Hidden information - Observable types

- Why salaries w_H and w_L must be positive?
  - To guarantee acceptance under observably, i.e., $w_i^* = v^{-1}(\bar{u} + g(e_i, \theta_i))$ only if $w_i^* > 0$ (This is true even if $\bar{u} = 0$).
- Why effort level must be positive?
  - Suppose otherwise, i.e., $e_H = 0$, Then FOC becomes

$$\begin{aligned} & \underbrace{p\pi'(0)}_{>0} - \gamma \cdot p \cdot v'(w - g(0, \theta_H)) \cdot \underbrace{g_e(0, \theta_H)}_{=0} \\ = \quad & p \cdot \pi'(0) > 0 \end{aligned}$$

    - Indicating that the marginal utility from increasing effort e_H away from zero is positive for the principal.
  - An analog argument applies if $e_L = 0$ in the fourth FOC.

Hidden information - Observable types

- Solving for γ in the two first FOCs and rearranging yields

$$v'\left(w_H^* - g(e_H^*, \theta_H)\right) = v'\left(w_L^* - g(e_L^*, \theta_L)\right)$$

  in terms of marginal utility, which also entails

$$w_H^* - g(e_H^*, \theta_H) = w_L^* - g(e_L^*, \theta_L)$$

  in terms of money, and

$$v\left(w_H^* - g(e_H^*, \theta_H)\right) = v\left(w_L^* - g(e_L^*, \theta_L)\right)$$

  in terms of total utility.
- Hence, the risk averse agent obtains the same money, utility and marginal utility of money across states of nature (risk insurance).

Hidden information - Observable types

- Given the P.C. must hold with equality (otherwise the principal could still reduce salaries retaining more profits)

$$pv\left(w_H^* - g(e_H^*, \theta_H)\right) + (1-p)v\left(w_L^* - g(e_L^*, \theta_L)\right) = \bar{u}$$

since $v\left(w_H^* - g(e_H^*, \theta_H)\right) = v\left(w_L^* - g(e_L^*, \theta_L)\right)$, the P.C. becomes

$$pv\left(w_H^* - g(e_H^*, \theta_H)\right) + (1-p)v\left(w_H^* - g(e_H^*, \theta_H)\right) = \bar{u}$$

or $v\left(w_H^* - g(e_H^*, \theta_H)\right) = \bar{u}$, i.e., we can find salary w_H^* by doing the inverse $w_H^* = v^{-1}\left(\bar{u} + g(e_H^*, \theta_H)\right)$.

- And similarly, $v\left(w_L^* - g(e_L^*, \theta_L)\right) = \bar{u}$.

Hidden information - Observable types

- After finding salaries, let's turn to effort levels.
- Combining the third FOC

$$p\pi'\left(e_H^*\right) - \gamma p v'\left(w_H^* - g(e_H^*, \theta_H)\right) g_e(e_H^*, \theta_H) = 0$$

with the first, $-p + \gamma p v'\left(w_H^* - g(e_H^*, \theta_H)\right) = 0$, yields

$$\pi'\left(e_H^*\right) = g_e(e_H^*, \theta_H)$$

- Similarly, combining the fourth and second FOCs obtains $\pi'\left(e_L^*\right) = g_e(e_L^*, \theta_L)$.
- In words, in state θ_i effort is increased until the point in which marginal profits equal marginal disutility from effort.

Hidden information - Observable types

- In particular, solving for $\gamma \cdot p \cdot v'(\cdot)$ in both the third and first FOC yields, respectively

$$\gamma \cdot p \cdot v'(\cdot) = \frac{p \cdot \pi'(e_H^*)}{g_e(e_H^*, \theta_H)}$$

- and

$$\gamma \cdot p \cdot v'(\cdot) = p$$

- Setting them equal to each other yields

$$\frac{p \cdot \pi'(e_H^*)}{g_e(e_H^*, \theta_H)} = p$$

- Which simplifies to

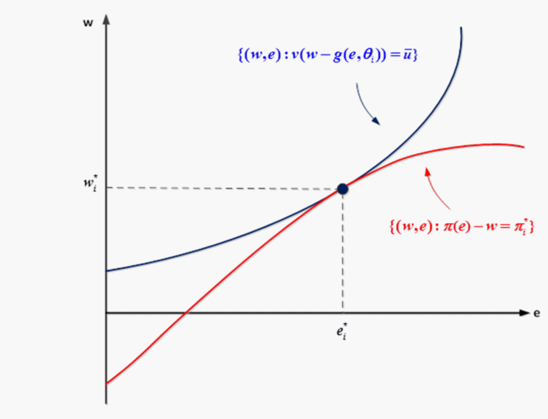
$$\pi'(e_H^*) = g_e(e_H^*, \theta_H)$$

Hidden information - Observable types

- Graphical representation of the previous results under observably
- Figure

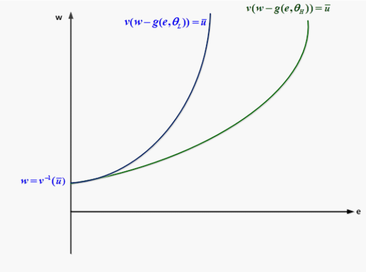
Hidden information - Observable types

- Optimal wage-effort pair (w_i^*, e_i^*) when state of nature is θ_i, where $i = \{H, L\}$.



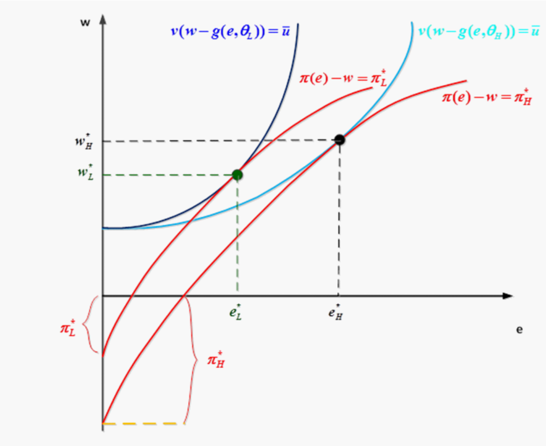
Hidden information - Observable types

- Let's now depict both wage effort pairs $\left(w_H^*, e_H^*\right)$ and $\left(w_L^*, e_L^*\right)$.



- since $g_{e\theta}(e,\theta) < 0$, the marginal disutility of effort satisfies $g_e(e,\theta_L) > g_e(e,\theta_H)$ implying that, in order to maintain utility level $\bar{u}$ unaffected, the manager needs to be more significantly compensated for each additional unit of effort when the state of nature is θ_L and θ_H.

Hidden information - Observable types



Hidden information - Unobservable types

- When only the manager observes the state θ, which contract will induce him to reveal to the principal the state θ voluntarily?
  - Can the principal still offer the same contract pairs (w_H, e_H) and (w_L, e_L), and achieve self-selection from the manager?
  - Not necessarily: In the previous figure, the manager in state θ_H would have incentives to lie saying that it is θ_L, since wage-effort pair (w_L, e_L) yields him a higher utility level.
  - In addition, such a lie reduces the principal's profits.

Hidden information - Unobservable types

- Given these problems, what optimal contract pairs should the principal offer?
  - We will have to redefine the principal's expected profit maximization problem, having P.C. conditions (as when θ was observable)...
  - but now add incentive compatibility (I.C., or self-selection) conditions.

Hidden information - Unobservable types

- In particular, the principal chooses contract pairs (w_H, e_H) and (w_L, e_L) to solve

$$\max \quad p\left[\pi\left(e_H\right) - w_H\right] + (1-p)\left[\pi\left(e_L\right) - w_L\right]$$

$$\text{subject to} \quad v\left(w_H - g(e_H, \theta_H)\right) \geq \bar{u} \qquad (\text{P.C.}_H)$$

$$v\left(w_L - g(e_L, \theta_L)\right) \geq \bar{u} \qquad (\text{P.C.}_L)$$

which we can alternatively write, using the inverse $v^{-1}()$ on both sides of the inequality, as $w_H - g(e_H, \theta_H) \geq v^{-1}(\bar{u})$, and $w_L - g(e_L, \theta_L) \geq v^{-1}(\bar{u})$, respectively.

Hidden information - Unobservable types

- What about the I.C. conditions?

$$w_H - g(e_H, \theta_H) \geq w_L - g(e_L, \theta_H) \qquad (\text{I.C.}_H)$$

  for the high type, where we have fixed θ_H on both sides of the inequality.

- And similarly for the low-type,

$$w_L - g(e_L, \theta_L) \geq w_H - g(e_H, \theta_L) \qquad (\text{I.C.}_L)$$

  where we fixed θ_L on both sides of the inequality.

- These incentive compatibility conditions are often referred to as *truth-telling* or *self-selection* conditions.

Hidden information - Unobservable types

- Let's start solving the above maximization problem for the principal.
- **First step:** We can first simplify the problem by noticing that constraint PC_H holds when all other constraints hold. In particular, from IC_H and PC_L we obtain

$$w_H - g(e_H, \theta_H) \underset{IC_H}{\geq} w_L - g(e_L, \theta_H)$$
$$\underset{\text{since } g(e_L,\theta_H)<g(e_H,\theta_H)}{\geq} w_L - g(e_L, \theta_L) \underset{PC_L}{\geq} v^{-1}(\bar{u})$$

implying that constraint PC_H $\left(w_H - g\left(e_H, \theta_H\right) \geq v^{-1}\left(\bar{u}\right)\right)$ must also hold.

Hidden information - Unobservable types

- The principal's problem can then be restated as follows

$$\max \quad p\left[\pi\left(e_H\right) - w_H\right] + (1-p)\left[\pi\left(e_L\right) - w_L\right]$$

$$\text{subject to} \quad w_L - g\left(e_L, \theta_L\right) \geq v^{-1}(\bar{u}) \qquad (\text{P.C.}_L)$$

$$w_H - g\left(e_H, \theta_H\right) \geq w_L - g\left(e_L, \theta_H\right) \qquad (\text{I.C.}_H)$$

$$w_L - g\left(e_L, \theta_L\right) \geq w_H - g\left(e_H, \theta_L\right) \qquad (\text{I.C.}_L)$$

Hidden information - Unobservable types

- Letting γ, ϕ_H and ϕ_L be the Lagrangian multipliers for the three constraints, PC$_L$, IC$_H$, and IC$_L$, respectively.
- Hence, Kuhn-Tucker conditions (with respect to w_H, w_L, e_H and e_L, respectively) are

$$\partial w_H: \quad -p+\phi_H-\phi_L=0 \tag{1}$$

$$\partial w_L: \quad -(1-p)+\gamma-\phi_H+\phi_L=0 \tag{2}$$

$$\partial e_H: \quad p\pi'(e_H)-\phi_H g_e(e_H,\theta_H)+\phi_L g_e(e_H,\theta_L)=0 \tag{3}$$

$$\partial e_L: \quad (1-p)\pi'(e_L)-(\gamma+\phi_L)g_e(e_L,\theta_L) + \phi_H g_e(e_L,\theta_H)=0 \tag{4}$$

Hidden information - Unobservable types

Step1B:

- Condition (1), $-p + \phi_H - \phi_L = 0$, can be written as $\phi_H = p + \phi_L$, which is positive (even if $\phi_L = 0$) since $p \in (0, 1)$.
  - Thus implying that the constraint associated to Lagrangian multiplier ϕ_H, IC_H, must hold with equality.
  - That is, $w_H - g(e_H, \theta_H) = w_L - g(e_L, \theta_H)$

Hidden information - Unobservable types

- **Second step:** Let us now use conditions (1) and (2). Adding them, we obtain

$$(-p + \phi_H - \phi_L) + (-(1-p) + \gamma - \phi_H + \phi_L) = 0$$

  which yields $\gamma = 1 > 0$.

  - Therefore, its associated constraint, i.e., PC$_L$, must hold with equality.
  - That is, $w_L - g(e_L, \theta_L) = v^{-1}(\bar{u})$
  - This result already helped us identify one of our unknowns: $w_L = v^{-1}(\bar{u}) + g(e_L, \theta_L)$.

Hidden information - Unobservable types

- **Third step:** Since two of the three Lagrangian multipliers are positive, $\phi_H > 0$ and $\gamma > 0$, then the remaining Lagrangian multiplier $\phi_L = 0$.
  - *Proof:* Suppose not, i.e., $\phi_L > 0$. Then, its associated constraint, IC_L, must be binding (holding with equality).
  - We can now show that we would reach a contradiction.
  - First, substitute for ϕ_H in condition (3) using the fact that $\phi_H = p + \phi_L$ from condition (1). In particular, we can rewrite (3) as

$$p\pi'(e_H) - \underbrace{(p + \phi_L)}_{\phi_H} g_e(e_H, \theta_H) + \phi_L g_e(e_H, \theta_L) = 0$$

or

$$p\left[\pi'(e_H) - g_e(e_H, \theta_H)\right] + \phi_L \left[g_e(e_H, \theta_L) + g_e(e_H, \theta_H)\right] = 0$$

Hidden information - Unobservable types

- If $\phi_L > 0$, then $\phi_L \left[g_e(e_H, \theta_L) - g_e(e_H, \theta_H) \right] > 0$ since the marginal cost of exerting e_H units of effort is larger for the manager who faces a state of nature θ_L than that facing θ_H, i.e., $g_e(e_H, \theta_L) > g_e(e_H, \theta_H)$.
- Therefore, the above condition entails

$$\underbrace{p\left[\pi'(e_H) - g_e(e_H, \theta_H)\right]}_{\text{Negative}} + \underbrace{\phi_L \left[g_e(e_H, \theta_L) - g_e(e_H, \theta_H) \right]}_{\text{Positive}} = 0$$

$$\Leftarrow$$

or $\pi'(e_H) - g_e(e_H, \theta_H) < 0$.

Hidden information - Unobservable types

- We can similarly use $\phi_H = p + \phi_L$ from condition (1), and $\gamma = 1$, to rewrite condition (4) as

$$(1-p)\pi'(e_L) - (1+\phi_L)g_e(e_L, \theta_L) + \underbrace{(p+\phi_L)}_{\phi_H} g_e(e_L, \theta_H) = 0$$

or

$$(1-p)\left[\pi'(e_L) - g_e(e_L, \theta_H)\right] + (1+\phi_L)\left[g_e(e_L, \theta_H) - g_e(e_L, \theta_L)\right] = 0$$

Hidden information - Unobservable types

- If $\phi_L > 0$, then $(1+\phi_L)\left[g_e(e_L,\theta_H) - g_e(e_L,\theta_L)\right] < 0$ since the marginal cost of exerting e_L units of effort is smaller for the manager who faces a state of nature θ_H than that facing θ_L, i.e., $g_e(e_L,\theta_H) < g_e(e_L,\theta_L)$.
- Therefore, the above condition entails

$$(1-p)\underbrace{\left[\pi'(e_L) - g_e(e_L,\theta_H)\right]}_{\text{Positive}} +$$

$$\underbrace{(1+\phi_L)\left[g_e(e_L,\theta_H) - g_e(e_L,\theta_L)\right]}_{\text{Negative}} = 0$$

or $\pi'(e_L) - g_e(e_L,\theta_H) > 0$.

Hidden information - Unobservable types

- We can, hence, summarize the conditions we obtained from rewriting (3) and (4) as follows

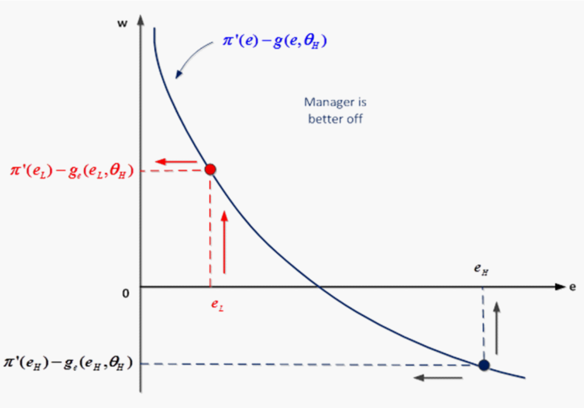
$$\pi'(e_L) - g_e(e_L, \theta_H) > 0 > \pi'(e_H) - g_e(e_H, \theta_H)$$

- In addition, since $\pi''(e) < 0$ and $g_{ee}(e, \theta_H) > 0$, thus implying that $\frac{\partial^2(\pi(e) - g(e, \theta_H))}{\partial e^2} < 0$, i.e., function $\pi(e) - g(e, \theta_H)$ is concave in e.
- Alternatively, its first-derivative is decreasing in e, entailing that

$$\pi'(e_L) - g_e(e_L, \theta_H) > \pi'(e_H) - g_e(e_H, \theta_H)$$

  to hold it must be that $e_H > e_L$.
- Figure.

Hidden information - Unobservable types



Hidden information - Unobservable types

- But if $e_H > e_L$ and IC_H binds (as we showed a few slides ago), then IC_L must also bind. In particular, from IC_H binding we know that

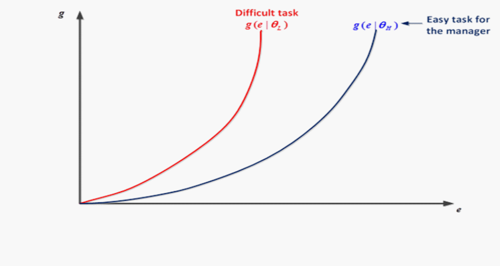
$$w_H - g_e(e_H, \theta_H) = w_L - g_e(e_L, \theta_H)$$

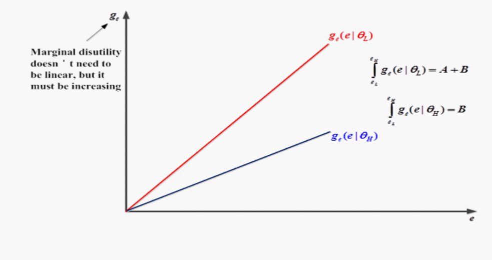
which can be expressed as

$$w_H - w_L = g_e(e_H, \theta_H) - g_e(e_L, \theta_H) = \int_{e_L}^{e_H} g_e(e, \theta_H) de$$

- Furthermore, $\int_{e_L}^{e_H} g_e(e, \theta_H) de < \int_{e_L}^{e_H} g_e(e, \theta_L) de$ since the marginal disutility of effort satisfies $g_e(e, \theta_H) < g_e(e, \theta_L)$ for all e.

Hidden information - Unobservable types





Hidden information - Unobservable types

- Hence, since $\int_{e_L}^{e_H} g_e(e, \theta_L) de = g_e(e_H, \theta_L) - g_e(e_L, \theta_L)$, we can rewrite the above expression as

$$w_H - w_L = g_e(e_H, \theta_H) - g_e(e_L, \theta_H) < g_e(e_H, \theta_L) - g_e(e_L, \theta_L)$$

or $w_H - w_L < g_e(e_H, \theta_L) - g_e(e_L, \theta_L)$, which entails

$$w_H - g_e(e_H, \theta_L) < w_L - g_e(e_L, \theta_L)$$

ultimately implying that constraint IC_L must be slack, i.e., $\phi_L = 0$, which is our desired contradiction.

Hidden information - Unobservable types

- **Fifth step:** After showing that $\phi_L = 0$, we can rewrite condition (1) as $\phi_H = p + \phi_L = p$.
- Substituting $\phi_L = 0$ and $\phi_H = p$ into conditions (3) and (4) yields

$$\pi'(e_H) - g_e(e_H, \theta_H) = 0 \tag{5}$$

and

$$\pi'(e_L) - g_e(e_L, \theta_L) + \frac{p}{1-p}\left[g_e(e_L, \theta_H) - g_e(e_L, \theta_L)\right] = 0 \tag{6}$$

- Optimal effort levels e_H and e_L solve conditions (5) and (6).
- Optimal wage levels w_H and w_L solve conditions PC_L and IC_H with equality.

- **Alternative approach** to solve the principal's problem:
  - Solve the principal's problem ignoring condition IC_L. Then, show that your result satisfies condition IC_L.

Hidden information - Comparison

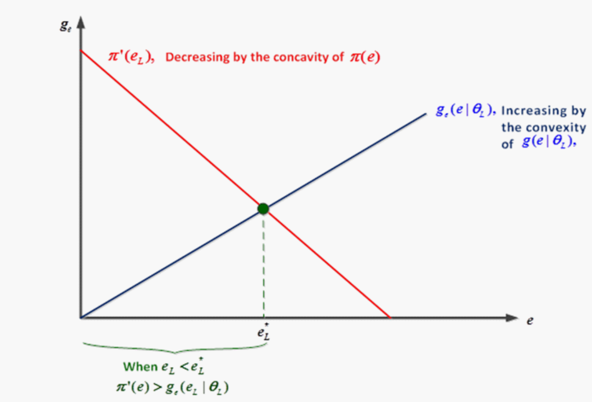
- Comparing optimal effort levels with/without observably of types θ:
  - *Similarity*: e_H coincides with/without observably $e_H = e_H^*$, i.e., it still solves $\pi'(e_H) = g_e(e_H, \theta_H)$.
  - *Difference*: e_L doesn't coincide with e_L^*. In particular, e_L^* solves $\pi'(e_L) = g_e(e_L, \theta_L)$ under observably, but now solves condition (6):

$$\Leftarrow$$

$$\overbrace{\left[\pi'(e_L) - g_e(e_L, \theta_L)\right]}^{+} + \frac{p}{1-p}\overbrace{\left[g_e(e_L, \theta_H) - g_e(e_L, \theta_L)\right]}^{-\text{ by definition}} = 0$$

  - The **first term** is only zero if $e_L = e_L^*$, but becomes positive for all $e_L < e_L^*$.
  - The **second term** is always negative since $g_e(e_L, \theta_H) < g_e(e_L, \theta_L)$ by definition.
  - Thus, we must have $e_L < e_L^*$ for condition (6) to be zero (inefficiency due to asymmetric information).

Hidden information - Comparison



Hidden information - Comparison

- This model helps us extend the model of competitive screening we analyzed in Chapter 13 (with firms offering contracts (w, t) to workers of two types) to...
  - settings in which a single firm offers contracts to the same two types of workers (high and low ability).
- For more details, see pages 500-501 in MWG.
  - It is just a matter of relabeling the principal's problem we described today.