Moral Hazard

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Moral Hazard

- **Reading materials:**
  - Start with Prajit Dutta, Chapter 19.
  - MWG, Chapter 14
  - Macho-Stadler and Perez-Castrillo, Chapter 3
  - Applications: Milgrom and Roberts (their book *Economics, Organization, and Management*), Chapters 6 and 7 (almost no math!)
  - More applications: Freixas and Rochet (their book *The Microeconomics of Banking*), Chapter 4.

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- Hidden actions:
  - As a employer, you cannot observe the effort that the manager you hired exerts...
  - but you can observe the profits of your firm as an **imperfect** indication of his effort.
- For this reason these models are often referred as principal-agent problems.
- Section 14.B in MWG

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- **Time structure:**
  - Principal (firm) offers a contract
  - Agent (worker) decides to accept or reject the contract
  - Upon acceptance, the agent exerts a non-observable effort level e.
  - Nature determines how effort transforms into profits.

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- The employee selects an effort level $e \in \mathbb{R}_+$
- Profits $\pi \in [\underline{\pi}, \overline{\pi}]$ are affected by effort e, as follows

$$f(\pi|e) > 0 \text{ for all } e > 0$$

thus indicating that a given profit value π can arise from any effort level e.

  - *Example:* A million US$ in profits can arise from a high effort (with perhaps a high conditional prob.), but also from a low effort level (lucky slacker!, although this occurs with a very low conditional prob.)

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- For simplicity, we restrict the effort level to be discrete $e \in \{e_L, e_H\}$
  - For the extension to the continuous case, where $e \in \mathbb{R}_+$, see App. A in MWG; or Macho-Stadler and Perez-Castrillo (posted on Angel).
- How to make sure there is a conflict between principal and manager's interests?
  - Assuming that a high effort is more likely to yield a high profit than a low effort.
  - The principal seeks to induce a high effort, while the manager would prefer a low effort (if he receives the same salary).

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- But how to put this assumption more formally?
  - Using an old friend: FOSD

$$F(\pi|e_H) \leq F(\pi|e_L) \text{ for all profits } \pi \in [\underline{\pi}, \overline{\pi}]$$

or

$$1 - F(\pi|e_H) > 1 - F(\pi|e_L) \text{ for all profits } \pi \in [\underline{\pi}, \overline{\pi}]$$

That is, the prob. that e_H induces profits equal to π or higher is larger than that of e_L.

- Figure

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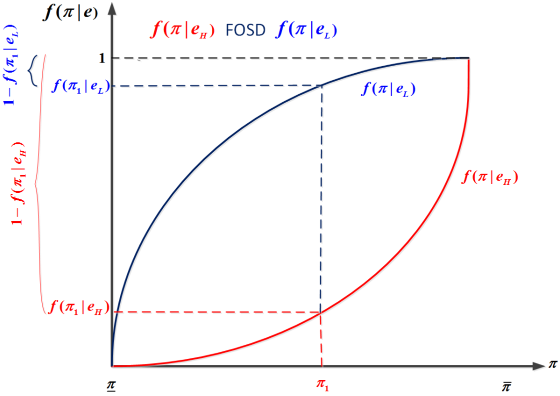

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- Note that the above condition can be written as

$$\int \pi f(\pi|e_H)d\pi > \int \pi f(\pi|e_L)d\pi$$

  In words, the expected profits that the principal obtains if the worker exerts a high effort are larger than when the worker exerts a low effort.

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- **Manager:**
  - His Bernouilli utility function is $u(w, e) = v(w) - g(e)$, where $v(w)$ represents his utility from the salary he receives whereas $g(e)$ indicates his disutility from effort.
  - In addition, $v'(w) > 0$ and $v''(w) \leq 0$; and $g(e_H) > g(e_L)$.
  - This entails that the manager is risk averse.
- **Principal:**
  - His Bernouilli utility function is $\pi - w$
  - Thus, the principal is risk neutral.
  - What if the principal is also risk-averse? See exercise 14.B.2

Benchmark - Effort is Observable

- The principal must offer at least a reservation utility level $\overline{u}$ to the manager.
- In particular, the principal's problem is

$$\max_{e \in \{e_L, e_H\}, w(\pi)} \int (\pi - w(\pi)) f(\pi|e) d\pi$$

$$\text{subject to} \int v(w(\pi)) f(\pi|e) d\pi - g(e) \geq \overline{u}$$

Benchmark - Effort is Observable

- Since

$$\int (\pi - w(\pi))\, f(\pi|e)d\pi = \int \pi f(\pi|e)d\pi - \int w(\pi)f(\pi|e)d\pi$$

then, for a given effort e, the above maximization problem is equivalent to the following minimization problem

$$\min_{w(\pi)} \int w(\pi)f(\pi|e)d\pi$$

$$\text{subject to } \int v(w(\pi))f(\pi|e)d\pi - g(e) \geq \overline{u}$$

Benchmark - Effort is Observable

- Taking first-order conditions with respect to w (for each level of π) yields

$$-f(\pi|e) + \gamma v'(w(\pi))f(\pi|e) = 0$$

or

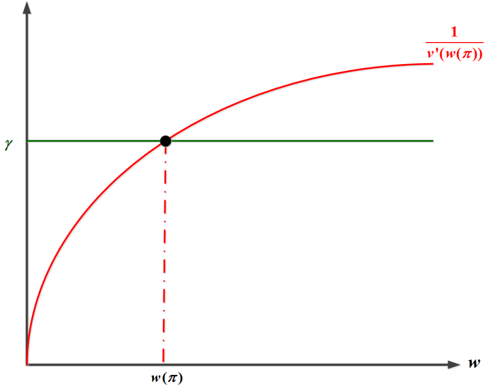
$$\underbrace{f\left(\pi|\, e\right)}_{+}\left[\gamma v'\left(w\left(\pi\right)\right) - 1\right] = 0$$

$$\frac{1}{v'(w(\pi))} = \gamma$$

- Figure

Benchmark - Effort is Observable

- $v'(w(\pi))$ is decreasing in w, i.e., $v'' < 0$, implying that its inverse, $\frac{1}{v'(w(\pi))}$, is increasing in w.



Benchmark - Effort is Observable

- The principal thus provides a fixed wage payment that solves $\frac{1}{v'(w(\pi))} = \gamma$.
- This is a standard risk-sharing result: the risk-neutral principal offers a contract to the risk-averse agent that guarantees him a fixed payoff w_e^* (which is still a function of the effort he exerts, which is observable in this setting, but it is unaffected by the profit realization).

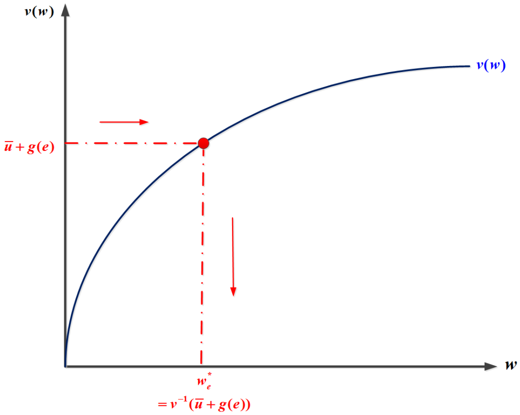
Benchmark - Effort is Observable

- Hence, the principal offers the minimum salary w_e^* that guarantees acceptance

$$\begin{aligned} v(w_e^*) - g(e) &= \bar{u} \\ &\iff v(w_e^*) = \bar{u} + g(e) \\ &\iff w_e^* = v^{-1}\left(\bar{u} + g(e)\right) \end{aligned}$$

  for every effort level e.
- Note that, rather than writing $\int v(w(\pi))f(\pi|e)d\pi$, we wrote $v(w_e^*)$ since the principal pays the same salary w_e^* for all profit levels.

Benchmark - Effort is Observable



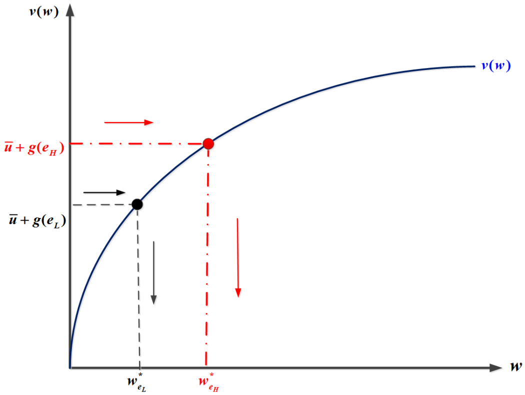
Benchmark - Effort is Observable

- In addition, since $g(e_H) > g(e_L)$, then

$$w^*_{e_H} = v^{-1}(\overline{u} + g(e_H)) > w^*_{e_L} = v^{-1}(\overline{u} + g(e_L))$$

  That is, the salary inducing a high effort level is larger than that inducing a low effort.
- Figure in next slide.

Benchmark - Effort is Observable



- Example: $v(w) = \sqrt{w}, g(e) = e^3, \bar{u} = 10$

Benchmark - Effort is Observable

- Using the above expression of salary w_e^*, the principal problem becomes the following unconstrained problem

$$\max_{e \in \{e_L, e_H\}} \int \pi f(\pi|e) d\pi - \underbrace{v^{-1}\left(\overline{u} + g(e)\right)}_{w_e^*}$$

which, in words, represents the expected profit the principal obtains minus the fixed salary he pays to the agent.

- *Which effort maximizes the above expression?*
    - It depends: if e_H increases expected profits by a larger extent than the increase in the necessary salary, then the principal chooses e_H. Otherwise, he induces e_L.

Benchmark - Effort is Not Observable

- We now need to make sure the agent receives sufficient incentives to select the effort level which is optimal for the principal.
- In particular, the principal's problem becomes

$$\min_{w(\pi)} \int w(\pi) f(\pi|e) d\pi$$

$$\text{subject to} \quad \int v(w(\pi)) f(\pi|e) d\pi - g(e) \geq \overline{u} \qquad \text{(P.C.)}$$

$$e \quad \text{solves} \quad \max_{\widetilde{e} \in \{e_L, e_H\}} \int v(w(\pi)) f(\pi|\widetilde{e}) d\pi - g(\widetilde{e}) \qquad \text{(I.C.)}$$

where P.C. denotes participation constraint condition; I.C. denotes incentive compatibility condition

Benchmark - Effort is Not Observable

- Before solving the problem, let's first try to get rid of some constraints by understanding:
  - which is the salary that induces effort e_L
  - which is the salary that induces effort e_H

Benchmark - Effort is Not Observable

- Which is the salary that induces effort e_L?
  - $w^*_{e_L} = v^{-1}\left(\overline{u} + g(e_L)\right)$.
  - It would not induce the alternative effort e_H (the salary is too low for the manager), thus satisfying IC.

$$\int v(w(\pi))f(\pi|e_L)d\pi - g(e_L) \geq \int v(w(\pi))f(\pi|e_H)d\pi - g(e_H)$$

Benchmark - Effort is Not Observable

- Which is the salary that induces effort e_L?
  - Note that this salary $w^*_{e_L} = v^{-1}(\overline{u} + g(e_L))$ coincides with the salary we found when effort is observable.
  - It also satisfies PC (recall our discussion when effort was observable).
  - It minimizes the salary expenses from the principal to the agent:
    - a higher salary could still be reduced achieving participation and an effort of e_L from the agent;
    - whereas a lower salary would deter the agent from accepting the contract.

Benchmark - Effort is Not Observable

- Which is the salary that induces effort e_H?
  - The agent chooses e_H rather than e_L if his incentive compatibility condition holds

$$\int v(w(\pi))f(\pi|e_H)d\pi - g(e_H) \geq \int v(w(\pi))f(\pi|e_L)d\pi - g(e_L) \tag{IC$_H$}$$

Benchmark - Effort is Not Observable

- Hence, the principal's optimization problem, when he seeks to induce e_H, becomes

$$\min_{w(\pi)} \int w(\pi) f(\pi|e_H) d\pi$$

$$\text{subject to} \int v(w(\pi)) f(\pi|e_H) d\pi - g(e_H) \geq \overline{u} \qquad (\text{PC}_H)$$

$$\int v(w(\pi)) f(\pi|e_H) d\pi - g(e_H) \geq \int v(w(\pi)) f(\pi|e_L) d\pi - g(e_L) \qquad (\text{IC}_H)$$

Benchmark - Effort is Not Observable

- Letting γ and μ be the Lagrangian multipliers of constraints PC and IC_H, respectively, the Kuhn-Tucker conditions (with respect to w) of this problem are

$$-f(\pi|e_H) + \gamma v'(w(\pi))f(\pi|e_H) + \mu v'(w(\pi))f(\pi|e_H)$$

$$-\mu v'(w(\pi))f(\pi|e_L) = 0$$

Rearranging,

$$\frac{1}{v'(w(\pi))} = \gamma + \underbrace{\mu\left[1 - \frac{f(\pi|e_L)}{f(\pi|e_H)}\right]}_{\text{New, relative to observable effort}}$$

Benchmark - Effort is Not Observable

- Both constraints bind, i.e., $\gamma > 0$ and $\mu > 0$. (Otherwise, the constraints would be superfluous.)
- We can now compare our FOCs with those under effort observably:

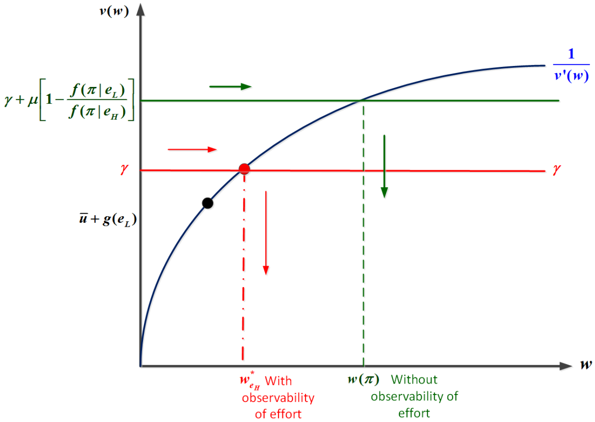
$$\text{if } f(\pi|e_L) < f(\pi|e_H), \text{ then } \frac{f(\pi|e_L)}{f(\pi|e_H)} < 1, \text{ and}$$

$$\frac{1}{v'(w(\pi))} = \gamma + \underbrace{\mu\left[1 - \frac{f(\pi|e_L)}{f(\pi|e_H)}\right]}_{\mu\times(0,1)} > \gamma$$

which implies that $w(\pi) > w^*_{e_H}$ (see next figure).

Benchmark - Effort is Not Observable

- Salary inducing effort e_H with/without observably of effort.



Benchmark - Effort is Not Observable

- Intuitively, $f(\pi|e_L) < f(\pi|e_H)$ implies a likelihood ratio $\frac{f(\pi|e_L)}{f(\pi|e_H)} < 1$,
  - indicating that a given profit level π is more likely to occur under effort e_H than under e_L.
- The opposite argument would apply if $f(\pi|e_L) > f(\pi|e_H)$.

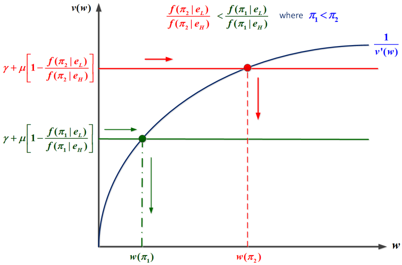
Benchmark - Effort is Not Observable

- Under which conditions is the optimal compensation scheme monotonically increasing in profits?
  - For that, we need the likelihood ratio $\frac{f(\pi|e_L)}{f(\pi|e_H)}$ to be *decreasing* in profits. In words, as profits increase the likelihood of obtaining profit π from e_H must increase faster than that from e_L.
  - This property is often referred as the *monotone likelihood ratio property* (MLRP); as introduced in EconS 501,
  - MLRP is generally expressed as

$$\frac{f(x)}{g(x)} < \frac{f(y)}{g(y)} \quad \text{where} \quad x > y.$$

Benchmark - Effort is Not Observable

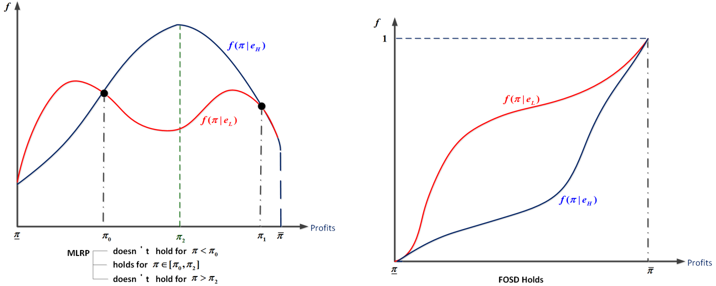
- What happens with the optimal salary under non-observability if the likelihood ratio $\frac{f(\pi|e_L)}{f(\pi|e_H)}$ decreases in profits, i.e., the MLRP holds?



- Hence, salary $w(\pi_2) > w(\pi_1)$ implying that if MLRP holds the salary of the agent exerting e_H is increasing in profits. (The salary of the agent exerting e_L is constant in profits.)

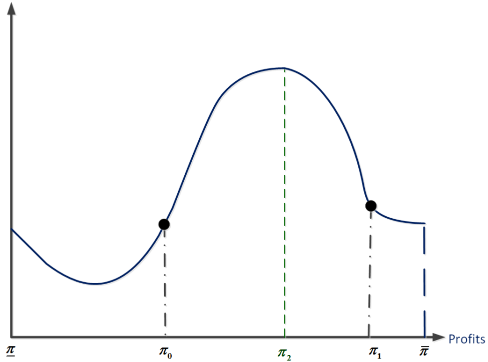
Benchmark - Effort is Not Observable

- Even if we impose FOSD as one of our initial assumption (left panel), MLRP doesn't necessarily holds (right panel)



Benchmark - Effort is Not Observable

- Optimal compensation scheme $w(\pi)$ is increasing in profits when MLRP holds, $f(\pi|\, e_H) > f(\pi|\, e_L)$, and the distance $f(\pi|\, e_H) - f(\pi|\, e_L) > 0$ grows, which only occurs in profits $\pi \in [\pi_0, \pi_2]$.



Benchmark - Effort is Not Observable

- **Given the above salaries for e_H and e_L, which effort should the principal implement?**
  - We know that salary $w_{e_L}^* = v^{-1}(\overline{u} + g(e_L))$ implements e_L, which coincides with that under effort observably.
  - We know that salary $w(\pi)$ implements e_H which, given the risk it introduces, must be higher than the fixed salary under effort observably $w_{e_H}^* = v^{-1}(\overline{u} + g(e_H))$, since the agent must be compensated for the risk he now bears.

Benchmark - Effort is Not Observable

- Hence, when deciding which effort to implement, the principal compares the effect that a larger effort entails:
- (1) on one hand, it increases the likelihood of higher profits; but
- (2) on the other hand, it is only induced with a higher salary.
  - The risk-premium that the principal must now offer the agent (relative to effort observably) makes e_H more costly to implement.
  - Thus, e_H is less likely to arise as optimal for the principal when effort is not observable than when effort is observable.

Benchmark - Effort is Not Observable

- **Alternative way to put it:**
  - If low effort e_L was optimal when effort is observable, then it also is when effort is non-observable.
  - In this case, nonobservability causes no losses.
  - However, if the high effort e_H was optimal under observability,
    - it might still be optimal under nonobservability, but at a higher cost for the principal; or
    - it might not be optimal under nonobservability.
    - Thus giving rise to inefficiencies in both cases.
- **Example:** Exercise 14.B.4 in MWG.