EconS 503 - Advanced Microeconomics II

Limit pricing models and PBE[1]

1 Model

Consider an entry game with an incumbent monopolist (Firm 1) and an entrant (Firm 2) who analyzes whether or not to join the market. The incumbent's marginal costs are either high H or low L, i.e., $c_1^H > c_1^L > 0$. We first examine the case where entrant and incumbent are informed about each others' marginal costs, and afterwards the case in which the entrant is uninformed about the incumbent's costs. Let us consider a two-stage game where, in the first stage, the incumbent has monopoly power and selects an output level q. In the second stage a potential entrant decides whether or not to enter. If entry occurs, agents compete as Cournot duopolists, simultaneously selecting production levels x_1 and x_2, for the incumbent and entrant, respectively. Otherwise, the incumbent maintains its monopoly power during both periods.

2 Complete information

We can apply backward induction to find the subgame perfect equilibrium of the complete information game.

Second period, No entry. During the second period, if entry does not occur, the incumbent chooses output x_1 which maximizes monopoly profits

$$\overline{M}_1^K \equiv \max_{x_1} p(x_1)\, x_1 - c_1^K x_1,$$

where $x_1^{K,m}$ is the profit-maximizing second-period output and $K = \{H, L\}$. The inverse demand function $p(\cdot)$ is decreasing and concave in output, satisfies $p(0) \geq c_1^K$, and is constant across periods.

Second period, Entry. If entry occurs, firms compete as Cournot duopolists in the second period, with associated equilibrium profits for the incumbent and entrant respectively,

$$D_1^K \equiv \max_{x_1} p(X)\, x_1 - c_1^K x_1 \quad \text{and} \quad D_2^K \equiv \max_{x_2} p(X)\, x_2 - c_2 x_2 - F$$

where X denotes aggregate second-period equilibrium output, $c_2 = c_1^H$ represents the entrant's marginal cost (we consider that the entrant lacks experience in the industry and thus has high costs), and F denotes the fixed entry cost. To make the entry decision interesting, assume that

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when the incumbent's costs are low, entry is unprofitable, whereas when they are high entry is profitable, i.e., $D_2^L < 0 < D_2^H$ for all q. (If we didn't have this assumption, entry would occur regardless of the incumbent's type, or entry would not entry regardless of the incumbent's type. In those settings, the uninformed potential entrant that we consider in the next section would not need to infer any information from the incumbent's first-period output because he would enter/not enter regardless of the incumbent's true type.)

First period. When the incumbent's costs are low, entry does not occur in the second period. Hence, the incumbent chooses the first-period output q that solves

$$\max_q \; p(q)q - c_1^L q + \delta \overline{M}_1^L,$$

where $\delta \in (0,1)$ denotes the incumbent's discount factor. Let $q^{L,Info}$ denote the solution to the above maximization problem with low costs.

When the incumbent's costs are high, entry follows in the second period. Hence, the incumbent chooses a first-period output q that solves

$$\max_q \; p(q)q - c_1^H q + \delta D_1^H.$$

Let $q^{H,Info}$ denote the solution to the above maximization problem with high costs.

Example. Consider a linear inverse demand function $p(q) = 1 - q$ and no discounting, $\delta = 1$. The incumbent's marginal costs satisfy $1 > c_1^H > c_1^L$. When no entry occurs, the incumbent selects output $x_1^{K,m} = \frac{1-c_1^K}{2}$ and $q^{K,Info} = \frac{1-c_1^K}{2}$ in the second and first period, respectively, for every incumbent type $K = \{H, L\}$. When entry occurs, incumbent and entrant choose $x_1^{K,d} = \frac{1+c_2-2c_1^K}{3}$ and $x_2^{K,d} = \frac{1-2c_2+c_1^K}{3}$ in the second period, while the incumbent selects $q^{K,Info} = \frac{1-c_1^K}{2}$ in the first period.

Importantly, under complete information, the high-cost incumbent cannot deter entry of potential competitors, while the low-cost incumbent doesn't need to select an output level different from $q^{L,Info}$ in order to deter entry. As we next examine, under incomplete information none of these results hold: the inefficient type of firm will be able to conceal its type from potential competitors and thus deter entry, whereas the low-cost firm might be in need of increasing its output level in order to convey its type to potential competitors, thus detering them from entering the industry.

3 Incomplete information

In this section we investigate the case where the incumbent is privately informed about its marginal costs, while the entrant only observes the incumbent's first-period output. The time structure of this signaling game is as follows.

1. Nature decides the realization of the incumbent's marginal costs, either high or low, with probabilities $p \in (0,1)$ and $1-p$, respectively. The incumbent privately observes this realization but the entrant does not.

2. The incumbent chooses its first-period output level, q.

3. Observing the incumbent's output decision, the entrant forms beliefs about the incumbent's initial marginal costs. Let $\mu(c_1^H|q)$ denote the entrant's posterior belief about the initial costs being high after observing a particular first-period output from the incumbent q.

4. Given the above beliefs, the entrant decides whether or not to enter the industry.

5. If entry does not occur, the incumbent maintains its monopoly power, whereas if entry occurs, both agents compete as Cournot duopolists and the entrant observes the incumbent's type.

3.1 Separating equilibrium

Let us next analyze the separating equilibrium where the incumbent selects a first-period output q^H when its costs are high, but chooses q^L when its costs are low.[2] Entrant's equilibrium beliefs after observing equilibrium output q^H and q^L are $\mu(c_1^H|q^H) = 1$ and $\mu(c_1^H|q^L) = 0$, respectively. The entrant enters (stays out) when it infers that the incumbent's initial cost are high (low, respectively). What about off-the-equilibrium beliefs, i.e., $\mu(c_1^H|q)$ after observing an output level $q \neq q^H \neq q^L$. For simplicity, we consider $\mu(c_1^H|q) = 1$, which attracts entry.[3] Figure 0 below depicts the entrant's beliefs in this separating strategy profile. Let us investigate the conditions that guarantee the existence of a separating equilibrium.

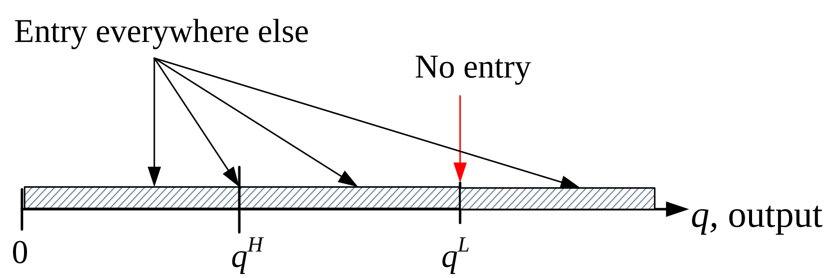

Fig 0. Output choices and entry in the separating PBE.

High-cost incumbent. When its marginal costs are high, the incumbent selects q^H and entry occurs. If the incumbent deviates towards the low-cost incumbent's output q^L, it deters entry.

[2] The separating output q^L is weakly higher than the production level that the low-cost incumbent selects under complete information, $q^{L,Info}$. Otherwise, the high-cost incumbent could be tempted to pool with the low-cost incumbent by also selecting q^L.

[3] We will later on confirm that these off-the-equilibrium beliefs are consistent with Cho and Kreps' (1987) Intuitive Criterion, which we apply in Proposition 2 to the set of separating PBEs.

Hence, the high-cost incumbent selects its equilibrium output q^H if $M_1^H(q^H) + \delta D_1^H \geq M_1^H(q^L) + \delta\overline{M}_1^H$. In addition, since off-the-equilibrium beliefs are $\mu(c_1^H|q) = 0$, the high-cost incumbent can anticipate that any output level different from q^L will induce entry. As a consequence, among all output levels inducing entry, the one maximizing this firm's profits, i.e., maximizing $M_1^H(q)+\delta D_1^H$, is $q^{H,Info}$. Thus, $M_1^H(q^{H,Info}) + \delta D_1^H \geq M_1^H(q^H) + \delta D_1^H$, and we can rewrite the above incentive compatibility condition as $M_1^H(q^{H,Info}) + \delta D_1^H \geq M_1^H(q^L) + \delta\overline{M}_1^H$, or equivalently,

$$M_1^H(q^{H,Info}) - M_1^H(q^L) \geq \delta\left[\overline{M}_1^H - D_1^H\right] \tag{C1}$$

Low-cost incumbent. Likewise, if the low-cost incumbent chooses the equilibrium output q^L, it deters entry. If instead the incumbent deviates towards the high-cost incumbent's output, q^H, it attracts entry. Conditional on entry, the low-cost incumbent can otherwise select the output $q^{L,Info}$ which maximizes its first-period profits, yielding $M_1^L(q^{L,Info}) + \delta D_1^L$. That is, represents the highest profits that the low-cost incumbent can obtain conditional on entry ensuing. Thus, the low-cost incumbent selects its equilibrium output of q^L if $M_1^L(q^{L,Info}) + \delta D_1^L \leq M_1^L(q^L) + \delta\overline{M}_1^L$, or equivalently,

$$M_1^L(q^{L,Info}) - M_1^L(q^L) \leq \delta\left[\overline{M}_1^L - D_1^L\right] \tag{C2}$$

Let us separately depict incentive compatibility condition C1 and C2. For the high-cost incumbent, C1 can be illustrated as in the figure 1a. Intuitively, the curve $M_1^H(q^{H,Info}) - M_1^H(q^L)$ represents the first-period profit loss that the high-cost incumbent experiences when deviating from the monopoly output level that maximizes its first-period profits, i.e., from selecting any q^L different from $q^{H,Info}$. When $q^L = q^{H,Info}$ such profit loss is minimized to zero, but any deviation from $q^{H,Info}$ (either lower or higher) entails a positive profit loss. In contrast, the flat line $\delta\left[\overline{M}_1^H - D_1^H\right]$ indicates the future gain that the high-cost incumbent can obtain if it successfully deters entry. For our previous parametric example, this line would be

$$\delta\left[\frac{\left(1-c_1^H\right)^2}{4} - \frac{\left(1-c_1^H\right)^2}{9}\right] = \delta\frac{5\left(1-c_1^H\right)^2}{36}$$

as $c_1^H = c_2$ thus yielding a Cournot competition between two symmetric firms in the second period if entry occurs, with profits $\frac{\left(1-c_1^H\right)^2}{9}$. Since we seek to find output levels that the high-cost incumbent doesn't profitably mimic, we need this firm's profit loss to exceed is future gain from deterring entry, as graphically depicted in figure 1a.

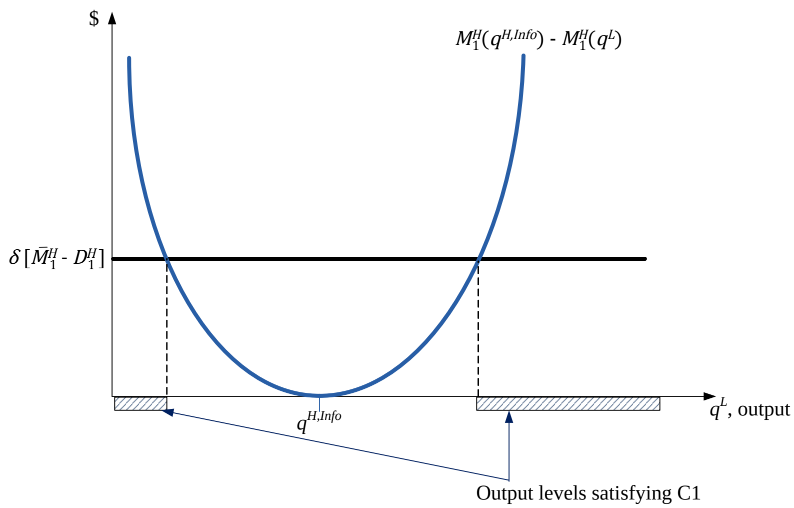

Fig 1a. Incentive compatibility condition C1.

The opposite argument applies for the low-cost incumbent in figure 1b: in order to have incentives to select an output level q_1^L, it must be that its future gain from deterring entry is larger than the profit loss from deviating from first-period profit-maximizing output $q^{L,Info}$; as depicted in the shaded set of output levels in figure 1b.

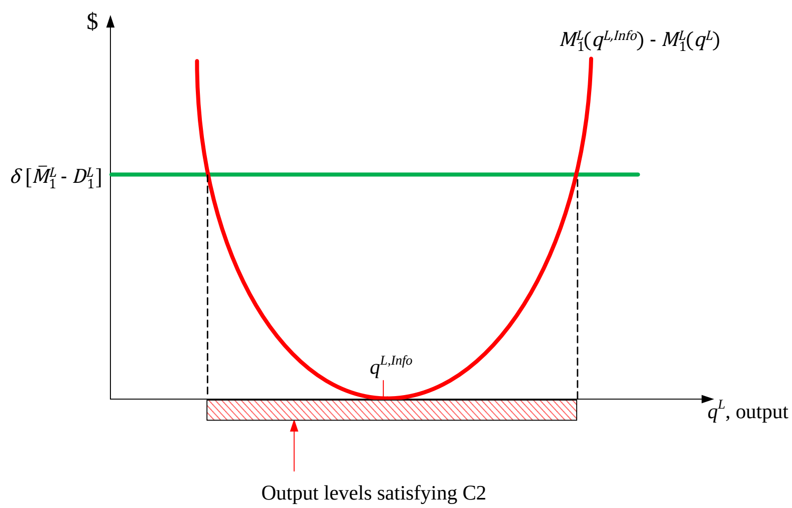

Fig 1b. Incentive compatibility condition C2.

Superimposing figures 1a and 1b, we can examine the set of output levels that simultaneously satisfy condition C1 (for the high-cost incumbent) and C2 (for the low-cost incumbent), as depicted in figure 2. In particular, the overlap between the range of outputs identified in figures 1a and 1b provides us with the set of output levels that constitute a separating PBE of the signaling game. Intuitively, the high-cost incumbent doesn't have incentives to mimic the output level chosen by the low-cost firm, i.e., $q^L \in [q^A, q^B]$. The low-cost firm, by contrast, has incentives to choose an output level in the interval $q^L \in [q^A, q^B]$, which is above the first-period output under complete information, $q^{L,Info} = \frac{1-c_1^L}{2}$. Thus, the low-cost incumbent increases its first-period output in order to communicate its efficient costs to the potential entrant, deterring entry as a result.

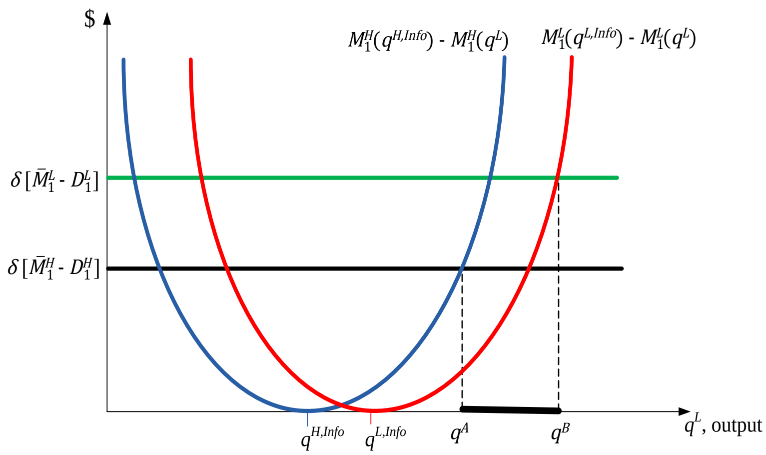

Fig 2. Separating equilibria in the limit pricing model.

Remark: Note that the figure depicts the profit-loss curves for the low- and high-cost firm as crossing only once. This single-crossing property holds under relatively general conditions. For instance, in our above parametric example, the profit-loss functions cross each other at the value of q^L that solves

$$M_1^L(q^{L,Info}) - M_1^L(q^L) = M_1^H(q^{H,Info}) - M_1^H(q^L)$$

or

$$\frac{\left(1-c_1^L\right)^2}{4} - \left[(1-q^L)q^L - c_1^L q^L\right] = \frac{\left(1-c_1^H\right)^2}{4} - \left[(1-q^L)q^L - c_1^H q^L\right]$$

(You can find, as a practice the value of q^L that solves this expression. For simplicity, you can assume the parameter values we consider below, $c_1^H = \frac{1}{2}$ and $c_1^L = \frac{1}{3}$. You will obtain a crossing point of $q^L = \frac{7}{24}$.) For more details about under which conditions the single-crossing property holds, see Tirole's textbook on Industrial Organization, pp. 367-374.

We can formally describe the set of separating PBEs in the limit pricing model as follows. Recall that in our description we must specify both equilibrium and off-the-equilibrium beliefs.

Proposition 1. *A separating strategy profile can be sustained as a Perfect Bayesian Equilibria (PBE) in the signaling game where:*

1. *In the first period, the high-cost incumbent selects $q^{H,Info}$ and the low-cost chooses $q^L \in$*

$[q^A, q^B]$, *where* q^A *solves condition C1 with equality, and* $q^A > q^{L,Info}$*; whereas* q^B *solves condition C2 with equality.*

2. *The entrant enters only after observing* $q^{H,Info}$*, given equilibrium beliefs* $\mu(c_1^H|q^{H,Info}) = 1$ *and* $\mu(c_1^H|q^L) = 0$ *after observing any* $q^L \in [q^A, q^B]$*. For every off-the-equilibrium output level* q*, where* $q \neq q^{H,Info} \neq q^L$*, entrant's beliefs are* $\mu(c_1^H|q) = 1$*; and*

3. *In the second period of the game, the incumbent selects an output* $x_1^{K,m}$ *if entry does not occur, and every firm* $i = \{1, 2\}$ *chooses* $x_i^{K,d}$ *if entry occurs.*

A natural question is which of the above separating PBEs survive the Cho and Kreps' (1987) Intuitive Criterion. The next proposition describes the unique separating PBE surviving this refinement criterion. (All proofs are relegated to the appendix.)

Proposition 2. *Among all separating PBEs identified in Proposition 1, only the following survives the Cho and Kreps' (1987) Intuitive Criterion.*

1. *In the first period, the high-cost incumbent selects* $q^{H,Info}$ *and the low-cost chooses* $q^L = q^A$*, where* q^A *solves condition C1 with equality, and* $q^A > q^{L,Info}$*;*

2. *The entrant enters only after observing* $q^{H,Info}$*, given equilibrium beliefs* $\mu(c_1^H|q^{H,Info}) = 1$ *and* $\mu(c_1^H|q^A) = 0$*. For any off-the-equilibrium output level* q*, where* $q \neq q^{H,Info} \neq q^A$*, entrant's beliefs are* $\mu(c_1^H|q) = 1$*, and prior probability* p *satisfies* $p > \overline{p}$*, where* $\overline{p} \equiv \frac{-D_2^L}{D_2^H - D_2^L}$*; and*

3. *In the second period of the game, the incumbent selects an output* $x_1^{K,m}$ *if entry does not occur, and every firm* $i = \{1, 2\}$ *chooses* $x_i^{K,d}$ *if entry occurs.*

Therefore the high-cost incumbent selects a first-period output $q^{H,Info}$, which coincides with its output under complete information; while the low-cost incumbent chooses q^A, which is larger than its output under complete information, $q^{L,Info}$, in order to convey its cost structure to the entrant, deterring it from entering. Intuitively, q^A is the least-costly separating output level (i.e., least-costly deviation from complete information strategies) that the low-cost firm must choose to reveal its type to the potential entrant.

A sensible measure of the overproduction that the low-cost incumbent practices in order to convey its type to the potential entrant (and thus deter entry) is the distance between its output choices under incomplete and complete information, $q^A - q^{L,Info}$, i.e., its "separating effort," as depicted in the next figure. We evaluate such separating effort in our next parametric example.

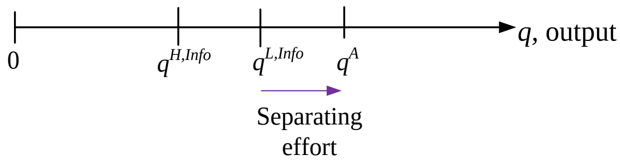

Fig 2b. Separating effort.

Example. Continuing with our above example, consider costs $c_1^H = \frac{1}{2}$ and $c_1^L = \frac{1}{3}$. In particular, in order to find the low-cost incumbent's separating output level q^A, we evaluate incentive compatibility condition C1 with equality

$$M_1^H(q^{H,Info}) - M_1^H(q^A) = \delta\left[\overline{M}_1^H - D_1^H\right] \tag{C1}$$

Using the functional forms of our example, yields

$$\underbrace{\frac{\left(1-c_1^H\right)^2}{4}}_{1/16 \text{ since } c_1^H=1/2} - \left[(1-q^A)q^A - \frac{1}{2}q^A\right] = \delta\left[\underbrace{\frac{\left(1-c_1^H\right)^2}{4}}_{1/16 \text{ since } c_1^H=1/2} - \underbrace{\frac{\left(1-c_1^H\right)^2}{9}}_{1/36 \text{ since } c_1^H=c_2=1/2}\right]$$

Rearranging, and assuming that $\delta = 1$, we obtain

$$\frac{1}{16} - \left[(1-q^A)q^A - \frac{1}{2}q^A\right] = \frac{1}{16} - \frac{1}{36}$$

$$\iff \frac{1}{36} - (1-q^A)q^A - \frac{1}{2}q^A = 0$$

or

$$-\left(q^A\right)^2 + \frac{1}{2}q^A - \frac{1}{36} = 0$$

and solving for output q^A yields $q^A = 0.55$ (the second root is negative). Summarizing, the low-cost incumbent raises its first-period output from $q^{L,Info} = \frac{1-c_1^L}{2} = \frac{1-\frac{1}{3}}{2} = 0.33$ under complete information to $q^A = 0.55$ under the separating equilibrium. Hence, the "separating effort" that this firm must exert in order to reveal its type to the potential entrant (and thus deter entry) is $q^A - q^{L,Info} = 0.55 - 0.33 = 0.22$.

Remark: Importantly, the low-cost incumbent chooses his output level q^A by considering the incentive compatibility condition of the high-cost incumbent (condition C1). The low-cost firm does that because that is the output level that would make the high-cost firm indifferent between mimicking its output decision q^A and produce $q^{H,Info}$. In other words, a higher output level than q^A would be strictly unprofitable to imitate by the high-cost firm, but output levels lower than q^A would be profitably mimicked by the high-cost incumbent. The low-cost firm then increases its output from $q^{L,Info}$ to q^A to successfully separate from the high-cost firm, thus conveying its type to the potential entrant, who is deterred from entering the market.

3.2 Pooling equilibrium

Let us now analyze the pooling strategy profiles where both types of incumbent select the same output level q.

Beliefs. In this case, equilibrium beliefs are hence $\mu\left(c_1^H|q\right) = p$ and $\mu\left(c_1^L|q\right) = 1-p$, which coincide with the prior probability distribution over types. In addition, off-the-equilibrium beliefs cannot be identified using Bayes' rule, and for simplicity let us assume that, after observing any output level $q' \neq q$, the entrant beliefs become $\mu\left(c_1^H|q'\right) = 1$, which induce the entrant to enter after observing the off-the-equilibrium output level q'.

Entrant's response. After observing the equilibrium output q, the entrant enters if and only if $pD_2^H + (1-p)D_2^L \geq 0$ or, solving for p,

$$p \geq \frac{-D_2^L}{D_2^H - D_2^L} \equiv \overline{p}$$

where $D_2^H > 0$, implying that $D_2^H - D_2^L \geq -D_2^L$, entailing that the probability cutoff $\overline{p}$ satisfies $\overline{p} \in (0,1)$. We can hence conclude that the entrant enters if $p \geq \overline{p}$, and stays out otherwise. Note that if entry occurs after q, this induces every type of incumbent to select $q^{K,Info}$. But since $q^{H,Info} \neq q^{L,Info}$ this strategy profile cannot be a pooling equilibrium where both types of incumbent choose the same output level. Hence, it must be that $p < \overline{p}$ inducing the entrant to stay out.

Incumbent's output level (i.e., incentive compatibily conditions). Let us check under which conditions the high-cost incumbent does not deviate from q. By selecting q, it deters entry obtaining $M_1^H(q) + \delta\overline{M}_1^H$. By deviating towards $q' \neq q$ it attracts entry, yielding a payoff of $M_1^H(q') + \delta D_1^H$, which is maximized at $q^{H,Info}$. Hence, the high-cost incumbent does not deviate from q if,

$$M_1^H(q) + \delta\overline{M}_1^H \geq M_1^H(q^{H,Info}) + \delta D_1^H$$

or equivalently,

$$M_1^H(q^{H,Info}) - M_1^H(q) \leq \delta\left[\overline{M}_1^H - D_1^H\right] \tag{C1'}$$

and similarly, for the low-cost incumbent,

$$M_1^L(q^{L,Info}) - M_1^L(q) \leq \delta\left[\overline{M}_1^L - D_1^L\right] \tag{C2'}$$

Hence, any q simultaneously satisfying both of the above C1' and C2' conditions for the high and low-cost incumbents constitutes a pooling equilibrium first-period output of the signaling game, which we refer as the pooling first-period output q. Intuitively, the above conditions indicate that both firms find that the future gain from deterring entry (flat lines in figure 3) is larger than the profit loss of deviating from the output level that maximizes first-period profits, $q^{K,Info}$ (as depicted in the curves of figure 3).

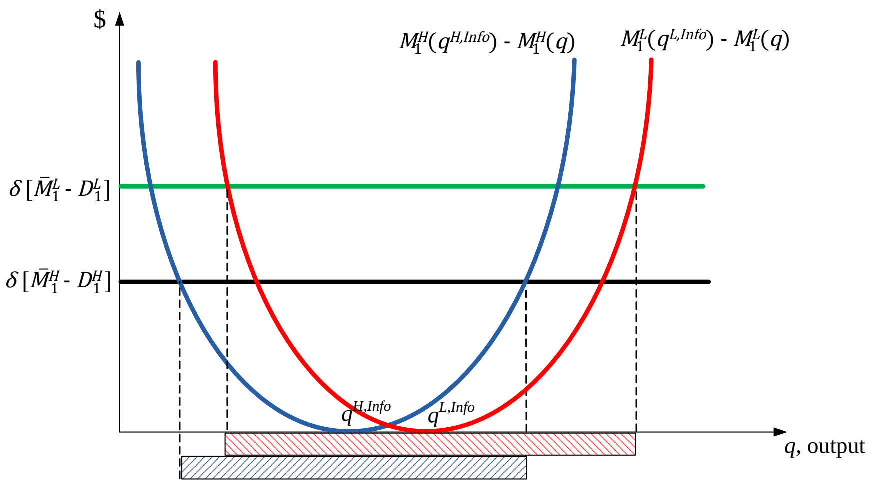

Fig 3. Output levels satisfying C1' and C2'.

Hence, the region of output levels that simultaneuosly satisfy conditions C1' and C2' are those in the overlap identified in figure 3, as summarized in figure 4. Therefore, in the pooling equilibrium both types of incumbent produce the same first-period output, q, which reveals no additional information to the entrant. Since, in addition, the entrant's equilibrium beliefs coincide with its priors, $\mu(c_1^H|q) = p$, and $p < \overline{p}$, the concealing strategy of the incumbent deters entry.

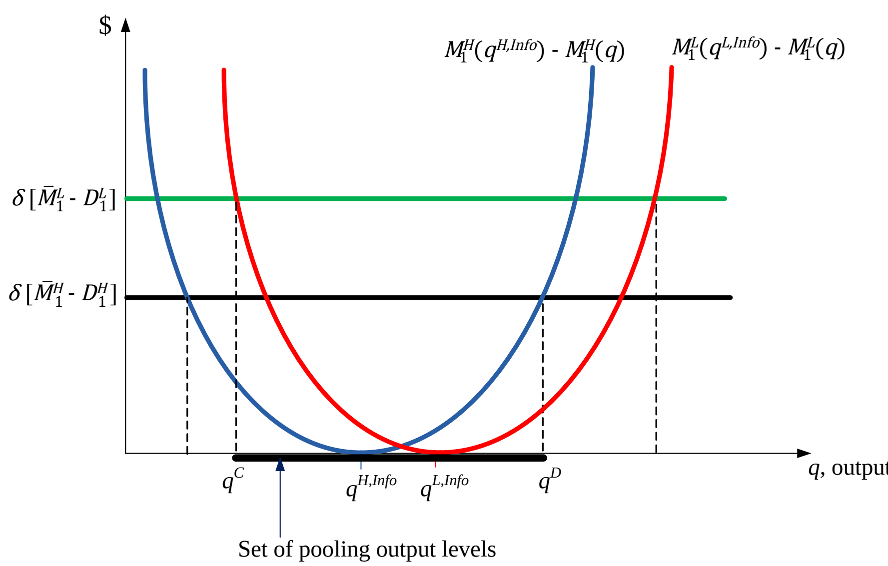

Fig 4. Set of pooling PBEs in the limit pricing model.

We can now summarize the pooling PBEs we found above in the following proposition. (We don't include the proof in the appendix as all steps were already shown above.)

Proposition 3. *The following strategy profiles can be sustained as pooling PBE in the limit pricing game:*

1. *In the first period, both types of incumbent select the same first-period output* $q \in \left[q^C, q^D\right]$*, where* q^C *solves condition C2' with equality, while* q^D *solves* $M_1^H(q^{H,Info}) - M_1^H(q) = \delta\left[\overline{M}_1^H - D_1^H\right]$.
2. *The entrant does not enter after observing the equilibrium output* $q \in \left[q^C, q^D\right]$*, but enters after observing off-the-equilibrium output* q'*,* $q' \neq q$*, given beliefs* $\mu(c_1^H|q^{L,Info}) = p < \overline{p}$ *and* $\mu(c_1^H|q') = 1$*; and*
3. *In the second period of the game, the incumbent selects* $x_1^{K,m}$ *if entry does not occur, and every firm* $i = \{1, 2\}$ *chooses* $x_i^{K,d}$ *if entry occurs.*

Similarly as we did for the separating PBE, we can now examine which of these pooling PBEs survive Cho and Kreps' (1987) Intuitive Criterion.

Proposition 4. *Among all pooling PBEs identified in Proposition 3, the only one surviving the Cho and Kreps' (1987) Intuitive Criterion is that in which both firms select a first-period output* $q = q^{L,Info}$ *which thus coincides with the output level that the low-cost firm would select under complete information.*

Hence, while the low-cost firm behaves as under complete information, i.e., it chooses $q^{L,Info}$, the high-cost firm must increase its output level, from $q^{H,Info}$ to $q^{L,Info}$, in order to mimic the production decision of the efficient firm, and thus conceal information from the potential entrant.

Example. Following the same example as in the separating equilibrium, note that the high-cost incumbent increases its output, from $q^{H,Info} = 0.25$ under complete information to $q^{L,Info} = 0.33$ in the pooling equilibrium. In other words, the high-cost incumbent needs to exert a "pooling effort" of $0.33 - 0.25 = 0.08$ in order to mimic the output level of the low-cost firm, conceal its type from the potential entrant, and thus deter entry.

4 Appendix

4.1 *Proof of Proposition 1*

First, note that entrant's beliefs become $\mu(c_1^H|q^H) = 1$ after observing an equilibrium output of q^H and $\mu(c_1^H|q^L) = 0$ after observing the equilibrium q^L, where $q^L \in \left[q^A, q^B\right]$. If the entrant observes an off-the-equilibrium output $q \neq q^H \neq q^L$, then Bayes' rule does not specify a particular off-the-equilibrium belief, i.e., $\mu(c_1^H|q) \in [0,1]$, and for simplicity we assume $\mu(c_1^H|q) = 1$.

High-cost incumbent. Let us now examine the high-cost incumbent. By selecting the equilibrium output $q^{H,Info}$, the high-cost incumbent obtains profits of $M_1^H(q^{H,Info}) + \delta D_1^H$. First, note that $q^{H,Info}$ maximizes $M_1^H(q)+\delta D_1^H$. Second, first-period output $q^{H,Info}$ coincides with the equilibrium output that the high-cost incumbent selects under complete information, yielding the same profits. By deviating towards the low-cost incumbent's equilibrium output, q^L, the high-cost firm obtains profits of $M_1^H(q^L) + \delta\overline{M}_1^H$. Hence, the high-cost incumbent prefers to produce an equilibrium output of $q^{H,Info}$ rather than deviating towards q^L if $M_1^H(q^{H,Info}) + \delta D_1^H \geq M_1^H(q^L) + \delta\overline{M}_1^H$, or alternatively,

$$M_1^H(q^{H,Info}) - M_1^H(q^L) \geq \delta\left[\overline{M}_1^H - D_1^H\right] \tag{C1}$$

If instead the high-cost incumbent deviates towards an off-the-equilibrium output of $q \neq q^H \neq q^L$ then entry follows, yielding profits of $M_1^H(q) + \delta D_1^H$, which do not exceed its equilibrium profits of $M_1^H(q^{H,Info}) + \delta D_1^H$.

Low-cost incumbent. Let us now turn to the low-cost incumbent. Selecting the equilibrium first-period output q^L yields $M_1^L(q^L)+\delta\overline{M}_1^L$. By deviating towards the high-cost incumbent's equilibrium output, $q^{H,Info}$, the low-cost incumbent attracts entry, obtaining profits of $M_1^L(q^{H,Info}) + \delta D_1^L$. Therefore, the low-cost incumbent selects an equilibrium output of q^L rather than deviating towards $q^{H,Info}$ if

$$M_1^L(q^L) + \delta\overline{M}_1^L \geq M_1^L(q^{H,Info}) + \delta D_1^L \tag{A.3}$$

If instead the low-cost incumbent deviates towards any off-the-equilibrium output of $q \neq q^H \neq q^L$ then entry follows and therefore the incumbent selects the value of q that maximizes profits after entry $M_1^L(q) + \delta D_1^L$. Let $q^{L,Info}$ denote the solution to this maximization problem, yielding profits of $M_1^L(q^{L,Info}) + \delta D_1^L$. Hence, the low-cost incumbent chooses an equilibrium output q^L rather than deviating towards $q^{L,Info}$ if

$$M_1^L(q^L) + \delta\overline{M}_1^L \geq M_1^L(q^{L,Info}) + \delta D_1^L \tag{A.4}$$

Note that condition A.4 implies A.3 since $M_1^L(q^{L,Info}) + \delta D_1^L > M_1^L(q^{H,Info}) + \delta D_1^L$, given that $q^{L,Info}$ maximizes the low-cost incumbent's profits across both periods given entry. Therefore, condition A.4 becomes the condition that must be satisfied in order to guarantee that the low-cost incumbent does not deviate from its equilibrium output of q^L. Let us rewrite this condition as follows

$$M_1^L(q^{L,Info}) - M_1^L(q^L) \leq \delta\left[\overline{M}_1^L - D_1^L\right] \tag{C2}$$

4.2 *Proof of Proposition 2*

Let us start considering the case in which the low-cost incumbent produces a first-period output of q^B. Let us first check if a deviation towards $q \in (q^A, q^B)$ is equilibrium dominated for either type of incumbent. On one hand, the highest profit that the high-cost incumbent can obtain deviating towards $q \in (q^A, q^B)$ occurs when entry does not follow. In such a case, the high-cost incumbent obtains $M_1^H(q) + \delta \overline{M}_1^H$. Hence, it deviates only if $M_1^H(q) + \delta \overline{M}_1^H > M_1^H(q^{H,Info}) + \delta D_1^H$. But condition C1 guarantees that this inequality does not hold for any $q \in (q^A, q^B)$. Hence, the high-cost incumbent does not have incentives to deviate from $q^{H,Info}$ to $q \in (q^A, q^B)$.

On the other hand, the highest profit that the low-cost incumbent can obtain from deviating towards $q \in (q^A, q^B)$ occurs when entry does not follow. In such case, the low-cost incumbent's payoff is $M_1^L(q)+\delta \overline{M}_1^L$, which exceeds its equilibrium profits of $M_1^L(q^B)+\delta \overline{M}_1^L$ since $M_1^L(q)+\delta \overline{M}_1^L$ reaches its maximum at $q^{L,Info}$ and $q^{L,Info} \leq q^L < q^B$. Therefore, the low-cost incumbent has incentive to deviate from q^B to $q \in (q^A, q^B)$.

Hence, after observing a first-period output of $q \in (q^A, q^B)$, the entrant concentrates its posterior beliefs on the incumbent's costs being low, i.e., $\mu(c_1^H|q) = 0$, and does not enter. Given these updated beliefs, the low-cost incumbent obtains $M_1^L(q) + \delta \overline{M}_1^L$ from selecting an output q, which exceeds its equilibrium profit from output q^B. Thus, the low-cost incumbent deviates from q^B, and the separating equilibrium in which it selects q^B violates the Intuitive Criterion. A similar argument is applicable for all separating equilibria in which the low-cost incumbent selects $q \in (q^A, q^B]$, concluding that all of them violate the Intuitive Criterion.

Finally, let us check if the separating equilibrium in which the low-cost incumbent chooses q^A survives the Intuitive Criterion. If the low-cost incumbent deviates towards $q \in (q^A, q^B]$ the highest profit that it can obtain is $M_1^L(q) + \delta \overline{M}_1^L$, which is lower than its equilibrium payoff of $M_1^L(q^A) + \delta \overline{M}_1^L$. If instead, it deviates towards $q < q^A$, the highest payoff that it can obtain is $M_1^L(q) + \delta \overline{M}_1^L$, which exceeds its equilibrium profit for all $q \in [q^{L,Info}, q^A)$. Hence, the low-cost incumbent has incentives to deviate. Let us now check if the high-cost incumbent also has incentives to deviate towards $q \in [q^{L,Info}, q^A)$. The highest profit that it can obtain is $M_1^H(q) + \delta \overline{M}_1^H$, which exceeds its equilibrium profit if $M_1^H(q) + \delta \overline{M}_1^H > M_1^H(q^{H,Info}) + \delta D_1^H$. This condition can be rewritten as

$$\delta \left[\overline{M}_1^H - D_1^H \right] > M_1^H(q^{H,Info}) - M_1^H(q)$$

which is satisfied for all $q < q^A$ since all separating equilibria satisfy condition C1. Hence, the high-cost incumbent also has incentives to deviate towards $q \in [q^{L,Info}, q^A)$.

This implies that, after a deviation in $q \in [q^{L,Info}, q^A)$, the entrant cannot update its prior beliefs, and chooses enter if its expected profit from entering satisfies $p \times D_2^H + (1-p) \times D_2^L > 0$ or

$$p \geq \frac{-D_2^L}{D_2^H - D_2^L} \equiv \overline{p}$$

where $\overline{p} \in (0, 1)$ by definition. Hence, if $p \geq \overline{p}$ entry occurs, yielding profits of $M_1^L(q) + \delta D_1^L$ for the low-cost incumbent. Such profits are lower than its equilibrium profits $M_1^L(q^A)+\delta \overline{M}_1^L$. Indeed, from C2 we know that $M_1^L(q^A) + \delta \overline{M}_1^L \geq M_1^L(q^{L,Info}) + \delta D_1^L$. Since, in addition, $q^{L,Info}$ is the argmax of $M_1^L(q) + \delta D_1^L$, then $M_1^L(q^A) + \delta \overline{M}_1^L \geq M_1^L(q) + \delta D_1^L$ for any deviation q. Therefore, the low-cost incumbent does not deviate from q^A. Regarding the high-cost incumbent, it obtains profits of

$M_1^H(q)+\delta D_1^H$ by deviating towards q, which are below its equilibrium profits of $M_1^H(q^{H,Info})+\delta D_1^H$ since $q^{H,Info}$ is the argmax of $M_1^H(q)+\delta D_1^H$. Hence, the high-cost incumbent does not deviate towards q either, and the separating equilibrium survives the Intuitive Criterion for $p > \overline{p}$.

If $p < \overline{p}$, then entry does not occur, yielding profits $M_1^L(q)+\delta\overline{M}_1^L$ for the low-cost incumbent, which exceed its equilibrium profits of $M_1^L(q^A)+\delta\overline{M}_1^L$ since $q \in [q^{L,Info}, q^A)$. Then, the separating equilibrium in which the low-cost incumbent selects q^A violates the Intuitive Criterion if $p < \overline{p}$. ■

4.3 *Proof of Proposition 4*

Case 1. Let us now check if the pooling first-period output $q = q^{L,Info}$ survives the Cho and Kreps' (1987) Intuitive Criterion. Let us first check if such output level is equilibrium dominated for either type of incumbent. On one hand, the low-cost incumbent obtains an equilibrium profit of $M_1^L\left(q^{L,Info}\right)+\delta\overline{M}_1^L$. By deviating towards an off-the-equilibrium output level q' such that $q' \neq q^{L,Info}$ the highest payoff that the low-cost incumbent can obtain occurs when entry is deterred, yielding payoffs of $M_1^L(q')+\delta\overline{M}_1^L$, which lies below its equilibrium profits since $M_1^L(q')+\delta\overline{M}_1^L$ reaches its maximum at exactly $q' = q^{L,Info}$. Hence, the low-cost incumbent does not have incentives to deviate from the pooling output $q = q^{L,Info}$. On the other hand, the high-cost incumbent obtains an equilibrium profit of $M_1^H\left(q^{L,Info}\right)+\delta\overline{M}_1^H$. By deviating towards $q' \neq q^{L,Info}$ the highest payoff that the high-cost incumbent can obtain occurs when entry is deterred, yielding payoffs of $M_1^H(q')+\delta\overline{M}_1^H$. Therefore, the high-cost incumbent does not have incentives to deviate if $M_1^H\left(q^{L,Info}\right)+\delta\overline{M}_1^H \geq M_1^H(q')+\delta\overline{M}_1^H$, which only holds for $q' \in (q^{H,Info}, q^{L,Info})$. Hence, the entrant assigns full probability to the cost being high for every deviation $q' \in (q^{H,Info}, q^{L,Info})$, i.e., $\mu\left(c_1^H|q'\right)=1$, whereas its updated beliefs are unaffected after observing any other deviation. Thus, after observing $q' \in (q^{H,Info}, q^{L,Info})$, the entrant believes that such deviation can only come from a high-cost incumbent and enters. The high-cost incumbent's profits from deviating towards q' are hence $M_1^H(q')+\delta D_1^H$, which are lower than its equilibrium profits if

$$M_1^H\left(q^{L,Info}\right)+\delta\overline{M}_1^H \geq M_1^H(q')+\delta D_1^H. \tag{A.5}$$

Note that deviation profits, $M_1^H(q')+\delta D_1^H$, are maximal at $q' = q^{H,Info}$, yielding profits of $M_1^H\left(q^{H,Info}\right)+\delta D_1^H$. Hence, if $M_1^H\left(q^{L,Info}\right)+\delta\overline{M}_1^H \geq M_1^H\left(q^{H,Info}\right)+\delta D_1^H$, condition A.5 holds for all deviations $q' \in (q^{H,Info}, q^{L,Info})$. Rearranging the last inequality,

$$M_1^H\left(q^{H,Info}\right)-M_1^H\left(q^{L,Info}\right) \leq \delta\left[\overline{M}_1^H - D_1^H\right]$$

which is satisfied since $q^{L,Info} < q^A$. Therefore, the high-cost incumbent does not have incentives to deviate either, and the pooling PBE in which $q = q^{L,Info}$ survives the Intuitive Criterion.

Case 2. Let us next check if the pooling first-period output $q > q^{L,Info}$ survives the Cho and Kreps' (1987) Intuitive Criterion. On one hand, the low-cost incumbent obtains $M_1^L(q)+\delta\overline{M}_1^L$ in equilibrium. By instead deviating towards $q^{L,Info}$, the highest profit that it can obtain occurs when entry is deterred yielding profits of $M_1^L\left(q^{L,Info}\right)+\delta\overline{M}_1^L$, which exceed its equilibrium profits if $M_1^L\left(q^{L,Info}\right)+\delta\overline{M}_1^L \geqslant M_1^L(q)+\delta\overline{M}_1^L$, which is true by concavity since $q > q^{L,Info}$. On the other hand, the high-cost incumbent obtains $M_1^H(q)+\delta\overline{M}_1^H$ in equilibrium. By deviating towards $q^{L,Info}$ the highest profit that it can obtain occurs after no entry, yielding profits of $M_1^H\left(q^{L,Info}\right)+\delta\overline{M}_1^H$, which exceed its equilibrium profits since $M_1^H(q)+\delta\overline{M}_1^H \leqslant M_1^H\left(q^{L,Info}\right)+\delta\overline{M}_1^H$, given that $q^{H,Info} < q^{L,Info} < q$ and concavity. Therefore, both types of incumbent have incentives to deviate towards $q^{L,Info}$ and entrant's beliefs cannot be updated, i.e., $\mu\left(c_1^H|q^{L,Info}\right)=p$ inducing no entry. Given these beliefs, both types of incumbent deviate toward $q^{L,Info}$, obtaining higher profits than in equilibrium. Hence, the pooling strategy profile in which both types select $q > q^{L,Info}$ violates the Intuitive Criterion.

Case 3. Let us finally check if the pooling first-period output $q < q^{L,Info}$ survives the Cho and Kreps' (1987) Intuitive Criterion. Let us first consider the case where $q < q^{H,Info} < q^{L,Info}$. On one hand, the low-cost incumbent obtains $M_1^L(q)+\delta\overline{M}_1^L$ in equilibrium. By instead deviating towards $q' \neq q$, the highest profit it can obtain is $M_1^L(q')+\delta\overline{M}_1^L$, which exceeds its equilibrium profit if $q' \in (q, q^{L,Info}]$ given the concavity of the $M_1^L(q')+\delta\overline{M}_1^L$ function with respect to q'. On the other hand, the high-cost incumbent obtains $M_1^H(q)+\delta\overline{M}_1^H$ in equilibrium. By instead deviating towards $q' \neq q$, the highest profit that it can obtain is $M_1^H(q')+\delta\overline{M}_1^H$, which exceeds its equilibrium profit if $q' \in (q, q^{H,Info}]$. Hence, beliefs can be restricted to $\mu\left(c_1^H|q'\right)=0$ after observing a deviation $q' \in (q^{H,Info}, q^{L,Info}]$. (Otherwise, entrant's beliefs are unaffected, since either both types of incumbent have incentives to deviate or none of them has.) Therefore, after observing a deviation $q' \in (q^{H,Info}, q^{L,Info}]$, the entrant believes that the incumbent's cost must be low, and does not enter. Under these updated beliefs, the low-cost incumbent's profit from deviating exceeds its pooling equilibrium profits. Hence, the low-cost incumbent deviates towards q'. Therefore, the pooling PBE where $q < q^{H,Info} < q^{L,Info}$ violates the Intuitive Criterion.

Let us check if the pooling first-period output q satisfying $q^{H,Info} < q < q^{L,Info}$ survives the Intuitive Criterion. On one hand, the highest payoff that the low-cost incumbent can obtain by deviating towards $q' \neq q$ is $M_1^L(q')+\delta\overline{M}_1^L$, which exceeds its equilibrium profit of $M_1^L(q)+\delta\overline{M}_1^L$ if $q' \in (q, q^{L,Info}]$. On the other hand, the highest payoff that the high-cost firm can obtain by deviating towards $q' \neq q$ is $M_1^H(q')+\delta\overline{M}_1^H$, which exceeds its equilibrium profit of $M_1^H(q)+\delta\overline{M}_1^H$ if $q' \in [q^{H,Info}, q)$. Hence, beliefs can be restricted to $\mu\left(c_1^H|q'\right)=0$ after observing a deviation $q' \in (q, q^{L,Info}]$, since only the low-cost incumbent has incentives to deviate to this range of output, inducing the entrant to not enter. Under these updated beliefs, the low-cost incumbent's profit from deviating exceeds its pooling equilibrium profits. Hence, the low-cost incumbent deviates towards q' and the pooling PBE where $q^{H,Info} < q < q^{L,Info}$ violates the Intuitive Criterion. ■