Legislative Bargaining

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A common scenario for bargaining is between a legislature attempting to allocate surplus through bills, budget agreements, or regulations.

In most political settings, the convention is that once a majority of players concur on an offer, the allocation is agreed upon despite the dismay of those who may oppose it.

Baron and Ferejohn (1989) built upon the Rubinstein-Ståhl model of bargaining to offer a framework for analyzing multilateral bargaining.
Assume that there are an odd number of \( N \) players bargaining over a "pie" of size 1 and one is randomly selected to be the proposer.

- Thus, \( \frac{N+1}{2} \) votes are needed to pass a proposal.
- If a proposal does not pass, the "pie" is discounted by \( \delta \), and another proposer is randomly drawn, and the game repeats.

Baron and Ferejohn (1989) look at two different bargaining techniques, closed-rule bargaining and open-rule bargaining.
In closed-rule bargaining, legislation cannot be modified once it has been proposed.

The proposer seeks to form a coalition with \( \frac{N-1}{2} \) other random voters to guarantee a majority will vote for the proposal.

Let \( v \) be the expected payoff for any player \( i \) to play the subgame-perfect equilibrium of this game at the beginning of any stage, and consider a player \( i \) who is choosing to respond to a proposal that awards him \( x_i \).

- Player \( i \) accepts if \( x_i \geq \delta v \), and thus the proposer will offer him \( x_i = \delta v \).
• The proposer’s payoff, $k$, becomes

\[
k = 1 - \left( \frac{N - 1}{2} \right) \delta v
\]
We can find a value for $v$, the expected payoff received by a responder.

At the beginning of any stage, each player $i$ has probability $\frac{1}{N}$ of becoming the proposer and proposing a payoff of $k$ for themself.

Likewise, they have probability $\frac{N-1}{N}$ of being a responder. In this subgroup, the have a probability of $\frac{1}{2}$ of being part of the coalition and receiving $\delta v$, and probability $\frac{1}{2}$ of being excluded and receiving nothing.

Thus, we can express $v$ as

$$v = \frac{k}{N} + \frac{N - 1}{2N} \delta v$$
Substituting the proposer’s payoff into this equation yields

\[ \nu = \frac{1}{N} \]

which makes sense since this is a symmetric equilibrium.

The proposer’s payoff becomes

\[ K = 1 - \delta \left( \frac{N - 1}{2N} \right) \]

which is decreasing in both \( \delta \) and \( N \).
The intuition behind $\delta$ is simple, since more patient responders will have to be given a larger share in order to vote to accept the proposal.

The intuition of $N$ is less so. With more voters, less must be given to each voter in order to get acceptance, but there are more voters to "buy off." The latter aspect tends to dominate the former, however.

It is interesting to note that while more responders are being bought off, the proposer’s payoff relative to the responders’ actually increases!
$k$ as a function of $N$ when $\delta = 1$
Now let’s look at open-rule bargaining. For simplicity, we are going to assume that $N = 3$.

In this case, once the legislation has been proposed, a second randomly chosen person can choose whether to "second" the legislation and send it to vote, or "amend" the legislation and change the allocations.

If the allocation is amended, the players vote between the original allocation and the new allocation, and whichever one wins becomes the new baseline proposal and is sent to the next round with a new player chosen as the amender.
The proposer can choose to proceed in two ways depending on his attitude toward risk.

- He can try to pay off all of the other voters to achieve *guaranteed success*.
- Or he can try to pay off just a simple majority, hoping that one of them is chosen as the amender, gaining *risky success*. 
For guaranteed success, the proposer will keep $k$ for himself, and offer $\frac{1-k}{2}$ to the other two players.

Let $\nu(k)$ be the equilibrium expected payoff of a player beginning an amendment stage with a previous proposal that offers $k$ to one of the players. The amending player will second the proposal if

$$\frac{1-k}{2} \geq \delta \nu(k)$$

Thus, the proposer will maximize his payoff and set $\frac{1-k}{2} = \delta \nu(k)$.

Furthermore, due to symmetry, each of the potential amenders could follow the same strategy to get the proposal adopted, with $k$ for himself. As such, it must be the case that $\nu(k) = k$.
Thus, in a symmetric subgame-perfect equilibrium,

\[
\frac{1-k}{2} = \delta k \implies k = \frac{1}{1 + 2\delta}
\]

Comparing this solution with its closed-rule counterpart, it’s straightforward to show that the proposer is better off under the closed-rule case. Intuitively, the proposer has to pay off everyone in this case whereas he only had to pay off \( \frac{N-1}{2} \) players in the closed-rule case.
If the proposer is feeling risky, he can attempt to only pay off \( \frac{N-1}{2} \) (1 in the case of \( N = 3 \)) voters in hopes that one of them will become the amender.

In this case, the proposer keeps \( k \) for himself, and offers \( 1 - k \) to one of the other voters.

If the player who was offered \( 1 - k \) is chosen as the amender, he will second the proposal if \( 1 - k \) is sufficiently high. However, if the player who was offered 0 is chosen as the amender, he will surely amend the proposal and offer 0 to the original proposer, \( k \) to himself, and \( 1 - k \) to the other player.
Thus, the proposer’s equilibrium expected payoff is

\[ v(k) = \frac{1}{2} k + \frac{1}{2} \delta v(0) \]

Paid off player chosen as responder \hspace{1cm} Non paid off player chosen as responder

Likewise, the payoff of the player who is offered 0 is

\[ v(0) = \frac{1}{2} \delta v(k) \]

Finally, the third player will accept the proposal without amendment if

\[ 1 - k \geq \delta v(k) \implies 1 - k = \delta v(k) \]
This system of equations can be solved, obtaining

\[ k = \frac{4 - \delta^2}{4 + 2\delta - \delta^2} \]

\[ v(k) = \frac{2}{4 + 2\delta - \delta^2} \]

\[ v(0) = \frac{\delta}{4 + 2\delta - \delta^2} \]
As before, the proposer is better off under the closed-rule bargaining than the risky success open-rule bargaining under the same logic as the guaranteed success case.

For different values of $\delta$, however, risky success could produce a higher share for the proposer than guaranteed success. We can solve for that as

$$\frac{2}{4 + 2\delta - \delta^2} > \frac{1}{1 + 2\delta}$$

$$\delta > \sqrt{3} - 1$$

Intuitively, this makes sense, as more patient individuals require higher payoffs to cooperate, so the proposer will be better off accepting the risk and only paying off a portion of the voters.
Let’s simultaneously depict all three cutoffs we obtained in the previous results.
As can be seen in the figure, the top line (blue) represents the equilibrium share for the closed-rule bargaining (when $N = 3$), where the proposer is able to take a much larger share for himself without the threat of amendment on the table.

In the middle (red) line, we have open-rule bargaining with guaranteed success.

- For low values of $\delta$, the proposer is able to take a large share due to the other voters being very impatient.
- As they become more patient, he is able to take less and less for himself.
Lastly, the bottom (purple) line represents the open-rule bargaining with risky success.

- For low values of $\delta$, the proposer takes half of $k$.
- As the other voters become more patient, he actually has to give who he hopes will be the amender more than he gives himself!
- For values of $\delta \geq \sqrt{3} - 1$, however, it is still a higher payoff than using guaranteed success.