Microeconomic Theory I
Assignment #9 - Due date: December 7th, 2016.

1. [**Private contributions to a public good**] Consider an economy with 2 consumers, Alessandro and Beatrice, $i = \{A, B\}$, one private good x, and one public good G. Let each consumer have an income of M. For simplicity, let the prices of both the public and private good to be 1. In addition, the utility functions of consumer A and B are:

$$U^A = \log(x^A) + \log(G), \quad \text{for individual } A, \text{ and}$$
$$U^B = \log(x^B) + \log(G), \quad \text{for individual } B$$

Assume that the public good G is only provided by the contributions of these two individuals, that is, $G = g^A + g^B$.

(a) Find Alessandro's best response function. Depict it in a figure with his contribution, g^A, on the vertical axis and Beatrice's contribution, g^B, on the horizontal axis.

(b) Identify Beatrice's best response function. Depict it in a figure with her contribution, g^B, on the horizontal axis and Alessandro's contribution, g^A, on the vertical axis.

(c) *Unregulated equilibrium.* Find the equilibrium contributions to the public good by Alessandro and Beatrice, that is, the Nash equilibrium of this public good game.

(d) *Social optimum.* Find the efficient (socially optimal) contribution to the public good by Alessandro and Beatrice.

(e) Use a figure to contrast the Pareto efficient level of private provision and the Nash equilibrium level of provision.

2. [**Production and Externalities**] According to some residents, a firm's production of paper at Lewiston, Idaho, generates a smelly gas as an unpleasant side product. Let $c(y, m; \mathbf{w})$ denote the (minimum) input cost of producing y tons of paper and m cubic meters of gas, where input prices are given by the vector $\mathbf{w} \gg \mathbf{0}$. Let $p > 0$ denote the market price of paper. Assume that the cost function satisfies $\frac{\partial c}{\partial y} > 0$ and $\frac{\partial c}{\partial m} < 0$, and that $c(y, m; \mathbf{w})$ is strictly convex in y and m. Let stars * denote solutions and assume throughout that the firm produces positive amounts of paper $y^* > 0$.

(a) Show that the cost function $c(y, m; \mathbf{w})$ is concave in input prices, $\mathbf{w}$.

(b) *Setting a quota.* Suppose that the government imposes a ceiling on gas emissions such that $m \leq \overline{m}$ (a quota). Assuming that this constraint binds, write down the firm's profit maximization problem with respect to y, and find necessary and sufficient conditions for the firm's cost-minimizing production, y^*.

(c) *Comparative statics.* Under which condition on the cost function $c(y, m; \mathbf{w})$ can we guarantee that an increase in the ceiling on gas emissions, $\overline{m}$, produces a raise in the firm's cost-minimizing production, y^*, whereby $\frac{\partial y^*}{\partial \overline{m}} > 0$?

(d) *Emission fee.* Suppose now that the government abandons its emissions ceiling and replaces it with a tax $t > 0$ on gas emissions. Thus, the new cost of producing (y, m) is given by $c(y, m; \mathbf{w}) + tm$. Show that maximized profits are convex in t, and that the firm's choice of pollution decreases in the pollution tax, i.e., $\frac{\partial m^*}{\partial t} \leq 0$.

3. [**Social planner prefering Cournot or Bertrand competition?**] Consider an industry with n symmetric firms, each facing a constant marginal cost $c > 0$ and inverse demand function $p(Q) = 1 - Q$, where $1 > c$. In addition, firms' production generates a linear environmental externality (damage) measured by $ED(Q) = d \times Q$.

(a) Assuming that firms compete a la Cournot, find their equilibrium individual and aggregate output, the equilibrium profits, the associated consumer surplus and overall social welfare.

(b) Assuming that firms compete a la Bertrand, find their equilibrium individual and aggregate output, the equilibrium profits, the associated consumer surplus and overall social welfare.

(c) Compare the social welfare arising when firms compete a la Cournot (found in part a) and a la Bertrand (found in part b). Under which conditions does the social planner prefer that firms compete a la Cournot? Interpret.