Microeconomic Theory I
Assignment #7 - Due date: November, 18th

1. **[Externalities and car accidents]** Consider an economy with two individuals $i = \{1, 2\}$ with the following quasi-linear utility function

$$u_i(s^i, q^i) = v^i(s^i) + \alpha w^i$$

where s^i denotes the speed at which individual i drives his car, w^i is his wealth, and $\alpha > 0$. The utility that individual i obtains from driving fast is $v^i(s^i)$, which is increasing but concave in speed, whereby $\frac{\partial v^i(s^i)}{\partial s^i} > 0$ and $\frac{\partial^2 v^i(s^i)}{(\partial s^i)^2} < 0$. Driving fast, however, increases the probability of suffering a car accident, represented by $\gamma(s^i, s^j)$. This probability is increasing both in the speed at which individual i drives, s^i, and the speed at which other individuals drive, s^j, where $j \neq i$. Hence, the speed of other individuals imposes a negative externality on driver i, since it increases his risk of suffering a car accident. If individual i suffers an accident, he bears a cost of $c^i > 0$, which intuitively embodies the cost of fixing his car, health-care expenses, etc.

   (a) *Unregulated equilibrium.* Set up individual i's expected utility maximization problem. Take first-order conditions with respect to s^i, and denote the (implicit) solution to this first-order condition as $\widehat{s}^i$.

   (b) *Social optimum.* Set up the social planner's expected welfare maximization problem. Take first-order conditions with respect to s^1 and s^2. Denote the (implicit) solution to this first-order condition as $\overline{s}^i$.

   (c) *Comparison.* Show that drivers have individual incentives to drive too fast, relative to the socially optimal speed, i.e., show that $\widehat{s}^i > \overline{s}^i$.

   (d) *Restoring the social optimum.* Let us now evaluate the effect of speeding tickets (fines) to individuals driving too fast, i.e., to those drivers with a speed $\widehat{s}^i$ satisfying, $\widehat{s}^i > \overline{s}^i$. What is the dollar amount of the fine m^i that induces every individual i to fully internalize the externality he imposes onto others?

   (e) Let us now consider that individuals obtain a utility from driving fast, $v^i(s^i)$, only in the case that no accident occurs. Repeat steps (a)-(c), finding the optimal fine m^i that induces individuals to fully internalize the externality.

   (f) *Parametric example.* Consider $v^i(s^i) = \sqrt{s^i}$ and $\gamma(s^i, s^j) = \beta_i s^i + \beta_j s^j$, $\beta_i = \beta_j = \frac{1}{2}$ and $c^i = c^j = \frac{2}{3}$. Evaluate your results of all previous parts of the exercise assuming these functional forms. Find the equilibrium speed, socially optimal speed, and the optimal fine.

2. **[Flexible and Inflexible Environmental Policy.]** Consider an industry with an incumbent monopolist in period $t = 1$ and a duopoly (i.e., the incumbent and an entrant) in period $t = 2$. For simplicity, assume that both firms face the same constant marginal cost $c > 0$, and a linear inverse demand curve $p(Q) = 1 - Q$, where Q represents aggregate output. Their output generates an environmental externality

given by the convex damage function $ED(Q) = d \cdot Q^2$, where $d > 0$. Assume that the social welfare function that the environmental protection agency (EPA) considers is

$$SW = CS + PS + T - ED$$

where CS (PS) denotes consumer (producer) surplus, respectively, and $T \equiv t \cdot Q$ represents total revenue from emission fees.[1]

(a) *Flexible policy.* Assume that the EPA can easily adjust emission fees between the first and second period. Find the emission fee it sets to the monopolist in period 1, t_1, and to the duopolists in period 2, t_2.

(b) *Inflexible policy.* Assume now that the EPA cannot adjust environmental regulation after industry conditions change. Such inflexibility may be due to the institutional setting requiring that changes in environmental regulation must be approved by Congress. Find the unique emission fee t that the EPA sets across both time periods. [*Hint*: The EPA anticipates that such a policy will generate inefficiencies in one (or both) periods, but seeks to minimize the sum of such inefficiencies. For simplicity, assume no time discounting.]

(c) *Comparison.* Compare the flexible emission fees you found in part (a) with the inflexible fee found in part (b). Interpret.

3. [**Entry in the commons**] Consider a common pool resource initially operated by a single firm during two periods, appropriating x_i units in the first period and q_i units in the second period. In particular, assume that its first-period cost function is $\frac{x_i^2}{\theta}$ where $\theta > 0$, while second-period cost function is $\frac{q_i^2}{\theta-(1-\beta)x_i}$. Intuitively, parameter θ reflects the initial abundance of stock, i.e., a large θ decreases the firms' first and second-period costs; while β denotes the regeneration rate of the resource. Hence, if regeneration is complete, $\beta = 1$, first- and second-period costs coincide. However, if regeneration is null, $\beta = 0$, second period costs become $\frac{q_i^2}{\theta - x_i}$ and thus every unit of first-period appropriation x_i increases the firm's second-period costs. For simplicity, assume that every unit of output is sold at a price of \$1 at the international market.[2]

(a) Assuming no entry during both periods (i.e., the incumbent operates alone in both periods), find the profit-maximizing second-period appropriation, q_i^{NE}, and its first-period appropriation, x_i^{NE}, where superscript NE denotes no entry. [*Hint*: Use backward induction.]

(b) Assume that entry occurs in the second period, and that the second-period cost function for both incumbent and entrant becomes $\frac{(q_i+q_j)q_i}{\theta-(1-\beta)x_i}$. Find the profit-maximizing second-period appropriation, q_i^E and q_j^E, and first-period appropriation, x_i^E, where superscript E denotes entry.

[1] This exercise is based on Espinola-Arredondo et al. (2014), which extend the model to a setting of incomplete information between firms in order to evaluate whether environmental regulation can entail more entry-deterring effects when such policy is flexible or inflexible.

[2] This exercise is based on Espinola-Arredondo and Munoz-Garcia (2013). The exercise, however, focuses on a complete information setting, whereas the article examines how the presence of incomplete information affect equilibrium appropriation, and ultimately welfare levels.