Microeconomic Theory I
Assignment #7 - Answer key

1. [**Menu pricing in monopoly**] Consider the example on second-degree price discrimination (see slides 91-93). To facilitate your calculations, assume $\theta_H = 5$, $\theta_L = 2$, and $c = 1$.

   (a) *Uniform pricing.* Assuming uniform pricing, find the monopolist's optimal price, output, and profits from serving both types of customers.

   (b) Still under a uniform price setting, if the monopolist were to focus on high-demand buyers alone, which is the optimal price, output and profits. For which values of probability γ does the firm prefer to serve both types of buyers? [*Hint*: Find the cutoff for γ, $\overline{\gamma}$, such that when γ satisfies $\gamma < \overline{\gamma}$ expected profits are higher serving both types of buyers.]

   (c) *Two-part tariff.* Consider now that the monopolist sets a two-part tariff (F_H, q_H) and (F_L, q_L). Find the optimal two-part tariff.

   (d) Consider a probability $\gamma = \frac{3}{4}$, which should satisfy the condition you found in part (b), $\gamma < \overline{\gamma}$. Hence, under a uniform price, the firm prefers to serve both types of buyers. Confirm that expected profits are higher when the firm practices a two-part tariff, followed by serving both types of customers under uniform pricing, and followed by serving high-type buyers alone under uniform pricing.

   - Please see answer key for this exercise in the handout on Two-part tariffs in the EconS 503 website (numerical example in pages 10-13).

2. [**Monopolist with intertemporal network effects.**] Consider a direct demand function

$$x(p, w) = \alpha - \beta p + \gamma q$$

where q represents the units of the good purchased in previous periods and $\gamma > 0$ denotes the network effects that exist in this industry. For instance, a larger pool of customers in previous periods makes the good more valuable for current customers, thus producing a rightward shift in the demand function. Network effects arise in industries such as operating systems, game consoles, etc. whereby the larger the population that uses a specific type of device the more useful it becomes for new users who will be able to exchange more files and design more programs. Assume that $\alpha, \beta > 0$. Hence, solving for p, we obtain the indirect utility function

$$p(q) = \frac{\alpha}{\beta} - \frac{1}{\beta}x + \frac{\gamma}{\beta}q$$

For compactness, let us denote $a \equiv \frac{\alpha}{\beta}, b \equiv \frac{1}{\beta}$, and $\lambda \equiv \frac{\gamma}{\beta}$, which reduces the above inverse demand function to the more familiar expression $p(q) = a - bx + \lambda q$, where now λ measures the network effects. Also assume that marginal costs c are constant and $c < a$.

(a) *Second period.* Determine a monopolist's optimal production level, and the resulting prices, if q units were sold in the market during the previous period. Find the monopoly profits as a function of q in the second period. [*Hint*: For simplicity, assume only two time periods.]

- We start by setting up a profit maximization problem. The monopolist's problem is

$$\max_x \quad \pi = p(q) \cdot x - cx = ax - bx^2 + \lambda qx - cx$$

With a first order condition,

$$\frac{\partial \pi}{\partial x} = a - 2bx + \lambda q - c = 0$$

Solving for x, we obtain second-period monopoly output

$$x^*(q) = \frac{(a-c) + \lambda q}{2b}$$

which increases in both q and λ. (Note that when $\lambda = 0$, this output level coincides with that of a standard monopolist without network effects, $\frac{a-c}{2b}$.)

- Plugging $x^*(q)$ into the inverse demand function, we find the second period price

$$p(q) = a - bx + \lambda q = a - b\left(\frac{(a-c) + \lambda q}{2b}\right) + \lambda q = \frac{a + \lambda q + c}{2}$$

And solving for the monopolist's profits,

$$\pi(q) = p(q) \cdot x(q) - cx(q) = \left(\frac{a + \lambda q + c}{2}\right)\left(\frac{(a-c) + \lambda q}{2b}\right) - c\left(\frac{(a-c) + \lambda q}{2b}\right)$$

$$= \frac{(a + \lambda q - c)^2}{4b}$$

(b) *First period.* If the monopolist anticipates that no firm will enter into the industry in future periods, how much does the monopolist produce in the first period, assuming a first-period inverse demand curve $p(q) = a - bx$.

- Setting up a profit maximization problem over two separate periods,

$$\max_q \quad \pi = p_1(q) \cdot q + p_2(q) \cdot x_2(q) - c(q + x_2(q))$$

$$= (a - bq)q + \left(\frac{a + \lambda q + c}{2}\right)\left(\frac{a + \lambda q - c}{2b}\right) - c\left[q + \left(\frac{a + \lambda q - c}{2b}\right)\right]$$

which simplifies to

$$\left(\frac{\lambda^2}{4b} - b\right) q^2 + \left((a-c)\left(1 + \frac{\lambda}{2b}\right)\right) q + \frac{(a-c)^2}{4b}$$

Taking first order conditions with respect to first-period output, q, yields,

$$\frac{\partial \pi}{\partial q} = 2\left(\frac{\lambda^2}{4b} - b\right) q + \left((a-c)\left(1 + \frac{\lambda}{2b}\right)\right) = 0$$

Solving for q, we obtain the monopolist's first-period output

$$q^* = \frac{a-c}{2b-\lambda}$$

- Plugging q^* into the second-period output function $x^*(q)$, we obtain

$$x^*(q) = \frac{(a-c) + \lambda\left(\frac{a-c}{2b-\lambda}\right)}{2b} = \frac{a-c}{2b-\lambda}$$

thus implying that the monopolist seeks to smooth its production decisions in both periods, by producing the same output level, $q^* = x(q^*) = \frac{a-c}{2b-\lambda}$, which is increasing in network effects, λ.

- Similarly as in part (a) of the exercise, note that if network effects are absent, $\lambda = 0$, the output level reduces to $q = \frac{a-c}{2b}$, which also coincides with the standard monopoly output, i.e., the monopolist would produce the same amount of output in both time periods $q^* = x^* = \frac{a-c}{2b}$.

(c) *Social optimum.* Assume that a social planner owned this monopoly. Considering that the social planner maximizes the sum of consumer and producer surplus in both periods, how much would it produce in each period? [*Hint*: Determine $x^{SO}(q)$ first, and then find q^{SO}.]

- *Second period.* Setting up the social planner's second-period maximization problem,

$$\max_x \; \frac{1}{2}\left[(a+\lambda q) - (a + \lambda q - bx)\right] x + \pi(x)$$
$$= \frac{1}{2}bx^2 + (a - bx + \lambda q)x - cx$$

where the first term represents the consumer surplus (which is independent on network effects) and the last two terms measure the monopolist's second-period profits. Importantly, this implies that a marginal increase in first-period output only raises the monopolist's revenues but does not affect consumer surplus. Simplifying,

$$= -\frac{1}{2}bx^2 + (a - c + \lambda q)x$$

Taking first order conditions with respect to x,

$$\frac{\partial S}{\partial x} = -bx + a - c + \lambda q = 0$$

Solving for x, we obtain the second-period socially optimal output as a function of q,

$$x^{SO}(q) = \frac{a - c + \lambda q}{b}$$

which, similarly to the second-period output function of the unregulated monopolist, it increases in first-period network effects (as measured by λq). Substituting into the inverse demand function,

$$p(q) = a - b\left(\frac{a-c+\lambda q}{b}\right) + \lambda q = c$$

which indicates that under a social optimum, the price charged for each unit of output would coincide with its marginal cost.

- *First period.* Setting up the social planner's first-period maximization problem,

$$\max_{q} \quad \frac{1}{2}[a-(a-bq)]q+\pi(q)+\frac{1}{2}\left[(a+\lambda q)-(a+\lambda q-bx^{SO}(q))\right]x^{SO}(q)+\pi(x^{SO}(q))$$

$$= \frac{1}{2}bq^2+(a-bq)q-cq+\frac{1}{2}b\left(\frac{a-c+\lambda q}{b}\right)^2+c\left(\frac{a-c+\lambda q}{b}\right)-c\left(\frac{a-c+\lambda q}{b}\right)$$

which also indicates that first-period consumer surplus is independent on λ. Simplifying, we obtain

$$\max_{q} \quad -\frac{1}{2}bq^2+(a-c)q+\frac{1}{2}b\left(\frac{a-c+\lambda q}{b}\right)^2$$

Taking first order conditions with respect to q,

$$\frac{\partial S}{\partial q} = -bq+a-c+b\left(\frac{a-c+\lambda q}{b}\right)\left(\frac{\lambda}{b}\right) = 0$$

Solving for q, we obtain the first-period social planner output

$$q^{SO} = \frac{a-c}{b-\lambda}$$

and an associated first-period equilibrium price of

$$p(q) = a - b\left(\frac{a-c}{b-\lambda}\right) = \frac{bc-a\lambda}{b-\lambda}$$

- Plugging q^{SO} into the second-period social planner's output function $x^{SO}(q)$, yields an output of

$$x^{SO}(q^{SO}) = \frac{a-c+\lambda\left(\frac{a-c}{b-\lambda}\right)}{b} = \frac{a-c}{b-\lambda}$$

which is increasing in network effects, λ. Hence, similarly as the unregulated monopolist in part (b) of the exercise, the regulator produces the same socially optimal output in both periods, i.e., $q^{SO} = x^{SO}(q^{SO})$, and this output is increasing in network effects. In addition, socially optimal output levels are strictly larger than those selected by the unregulated monopolist, i.e., $\frac{a-c}{b-\lambda} > \frac{a-c}{2b-\lambda}$, for all parameter values.

(d) *Numerical example.* Consider parameter values $a = b = 1$ and $c = 0$. Fidn output q^* and $x(q^*)$ as a function of λ. Find the social optimum q^{SO} and $x^{SO}(q^{SO})$. Compare.

- Evaluating our above results in these parameter values, we obtain the monopolist produces $q^* = x(q^*) = \frac{1}{2-\lambda}$. In contrast, the social planner would select $q^{SO} = x^{SO}(q^{SO}) = \frac{1}{1-\lambda}$. Hence, the socially optimal output is higher than the output that the monopolist would choose for all network effects λ. Figure 1 depicts the first- and second-period output that the unregulated monopolist selects, along with the output that social planner would select.

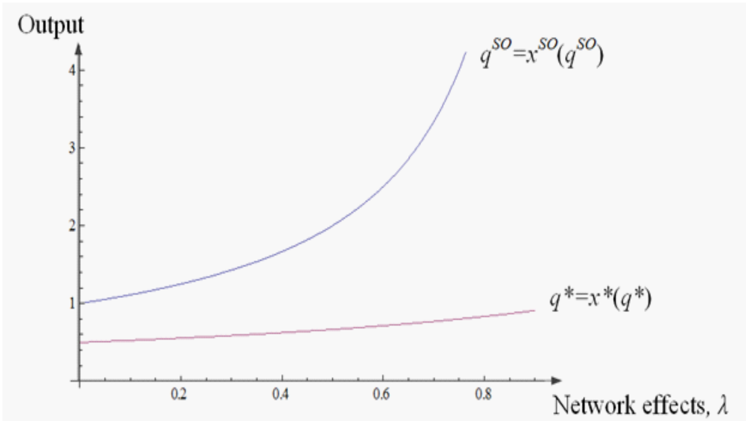

Figure 1. Output decisions under network effects.

3. [**Advertising in monopoly**] Consider a monopolist facing inverse demand curve $p(q) = a - bq$, where $a, b > 0$. The vertical intercept$a$ satisfies $a = \alpha\sqrt{A}$ where $A \geq 0$ represents the monopolist's advertising expenditure, and $\alpha \in [0, 1]$ denotes the sensitivity of demand to one additional dollar of advertising. In addition, assume that the monopolist's production cost is $TC(q) = c_q q$, where $c_q > 0$; and its advertising cost is $TC(A) = c_A\sqrt{A}$, where $c_A > 0$.

(a) Assuming that the monopolist chooses q and A simultaneously, write down the monopolist's profit-maximization problem. Find the optimal output and advertising level.

- Since $a = \alpha\sqrt{A}$, the inverse demand function can be expressed as $p(q) = a - bq = \alpha\sqrt{A} - bq$. In this setting, the monopolist solves

$$\max_{q,A} \left[\alpha\sqrt{A} - bq\right] q - c_q q - c_A\sqrt{A}$$

Taking FOC with respect to q and A, we obtain

$$\begin{aligned} \alpha\sqrt{A} - 2bq - c &= 0, \text{ and} \\ \frac{\alpha q - c_A}{2\sqrt{A}} &= 0 \end{aligned}$$

Rearranging, and solving for q and A, we find

$$q = \frac{2c_A}{4bc_A - \alpha^2}c \text{ and } A = \left(\frac{\alpha c}{\alpha^2 - 4bc_A}\right)^2$$

(b) Assume now that the monopolist chooses q and A sequentially (first A, and then q). Write down the monopolist's profit-maximization problems (one for A, and one of q). Using backward induction, find the optimal output and advertising level.

- *Second stage.* We analyze the last stage, when the monopolist chooses its output q for any given value of advertising A selected in the first stage. The monopolist finds the optimal q, by solving

$$\max_{q} \left[\alpha\sqrt{A} - bq\right] q - c_q q - c_A A^2$$

Taking FOC with respect to q, we obtain

$$\alpha\sqrt{A} - 2bq - c = 0$$

Rearranging, and solving for q, we find the function

$$q(A) = \frac{\alpha\sqrt{A} - c}{2b}.$$

- *First stage.* The monopolist anticipates the optimal value of $q(A)$ that it will choose in the subsequent stage. Hence, the firm can insert $q(A)$ into the profit function of the first stage, as follows

$$\max_{A} \left[\alpha\sqrt{A} - bq(A)\right] q(A) - c_q q(A) - c_A\sqrt{A}$$

thus obtaining a profit function that depends on advertising A alone. Taking FOC with respect to A, and solving for A, we find

$$A^* = \left(\frac{\alpha c}{4bc_A}\right)^2$$

- Hence, the optimal output evaluated at $A^* = \left(\frac{\alpha c}{4bc_A}\right)^2$ is $q(A^*) = \left(\frac{\alpha^2 - 4bc_A}{8b^2 c_A}\right) c$

(c) Compare your results in parts (a) and (b).

- Comparing advertising, note that advertising is larger in the simultaneous game than in its sequential version

$$\left(\frac{\alpha c}{4bc_A - \alpha^2}\right)^2 > \left(\frac{\alpha c}{4bc_A}\right)^2$$

since $\alpha > 0$. Last, because output is increase in A, output is also larger in the simultaneous game than in its sequential version.

4. **[Regulating a natural monopoly]** A water supply company provides water to Pullman. The demand for water in Pullman is $p(q) = 10 - q$, and this company's costs are $c(q) = 1 + 2q$.

(a) Depict the following in a figure: the demand curve $p(q)$, the associated marginal revenue $MR(q)$, the marginal cost of production $MC(q)$ and the average cost of production $AC(q)$. Discuss why this situation illustrates a "natural monopoly."

- Figure 2 depicts the information provided in the exercise. The average cost curve is decreasing in output, implying that multiple producers are more costly than a single monopolist, i.e., the sum of their average costs will be larger than the monopolist's average costs. Then, a monopolist naturally becomes more cost-efficient than having several producers with a similar cost structure as the monopolist.

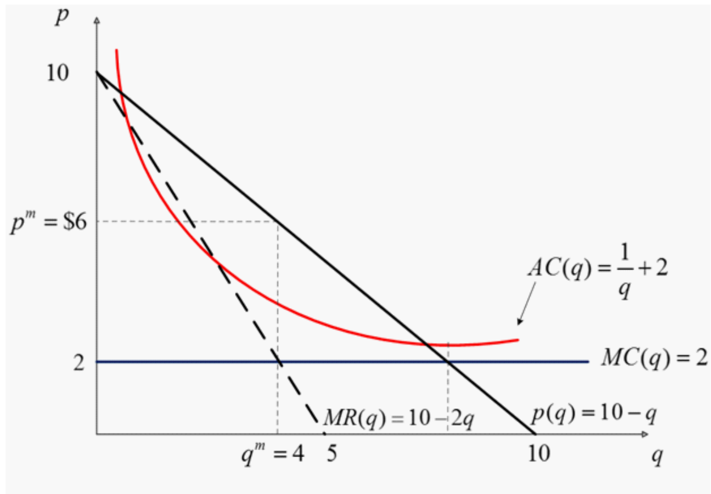

Figure 2. Natural monopoly.

(b) *Unregulated monopolist.* Find the amount of water that this firm will produce if left unregulated as a monopolist. Determine the corresponding prices and profits for the firm.

- The monopolist maximizes

$$\max_q \ (10 - q)q - (1 + 2q)$$

Taking first order conditions with respect to q, we find $10 - 2q^m - 2 = 0$. Solving for q we obtain a monopoly output of $q^m = 4$ units, which are sold at a price of $p^m = 10 - 4 = \$6$, with associated monopoly profits of

$$\pi^m = (10 - 4)4 - (1 + (2 \times 4)) = \$15.$$

(c) *Marginal cost pricing.* Determine the amount of water that this firm will produce if a regulatory agency in Pullman forces the firm to price according to marginal cost (i.e., to produce an amount of output q^* that solves $p(q^*) = MC(q^*)$). Find the corresponding prices and profits for the firm.

- In that case, the monopolist sets $10 - q^* = 2$, i.e., $q^* = 8$ units, at a price $p^* = 10 - 8 = \$2$, with corresponding losses of

$$\pi = (10 - 8)8 - (1 + (2 \times 8)) = -1.$$

This result arises because the presence of decreasing average costs, i.e., in figure 6.5 the production of $q^* = 8$ units yields per unit losses of

$$AC(8) - MC(8) = \left(\frac{1}{8} + 2\right) - 2 = \$0.125.$$

(d) *Price discrimination.* Consider now that the regulatory agency allows the monopoly to charge two different prices: a high price p_1 for the first q_1 units, and a low price $p(q^*)$ for the remaining $(q^* - q_1)$ (i.e., the units from q_1 up to the output level you found in part (c), q^*). In addition, the regulatory agency imposes the condition that the firm cannot make any profits, $\pi = 0$, when charging these two prices.

1. Find the value of q_1 and the associated value of $p(q_1)$.
   - First, note that the value of q_1 must satisfy the "no profits" condition, that is

     $$\pi = [p(q_1) - AC(q_1)]\, q_1 + [p(q^*) - AC(q^*)]\, (q^* - q_1) = 0$$

     and since we know from part (c) that $q^* = 8$ units, and that $p(q) = 10 - q$ and $AC(q) = \frac{1+2q}{q} = \frac{1}{q} + 2$, we can rewrite the above condition as

     $$\pi = \left[(10 - q_1) - \left(\frac{1}{q_1} + 2\right)\right] q_1 + \left[(10 - 8) - \left(\frac{1}{8} + 2\right)\right] (8 - q_1) = 0$$

     which simplifies to

     $$-q^2 + \frac{65}{8}q - 2 = 0$$

     Solving for q_1 we obtain two solutions:
   - *First solution:* $q_1 = 0.82$ with a corresponding price of $p_1 = \$9.18$. This means that the first 0.82 units are sold at \$9.18 each, while the remaining 7.18 (up to 8 units) are sold at a price of \$2.
   - *Second solution:* $q_1 = 0.30$ with a corresponding price of $p_1 = \$9.7$. This means that the first 0.30 units are sold at \$9.7 each, while the remaining 9.7 (up to 8 units) are sold at a price of \$2.
2. Depict these two prices and quantities in a figure and shade the areas of benefits and losses for the firm.
   - Figure 3 depicts out above results: for the first q_1 units, the monopolist makes a profit of $p(q_1) - AC(q_1)$ per unit. For instance, if $q_1 = 0.82$, then the monopolist makes a profit of $(10 - 0.82) - \left(\frac{1}{0.82} + 2\right) = \$5.97$ per unit. In contrast, for the remaining $8 - q_1$ units, the monopolist incurs a loss measured by the distance between the average and marginal cost curves, i.e.,

     $$AC(8) - MC(8) = \left(\frac{1}{8} + 2\right) - 2 = \$0.125$$

per unit.

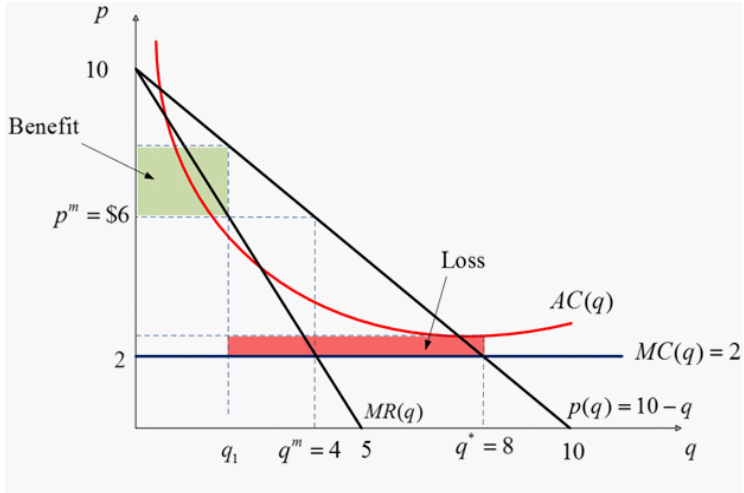

Figure 3. Discriminating natural monopoly.