1. **[Menu pricing in monopoly]** Consider the example on second-degree price discrimination discussed in class (see slides 91-93, Chapter 7). To facilitate your calculations, assume $\theta_H = 5$, $\theta_L = 2$, and $c = 1$.

(a) *Uniform pricing.* Assuming uniform pricing, find the monopolist’s optimal price, output, and profits from serving both types of customers.

(b) Still under a uniform price setting, if the monopolist were to focus on high-demand buyers alone, which is the optimal price, output and profits. For which values of probability $\gamma$ does the firm prefer to serve both types of buyers? [Hint: Find the cutoff for $\gamma$, $\gamma$, such that when $\gamma$ satisfies $\gamma < \gamma$ expected profits are higher serving both types of buyers.]

(c) *Two-part tariff.* Consider now that the monopolist sets a two-part tariff $(F_H, q_H)$ and $(F_L, q_L)$. Find the optimal two-part tariff.

(d) Consider a probability $\gamma = \frac{3}{4}$, which should satisfy the condition you found in part (b), $\gamma < \gamma$. This condition means that, under a uniform price, the firm prefers to serve both types of buyers. Confirm that expected profits are higher when the firm practices a two-part tariff, followed by serving both types of customers under uniform pricing, and followed by serving high-type buyers alone under uniform pricing.

2. **[Monopolist with intertemporal network effects.]** Consider a direct demand function

$$x(p, w) = \alpha - \beta p + \gamma q$$

where $q$ represents the units of the good purchased in previous periods and $\gamma > 0$ denotes the network effects that exist in this industry. For instance, a larger pool of customers in previous periods makes the good more valuable for current customers, thus producing a rightward shift in the demand function. Network effects arise in industries such as operating systems, game consoles, etc. whereby the larger the population that uses a specific type of device the more useful it becomes for new users who will be able to exchange more files and design more programs. Assume that $\alpha, \beta > 0$. Hence, solving for $p$, we obtain the indirect utility function

$$p(q) = \frac{\alpha}{\beta} - \frac{1}{\beta} x + \frac{\gamma}{\beta} q$$

For compactness, let us denote $a \equiv \frac{\alpha}{\beta}$, $b \equiv \frac{1}{\beta}$, and $\lambda \equiv \frac{\gamma}{\beta}$, which reduces the above inverse demand function to the more familiar expression $p(q) = a - bx + \lambda q$, where now $\lambda$ measures the network effects. Also assume that marginal costs $c$ are constant and $c < a$. 

(a) **Second period.** Determine a monopolist’s optimal production level, and the resulting prices, if \( q \) units were sold in the market during the previous period. Find the monopoly profits as a function of \( q \) in the second period. [**Hint:** For simplicity, assume only two time periods.]

(b) **First period.** If the monopolist anticipates that no firm will enter into the industry in future periods, how much does the monopolist produce in the first period, assuming a first-period inverse demand curve \( p(q) = a - bx \).

(c) **Social optimum.** Assume that a social planner owned this monopoly. Considering that the social planner maximizes the sum of consumer and producer surplus in both periods, how much would it produce in each period? [**Hint:** Determine \( x^{SO}(q) \) first, and then find \( q^{SO} \).]

(d) **Numerical example.** Consider parameter values \( a = b = 1 \) and \( c = 0 \). Find output \( q^* \) and \( x(q^*) \) as a function of \( \lambda \). Find the social optimum \( q^{SO} \) and \( x^{SO}(q^{SO}) \). Compare.

### 3. Advertising in monopoly

Consider a monopolist facing inverse demand curve \( p(q) = a - bq \), where \( a, b > 0 \). The vertical intercept satisfies \( a = \alpha \sqrt{A} \) where \( A \geq 0 \) represents the monopolist’s advertising expenditure, and \( \alpha \in [0, 1] \) denotes the sensitivity of demand to one additional dollar of advertising. In addition, assume that the monopolist’s production cost is \( TC(q) = c_q q \), where \( c_q > 0 \); and its advertising cost is \( TC(A) = c_A A^2 \), where \( c_A > 0 \).

(a) **Simultaneous choice.** Assuming that the monopolist chooses \( q \) and \( A \) simultaneously, write down the monopolist’s profit-maximization problem. Find the optimal output and advertising level.

(b) **Sequential choice.** Assume now that the monopolist chooses \( q \) and \( A \) sequentially (first \( A \), and then \( q \)). Write down the monopolist's profit-maximization problems (one for \( A \), and one of \( q \)). Using backward induction, find the optimal output and advertising level.

(c) Compare your results in parts (a) and (b).

### 4. Regulating a natural monopoly

A water supply company provides water to Pullman. The demand for water in Pullman is \( p(q) = 10 - q \), and this company’s costs are \( c(q) = 1 + 2q \).

(a) Depict the following in a figure: the demand curve \( p(q) \), the associated marginal revenue \( MR(q) \), the marginal cost of production \( MC(q) \) and the average cost of production \( AC(q) \). Discuss why this situation illustrates a “natural monopoly.”

(b) **Unregulated monopolist.** Find the amount of water that this firm will produce if left unregulated as a monopolist. Determine the corresponding prices and profits for the firm.

(c) **Marginal cost pricing.** Determine the amount of water that this firm will produce if a regulatory agency in Pullman forces the firm to price according to marginal cost (i.e., to produce an amount of output \( q^* \) that solves \( p(q^*) = MC(q^*) \)). Find the corresponding prices and profits for the firm.
(d) *Price discrimination.* Consider now that the regulatory agency allows the monopoly to charge two different prices: a high price $p_1$ for the first $q_1$ units, and a low price $p(q^*)$ for the remaining $(q^* - q_1)$ (i.e., the units from $q_1$ up to the output level you found in part (c), $q^*$). In addition, the regulatory agency imposes the condition that the firm cannot make any profits, $\pi = 0$, when charging these two prices.

1. Find the value of $q_1$ and the associated value of $p(q_1)$.
2. Depict these two prices and quantities in a figure and shade the areas of benefits and losses for the firm.