Microeconomic Theory I
Assignment #7 - Due date: November, 18th

1. [**Menu pricing in monopoly**] Consider the example on second-degree price discrimination discussed in class (see slides 91-93, Chapter 7). To facilitate your calculations, assume $\theta_H = 5$, $\theta_L = 2$, and $c = 1$.

   (a) *Uniform pricing.* Assuming uniform pricing, find the monopolist's optimal price, output, and profits from serving both types of customers.

   (b) Still under a uniform price setting, if the monopolist were to focus on high-demand buyers alone, which is the optimal price, output and profits. For which values of probability γ does the firm prefer to serve both types of buyers? [*Hint*: Find the cutoff for γ, $\overline{\gamma}$, such that when γ satisfies $\gamma < \overline{\gamma}$ expected profits are higher serving both types of buyers.]

   (c) *Two-part tariff.* Consider now that the monopolist sets a two-part tariff (F_H, q_H) and (F_L, q_L). Find the optimal two-part tariff.

   (d) Consider a probability $\gamma = \frac{3}{4}$, which should satisfy the condition you found in part (b), $\gamma < \overline{\gamma}$. This condition means that, under a uniform price, the firm prefers to serve both types of buyers. Confirm that expected profits are higher when the firm practices a two-part tariff, followed by serving both types of customers under uniform pricing, and followed by serving high-type buyers alone under uniform pricing.

2. [**Monopolist with intertemporal network effects.**] Consider a direct demand function

$$x(p, w) = \alpha - \beta p + \gamma q$$

where q represents the units of the good purchased in previous periods and $\gamma > 0$ denotes the network effects that exist in this industry. For instance, a larger pool of customers in previous periods makes the good more valuable for current customers, thus producing a rightward shift in the demand function. Network effects arise in industries such as operating systems, game consoles, etc. whereby the larger the population that uses a specific type of device the more useful it becomes for new users who will be able to exchange more files and design more programs. Assume that $\alpha, \beta > 0$. Hence, solving for p, we obtain the indirect utility function

$$p(q) = \frac{\alpha}{\beta} - \frac{1}{\beta}x + \frac{\gamma}{\beta}q$$

For compactness, let us denote $a \equiv \frac{\alpha}{\beta}, b \equiv \frac{1}{\beta}$, and $\lambda \equiv \frac{\gamma}{\beta}$, which reduces the above inverse demand function to the more familiar expression $p(q) = a - bx + \lambda q$, where now λ measures the network effects. Also assume that marginal costs c are constant and $c < a$.

(a) *Second period.* Determine a monopolist's optimal production level, and the resulting prices, if q units were sold in the market during the previous period. Find the monopoly profits as a function of q in the second period. [*Hint*: For simplicity, assume only two time periods.]

(b) *First period.* If the monopolist anticipates that no firm will enter into the industry in future periods, how much does the monopolist produce in the first period, assuming a first-period inverse demand curve $p(q) = a - bx$.

(c) *Social optimum.* Assume that a social planner owned this monopoly. Considering that the social planner maximizes the sum of consumer and producer surplus in both periods, how much would it produce in each period? [*Hint*: Determine $x^{SO}(q)$ first, and then find q^{SO}.]

(d) *Numerical example.* Consider parameter values $a = b = 1$ and $c = 0$. Fidn output q^* and $x(q^*)$ as a function of λ. Find the social optimum q^{SO} and $x^{SO}(q^{SO})$. Compare.

3. [**Advertising in monopoly**] Consider a monopolist facing inverse demand curve $p(q) = a - bq$, where $a, b > 0$. Tthe vertical intercept$a$ satisfies $a = \alpha\sqrt{A}$ where $A \geq 0$ represents the monopolist's advertising expenditure, and $\alpha \in [0, 1]$ denotes the sensitivity of demand to one additional dollar of advertising. In addition, assume that the monopolist's production cost is $TC(q) = c_q q$, where $c_q > 0$; and its advertising cost is $TC(A) = c_A A^2$, where $c_A > 0$.

   (a) *Simultaneous choice.* Assuming that the monopolist chooses q and A simultaneously, write down the monopolist's profit-maximization problem. Find the optimal output and advertising level.

   (b) *Sequential choice.* Assume now that the monopolist chooses q and A sequentially (first A, and then q). Write down the monopolist's profit-maximization problems (one for A, and one of q). Using backward induction, find the optimal output and advertising level.

   (c) Compare your results in parts (a) and (b).

4. [**Regulating a natural monopoly**] A water supply company provides water to Pullman. The demand for water in Pullman is $p(q) = 10 - q$, and this company's costs are $c(q) = 1 + 2q$.

   (a) Depict the following in a figure: the demand curve $p(q)$, the associated marginal revenue $MR(q)$, the marginal cost of production $MC(q)$ and the average cost of production $AC(q)$. Discuss why this situation illustrates a "natural monopoly."

   (b) *Unregulated monopolist.* Find the amount of water that this firm will produce if left unregulated as a monopolist. Determine the corresponding prices and profits for the firm.

   (c) *Marginal cost pricing.* Determine the amount of water that this firm will produce if a regulatory agency in Pullman forces the firm to price according to marginal cost (i.e., to produce an amount of output q^* that solves $p(q^*) = MC(q^*)$). Find the corresponding prices and profits for the firm.

(d) *Price discrimination.* Consider now that the regulatory agency allows the monopoly to charge two different prices: a high price p_1 for the first q_1 units, and a low price $p(q^*)$ for the remaining $(q^* - q_1)$ (i.e., the units from q_1 up to the output level you found in part (c), q^*). In addition, the regulatory agency imposes the condition that the firm cannot make any profits, $\pi = 0$, when charging these two prices.

1. Find the value of q_1 and the associated value of $p(q_1)$.
2. Depict these two prices and quantities in a figure and shade the areas of benefits and losses for the firm.