

Micro Theory I
Assignment #5 - Due date: October 28th, 2016.

1. **Exercises from MWG (Chapter 6):**

(a) Exercises 6.B.1 and 6.B.4.

2. **Exercises from Rubinstein:**

(a) Lecture 7 (Expected utility): Exercises 4 and 5.

(b) Lecture 8 (Risk): Exercise 6.

3. **[Investment]** An individual is an expected utility maximizer described by the intertemporally additive preference-scaling function

$$u(c_0) + \beta u(c_1)$$

where $u(\cdot)$ is a strictly concave function with $u''(\cdot)$. The individual has current income I_0 . The individual can buy bonds at unit price $p = \beta$ which pay out in the next period one unit of consumption per unit of bond held.

(a) Compare the individual's demand for bonds in the case where her future income is certain and equal to I_0 , and the situation in which there is a 50% chance that her future income is $I_0 - \varepsilon$ and a 50% chance that her future income is $I_0 + \varepsilon$.

(b) Show that the individual's demand for bonds (i.e., for "saving") is greater when she faces uncertain future income than when she faces certain future income.

4. **[Hyperbolic Absolute Risk Aversion, HARA]** Consider the family of utility functions with Hyperbolic Absolute Risk Aversion (HARA) as follows

$$u(x) = \frac{1}{\beta - 1} (\alpha + \beta x)^{\frac{\beta - 1}{\beta}},$$

where $\beta \neq 0$ and $\beta \neq 1$. Find the Arrow-Pratt coefficient of absolute risk-aversion, $r_A(x, u)$. Describe how it varies in parameters α and β .

5. **[Non-constant coefficient of absolute risk aversion]** Suppose that the utility function is given by

$$u(w) = aw - bw^2,$$

where $a, b > 0$, and $w > 0$ denotes income.

(a) Find the coefficient of absolute risk-aversion, $r_A(w, u)$. Does it increase or decrease in wealth? Interpret.

- (b) Let us now consider that this decision maker is deciding how much to invest in a risky asset. This risky asset is a random variable R , with mean $\bar{R} > 0$ and variance σ_R^2 . Assuming that his initial wealth is W , state the decision maker's expected utility maximization problem, and find first order conditions.
- (c) What is the optimal investment in risky assets?
- (d) Show that the optimal amount of investment in risky assets is a decreasing function in wealth. Interpret.