Microeconomic Theory I
Assignment #2 - Due date: September, 16th, in class.

1. **[Checking WARP].** For each of the following demand functions, check whether they satisfy the weak axiom of revealed preference (WARP).

   (a) "Random demand": For any pair of prices p_1 and p_2 and wealth w, the consumer randomizes uniformly over all points in the budget frontier.

   (b) "Average demand": The expected "random demand" given p_1, p_2 and w.

   (c) "Conspicuous demand": For any p_1, p_2 and w,

$$x_i(p_1, p_2, w) = \begin{cases} \frac{w}{p_i} & \text{if } \frac{w}{p_i} = \min\left\{\frac{w}{p_1}, \frac{w}{p_2}\right\} \text{ and } \frac{w}{p_1} \neq \frac{w}{p_2} \\ \frac{w}{p_i} & \text{if } \frac{w}{p_1} = \frac{w}{p_2} \text{ and } i = 1 \\ 0 & \text{otherwise} \end{cases}$$

2. **[Stone-Geary utility function].** Exercise 4.12 from Nicholson and Snyder.

3. **[UMP, EMP, and Duality].** A consumer has a Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha},$$

   where $\alpha > 0$ and $x_1, x_2 \in \mathbb{R}_+$. Assume that the price vector satisifes $p \equiv (p_1, p_2) >> 0$, and wealth $w > 0$.

   (a) Write the consumer's utility maximization problem. Find Walrasian demands, and the indirect utility function function $v(p_1, p_2, w)$.

   (b) Write the consumer's expenditure minimization problem. Find Hicksian demands, and the expenditure function $e(p_1, p_2, u)$.

   (c) Evaluate the Walrasian demands $x(p_1, p_2, w)$ at $w = e(p_1, p_2, u)$, and show that Walrasian and Hicksian demands coincide, that is,

$$x(p_1, p_2, e(p_1, p_2, u)) = h(p_1, p_2, u).$$

   (d) Evaluate the Hicksian demands $h(p_1, p_2, u)$ at $u = v(p_1, p_2, w)$, and show that Hicksian and Walrasian demands coincide, that is,

$$h(p_1, p_2, v(p_1, p_2, w)) = x(p_1, p_2, u).$$

   (e) Evaluate the indirect utility function $v(p_1, p_2, w)$ at $w = e(p_1, p_2, u)$, and show that

$$v(p_1, p_2, e(p_1, p_2, u)) = u.$$

   (f) Evaluate the expenditure function $e(p_1, p_2, u)$ at $u = v(p_1, p_2, w)$, and show that

$$e(p_1, p_2, v(p_1, p_2, w)) = w.$$

4. **[Choosing between two types of labor.]** Consider an individual who is endowed with one unit of time which can be allocated to leisure (l) or to either of two types of labor. Generally, the person uses its labor income to purchase consumption goods c at the price p. The first type of labor, L_1, pays a lower wage (w_1) but is easier to perform, whereas the second type, L_2, pays a higher wage ($w_2 > w_1$) but is more difficult. The person's preferences are given by the strictly concave utility function

$$u(c, l, L_1, L_2)$$

where the partial derivatives are $u_c > 0$, $u_l > 0$, and $u_{L_i} < 0$ for both types of labor $i = \{1, 2\}$. In terms of these partial derivatives, one way to formalize the notion that "L_1 is easier than L_2" by wrting that, when the individual works the same amount of time in both types of labor, $L_1 = L_2$, he has $u_{L_1} > u_{L_2}$ implying that, since $u_{L_i} < 0$, the marginal disutility from L_2 is larger (i.e., more negative) than that from L_1.

(a) Assume the person can work on either L_1 or L_2 but not both, and that it is only possible to choose $L_i \in \left\{0, \frac{1}{3}\right\}$; that is, labor can only be supplied in the discrete amounts 0 or 1/3. Explain how the individual would choose which job, if either, to perform.

(b) Now assume that labor can be supplied continuously but that again it is not possible to do both jobs. In this case, explain how the individual would choose which job to accept.

(c) Next, assume it is possible to supply both types of labor. Formulate the individual's decision problem and characterize the solution in terms of the appropriate first order conditions.

(d) Prove that in order for the agent to supply positive quantities of both types of labor, it is necessary that L_2 should be more difficult than L_1 according to the definition of "L_1 is easier than L_2" we described at the beginning of the exercise.

(e) Finally, suppose that while L_1 is easier than L_2 (i.e., entails less effort) it is also more boring. As presently formulated, is the above model sufficient to allow these two factors (effort and boring) to be taken into consideration? If so, explain. Otherwise, discuss how the model might be extended to incorporate them.