Microeconomic Theory I
Assignment #1 - Due date: September, 5th, in class.

1. **"Liking more of everything"** Consider a set $X = \mathbb{R}^L$, and define that, for every two bundles $x, y \in X$,

$$x \succsim y, \text{ if and only if } x_k \geq y_k \text{ for every component } k,$$

that is, bundle x is at least as good as bundle y if the former contains more units than the latter in each of its components. Check if this preference relation satisfies (a) completeness, (b) transitivity, (c) strong monotonicity, and (d) strict convexity.

2. **Checking properties of a preference relation.** Consider a consumer with the following preference relation: he weakly prefers (x_1, x_2) to (y_1, y_2), i.e., $(x_1, x_2) \succsim (y_1, y_2)$, if and only if $\max\{x_1, x_2\} \geqslant \min\{y_1, y_2\}$.

   (a) Provide a verbal description of his preference relation.

   (b) Check whether this preference relation is complete, transitive, monotone, convex, and locally nonsatiated.

3. **Strictly Convex Preferences.** Consider preferences defined on the consumption set $X = \mathbb{R}^2_+$.

   (a) Suppose Alex has a utility function $U(x) = (1 + x_1)(1 + x_2)$. Show that his preferences are convex. Are his preferences strictly convex?

   (b) Barbara has a utility function $U(x) = x_1 x_2$. Are her preferences convex or strictly convex?

4. **Quasi-Linear Preference.** Write the $(n + 1)-$dimensional consumption vector x as (y, z) where y is a scalar and z is an $n-$dimensional consumption vector. A utility function $U(x)$ is quasi-linear if it can be written as follows $U(x) = \alpha y + V(z)$. Consider that the consumption set is $X = \mathbb{R}^{n+1}_+$, where $y \in \mathbb{R}_+$ and $z \in \mathbb{R}^n_+$.

   (a) Show that if V is concave, U is quasi-concave.

   (b) Show that if U is quasi-concave, V is concave.

      - *Hint*: Suppose that for some x^0, x^1, x^λ, concavity fails; that is, $V(x^\lambda) < (1 - \lambda)V(x^0) + \lambda V(x^1)$. Choose y^0, y^1 such that $U(x^0) = U(x^1)$ and show that $U(x^\lambda) < U(x^0)$.

5. **Rubinstein.** Problem set 2, exercises 3, 4, 5, and 6.