Comparative Statics

- What happens if...
- the price of one good increases, or if
- the endowment of one input increases?
  - *Reading:* MWG pp. 534-537.

Comparative Statics

- Consider a setting with two goods, each being produced by two factors 1 and 2 under constant returns to scale (CRS).
- Given CRS, a necessary condition for input prices (w_1^*, w_2^*) to be in equilibrium is

$$c_1(w_1, w_2) = p_1 \quad \text{and} \quad c_2(w_1, w_2) = p_2$$

- Let $a_{1j}(w)$ denote firm j's demand for factor 1, and $a_{2j}(w)$ be its demand for factor 2.
  - This is equivalent to the factor demand correspondences $z(w, q)$ in the chapter on production theory where, for simplicity, we consider the production of one unit of output $q = 1$, which helps us ignore the second argument.

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- Hence, we say that the production of good 1 is relatively more intense in factor 1 than is the production of good 2 if

$$\frac{a_{11}(w)}{a_{21}(w)} > \frac{a_{12}(w)}{a_{22}(w)}$$

where $\frac{a_{1j}(w)}{a_{2j}(w)}$ represents firm j's demand for input 1 relative to that of input 2.

Comparative Statics

- *First comparative statics question*:
  - How does a change in the price of one of the outputs, say p_1, affect the equilibrium factor prices and the factor allocations?
- Our answer comes with the **Stolper-Samuelson theorem:**
  - In the 2x2 production model with the factor intensity assumption, if p_j increases:
  - Then the equilibirum price of the factor more intensively used in the production of good j increases,
  - while the price of the other factor decreases.

Comparative Statics

- **Stolper-Samuelson theorem (Proof)**
  - Take the equilibrium conditions

$$c_1(w_1, w_2) = p_1 \quad \text{and} \quad c_2(w_1, w_2) = p_2$$

  - Differentiating them yields

$$\begin{aligned} \frac{\partial c_1(w_1, w_2)}{\partial w_1} dw_1 + \frac{\partial c_1(w_1, w_2)}{\partial w_2} dw_2 &= dp_1, \quad \text{and} \\ \frac{\partial c_2(w_1, w_2)}{\partial w_1} dw_1 + \frac{\partial c_2(w_1, w_2)}{\partial w_2} dw_2 &= dp_2 \end{aligned}$$

Comparative Statics

- **Stolper-Samuelson theorem (Proof)**
  - Applying Shephard's lemma, we obtain

$$\begin{aligned} a_{11}(w)dw_1 + a_{12}(w)dw_2 &= dp_1, \text{ and} \\ a_{21}(w)dw_1 + a_{22}(w)dw_2 &= dp_2 \end{aligned}$$

  - Hence, if only p_1 varies, $dp_2 = 0$. We can rewrite the second condition as

$$dw_1 = -\frac{a_{22}}{a_{21}} dw_2$$

Comparative Statics

- **Stolper-Samuelson theorem (Proof)**
  - We can now use the first condition.
  - Solving for $\frac{dw_1}{dp_1}$ yields

$$\frac{dw_1}{dp_1} = \frac{a_{22}}{a_{11}a_{22} - a_{12}a_{21}}$$

  - Solving, instead, for $\frac{dw_2}{dp_1}$ yields

$$\frac{dw_2}{dp_1} = -\frac{a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

- **Stolper-Samuelson theorem (Proof)**
  - From the intensity of use condition $\frac{a_{11}(w)}{a_{21}(w)} > \frac{a_{12}(w)}{a_{22}(w)}$, we know that $a_{11}a_{22} - a_{12}a_{21} > 0$ (the denominator in the previous expressions is positive).
  - Hence, since the numerator is also positive (they are just factor demands, not derivatives), the overall sign of the previous expressions is
$$\frac{dw_1}{dp_1} > 0 \text{ and } \frac{dw_2}{dp_1} < 0$$
Intuitively, if p_1 increases, the price of input 1, w_1, increases, and that of the other input, w_2, decreases (as required).

Comparative Statics

- *Second comparative statics question*:
  - How does a change in the endowment of one input, say input 1, affect the equilibrium output of each good?
- Our answer comes with the **Rybcszynski theorem:**
  - In the 2x2 production model with the factor intensity assumption, if the endowment of a factor increases...
  - Then the production of the good that uses this facor more intensively increases,
  - while the production of the other good decreases.

Comparative Statics

- **Rybcszynski theorem (Proof):**
  - Consider a economy with two factors, labor and capital, and two goods, 1 and 2.
  - The amount of labor used in the production of goods 1 and 2 is

    $$\overline{L} = L_1 + L_2$$

    while that of capital is

    $$\overline{K} = K_1 + K_2$$

Comparative Statics

- **Rybcszynski theorem (Proof):**
  - We can now divide $\frac{\overline{L}}{\overline{K}}$ to obtain the relative use of factors in this economy,
  $$\frac{\overline{L}}{\overline{K}} = \frac{L_1}{\overline{K}} + \frac{L_2}{\overline{K}}$$
  which is equivalent to
  $$\frac{\overline{L}}{\overline{K}} = \frac{L_1}{K_1}\frac{K_1}{\overline{K}} + \frac{L_2}{K_2}\frac{K_2}{\overline{K}}$$

Comparative Statics

- **Rybcszynski theorem (Proof):**
  - A few things to note on this expression:

$$\frac{\overline{L}}{\overline{K}} = \frac{L_1}{K_1}\frac{K_1}{\overline{K}} + \frac{L_2}{K_2}\frac{K_2}{\overline{K}}$$

  - 1) ratio $\frac{\overline{L}}{\overline{K}}$ increases if the endowment of labor in the economy increases while that of capital remains unaffected.
  - 2) the capital-labor ratio used by firm 1, $\frac{L_1}{K_1}$, and by firm 2, $\frac{L_2}{K_2}$, must remain the same since input prices have not changed (output prices have not changed either).
  - 3) the factor intensity property in this setting can be written as $\frac{L_1}{K_1} > \frac{L_2}{K_2}$.

Comparative Statics

- **Rybczynski theorem (Proof):**
  - The only way to reconcile (1), (2) and (3) is by increasing $\frac{K_1}{K}$ and decreasing $\frac{K_2}{K}$.
  - That is,

$$\frac{\overline{L}}{\overline{K}}\Big\uparrow = \underbrace{\frac{L_1}{K_1}}_{\text{constant}} \frac{K_1}{\overline{K}}\Big\uparrow + \underbrace{\frac{L_2}{K_2}}_{\text{constant}} \frac{K_2}{\overline{K}}\Big\downarrow$$

  - In words, if the production of good 1 uses labor more intensively than good 2, i.e., $\frac{L_1}{K_1} > \frac{L_2}{K_2}$, an increase in the endowment of labor, i.e., an increase in ratio $\frac{\overline{L}}{\overline{K}}$, yields a larger use of the aggregate amount of capital by firm 1 (and a consequent decrease in the use of capital by firm 2).
    - As a result, firm 1 increases its output, while that of firm 2 decreases.