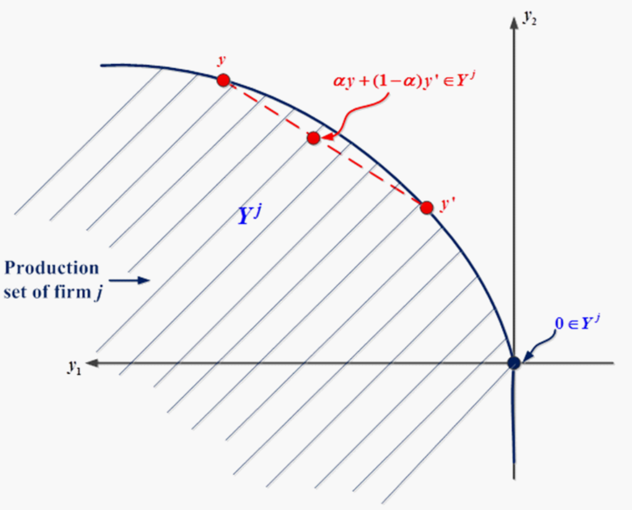
Production

- Assume J firms in the economy, each with production set Y^j satisfying:
  - $\mathbf{0} \in Y^j$
  - Y^j is closed and bounded.
  - Y^j is strictly convex.
- Figure about a production plan satisfying these assumptions.

Production



y_2
y
$\alpha y + (1-\alpha)y' \in Y^j$
y'
Y^j
Production set of firm j
$0 \in Y^j$
y_1

Production

- In a perfectly competitive market, every firm j faces a fixed price vector $\mathbf{p} >> \mathbf{0}$ and independently solves the PMP

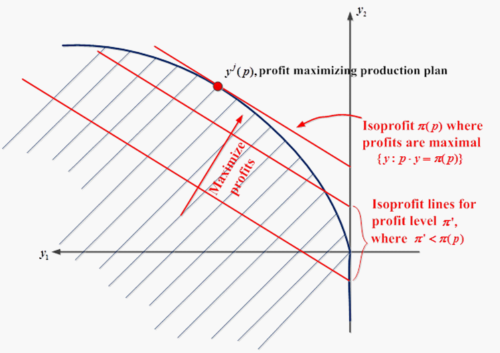
$$\max_{y^j \in Y^j} \mathbf{p} \cdot y_j$$

- Given our above assumptions, a profit-maximizing production plan $y^j(\mathbf{p})$ exists for every firm j, and it is unique.
    - In addition, by the theorem of the maximum (see Simon and Blume), both the argmax, $y^j(\mathbf{p})$, and the value function, $\pi^j(\mathbf{p}) \equiv \mathbf{p} \cdot y^j(\mathbf{p})$, are continuous in p.

Production

Since profits are $\pi^0 = p_2 y_2 - p_1 y_1$, then solving for y_2 yields

$$y_2 = \underbrace{\frac{\pi^0}{p_2}}_{\text{vertical intercept of isoprofit lines}} + \underbrace{\frac{p_1}{p_2}}_{\text{slope of isoprofit lines}} \cdot y_1$$



Production

- Aggregate production set

$$Y = \left\{ \mathbf{y} \mid \mathbf{y} = \sum_{j=1}^{J} y^j \text{ where } y^j \in Y^j \right\}$$

- The production plan $\mathbf{y} \in Y$ that maximizes aggregate profits can be decomposed into the production plans $y^j(\mathbf{p})$ that maximize the individual profits of every firm j.
  - (We already showed this result at the end of the chapter on Production theory. If you cannot find it there, see pages 222-223 in JR.)

Production

- Consumers own shares of firms, which affects every consumer i's budget constraint as follows

$$\mathbf{p} \cdot \mathbf{x}^i \leq \mathbf{p} \cdot \mathbf{e}^i + \underbrace{\sum_{j=1}^{J} \theta^{ij} \pi^j(\mathbf{p})}_{\text{New term}}$$

where θ^{ij} denotes consumer i's share in firm j, where

- $0 \leq \theta^{ij} \leq 1$ and,
- for a given firm j, $\sum_{i=1}^{I} \theta^{ij} = 1$ when we sum across all I consumers.

Production

- We can more compactly express the budget constraint as

$$\mathbf{p} \cdot \mathbf{x}^i \leq \underbrace{\mathbf{p} \cdot \mathbf{e}^i + \sum_{j=1}^{J} \theta^{ij} \pi^j(\mathbf{p})}_{m^i(\mathbf{p})} \implies \mathbf{p} \cdot \mathbf{x}^i \leq m^i(\mathbf{p})$$

  where $m^i(\mathbf{p})$ denotes all the resources of individual i, either originating from the market value of his initial endowment or the profits he makes in the firms he owns.

- Positive profits: Our previous assumptions on the production sets Y^j imply that the profit-maximizing plans entail a positive profit. As a consequence, $m^i(\mathbf{p}) > 0$.

Equilibrium with production

- Let us first define the excess demand function for good k

$$z_k(\mathbf{p}) \equiv \sum_{i=1}^{I} x_k^i\left(\mathbf{p}, m^i(\mathbf{p})\right) - \sum_{i=1}^{I} e_k^i - \underbrace{\sum_{j=1}^{J} y_k^j(\mathbf{p})}_{\text{New}}$$

  where $\sum_{j=1}^{J} y_k^j(\mathbf{p})$ is a new term we didn't include in our analysis of general equilibrium without production.

- Hence, the aggregate demand vector is

$$\mathbf{z}(\mathbf{p}) \equiv \left(z_1(\mathbf{p}), z_2(\mathbf{p}), ..., z_n(\mathbf{p})\right)$$

Equilibrium with production

- We are now ready to use $\mathbf{z}(\mathbf{p})$ in order to find a WEA with production.
- **Existence of WEA with production:**
  - Under the previous assumptions on consumers and producers, and $\sum_{i=1}^{I} \mathbf{e}^i >> 0$, then there exists a price vector $\mathbf{p}^* >> 0$ such that $\mathbf{z}(\mathbf{p}) = \mathbf{0}$.

Equilibrium with production

- **Parametric example:**
- Worked-out example of how to solve General Equilibrium problems with production:
  - Example 5.2 in JR (pages 226-231) is strongly recommended.

Equilibrium with production - Welfare

- In this subsection we seek to extend the First and Second Welfare Theorems to economies with production.
- Let us first define a **WEA in a production economy**:
    - If $\mathbf{p}^* >> 0$, a pair $(\mathbf{x}(\mathbf{p}^*), \mathbf{y}(\mathbf{p}^*))$ is a WEA if:
        - 1) each consumer i solves his UMP with the ith entry of $\mathbf{x}(\mathbf{p}^*)$, i.e., $\mathbf{x}^i(\mathbf{p}^*, m^i(\mathbf{p}^*))$.
        - 2) each firm j solves its PMP with the jth entry of $\mathbf{y}(\mathbf{p}^*)$, i.e., $\mathbf{y}^j(\mathbf{p}^*)$.
        - 3) demand equals supply

$$\sum_{i=1}^{I} \mathbf{x}^i \left(\mathbf{p}^*, m^i(\mathbf{p}^*) \right) = \sum_{i=1}^{I} \mathbf{e}^i + \sum_{j=1}^{J} \mathbf{y}^j(\mathbf{p}^*)$$

which states the market clearing condition (or feasibility when expressed for any $\mathbf{p}$).

Equilibrium with production - Welfare

- Before stating the First Welfare Theorem, let us define what do we mean by a Pareto-efficient allocation in economies with production:
  - The feasible allocation $(\mathbf{x}, \mathbf{y})$ is Pareto efficient if there is no other feasible allocation $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ such that

$$u^i(\bar{\mathbf{x}}^i) \geq u^i(\mathbf{x}^i)$$

    for every consumer $i \in I$, with $u^i(\bar{\mathbf{x}}^i) > u^i(\mathbf{x}^i)$ for at least one consumer.
- That is, a feasible allocation of bundles to consumers and production plans to firms is Pareto efficient if there is no other feasible allocation that makes at least one consumer strictly better off and no consumer worse off.

Equilibrium with production - Welfare

- We can now state the First Welfare Theorem in economies with production.
- **First Welfare Theorem with production:**
  - If each utility function u^i is strictly increasing, then every WEA is Pareto efficient.
  - (Proof in the next slides.)

Equilibrium with production - Welfare

- **First Welfare Theorem with production:**
  - **Proof:** By contradiction, suppose that $(\mathbf{x}, \mathbf{y})$ is a WEA at prices $\mathbf{p}^*$, but is not Pareto efficient.
  - Because $(\mathbf{x}, \mathbf{y})$ is a WEA, then it must be feasible:

$$\sum_{i=1}^{I} \mathbf{x}^i = \sum_{i=1}^{I} \mathbf{e}^i + \sum_{j=1}^{J} \mathbf{y}^j \qquad \text{(A)}$$

  - In addition, because $(\mathbf{x}, \mathbf{y})$ is not Pareto efficient, there exists some other feasible allocation $(\widehat{\mathbf{x}}, \widehat{\mathbf{y}})$ such that $u^i(\widehat{\mathbf{x}}^i) \geq u^i(\mathbf{x}^i)$ for every consumer $i \in I$, with $u^i(\widehat{\mathbf{x}}^i) > u^i(\mathbf{x}^i)$ for at least one consumer.
  - But this implies that $\overline{\mathbf{x}}^i$ is more costly than $\mathbf{x}^i$, i.e., $\mathbf{p}^* \cdot \widehat{\mathbf{x}}^i \geq \mathbf{p}^* \cdot \mathbf{x}^i$ for all i (with at least one strict inequality).

Equilibrium with production - Welfare

- **First Welfare Theorem with production:**
  - **Proof (cont'd):** Summing over all consumers yields

$$\mathbf{p}^* \cdot \sum_{i=1}^{I} \widehat{\mathbf{x}}^i > \mathbf{p}^* \cdot \sum_{i=1}^{I} \mathbf{x}^i \qquad \text{(B)}$$

  - Combining A and B with the feasibility of $(\widehat{\mathbf{x}}, \widehat{\mathbf{y}})$ yields

$$\mathbf{p}^* \cdot \left(\sum_{i=1}^{I} \mathbf{e}^i + \sum_{j=1}^{J} \widehat{\mathbf{y}}^j \right) > \mathbf{p}^* \cdot \left(\sum_{i=1}^{I} \mathbf{e}^i + \sum_{j=1}^{J} \mathbf{y}^j \right)$$

or rearranging

$$\mathbf{p}^* \cdot \sum_{j=1}^{J} \widehat{\mathbf{y}}^j > \mathbf{p}^* \cdot \sum_{j=1}^{J} \mathbf{y}^j$$

  - However, this result implies that $\mathbf{p}^* \cdot \widehat{\mathbf{y}}^j > \mathbf{p}^* \cdot \mathbf{y}^j$ for some firm j, thus indicating that production plan $\mathbf{y}^j$ was not profit-maximizing and, as a consequence, it cannot be part of a WEA.

Equilibrium with production - Welfare

- **Second Welfare Theorem with production:**
  - Consider the assumptions on consumers and producers described above.
  - Then, for every Pareto efficient allocation $(\widehat{\mathbf{x}}, \widehat{\mathbf{y}})$ there are: (1) income transfers $(T_1, T_2, ..., T_I)$ satisfying $\sum_{i=1}^{I} T_i = 0$, and (2) a price vector $\overline{\mathbf{p}}$ such that:
    - a) $\widehat{\mathbf{x}}^i$ solves the UMP

      $$\max_{\mathbf{x}^i} u^i(\mathbf{x}^i) \text{ subject to } \overline{\mathbf{p}} \cdot \mathbf{x}^i \leq m^i(\overline{\mathbf{p}}) + T_i \text{ for every } i \in I$$

    - b) $\widehat{\mathbf{y}}^j$ solves the PMP

      $$\max_{\mathbf{y}^j} \overline{\mathbf{p}} \cdot \mathbf{y}^j \text{ subject to } \mathbf{y}^j \in Y^j \text{ for every firm } j \in J$$

    - Proof (see pages 234-236 in JR)

Contingent commodities

- We could extend our model to consider time-dependent commodities (bread exchanged today, or bread exchanged tomorrow), or
- State-dependent commodities (you provide me with an umbrella if it rains, but no umbrella if it doesn't).
- Analysis is analog to that we just described, but using one more subscript for either time or state.
- Short section in JR (Section 5.4, pages 236-239).
    - On your own.