ECONS 301 – INTERMEDIATE MICROECONOMICS

Final exam – *Answer key*

Exercise #1 – Capturing surplus using a block pricing

Consider a monopolist facing a linear demand $p(q) = 20 - q$, and marginal cost $MC = 2$.

a) If the monopolist charges the same price for all units purchased (standard monopoly pricing), which is its optimal output and price? Which are its profits?
b) Assume now that the monopolist uses block pricing, i.e., charges a price p_1 for the first q_1 units sold, and a price p_2 for the remaining $(q_2 - q_1)$ units sold. Set up the monopolist profit-maximizing problem.
c) Differentiate with respect to q_1 and q_2, and find the optimal values of q_1 and q_2.
d) Substitute the values of q_1 and q_2 you found in part (c) into the demand function to obtain the optimal prices p_1 and p_2.
e) Find the profits that the monopolist makes under block pricing you identified in parts (c)-(d), and compare them against the profits you found in part (a) when the monopolist charges the same price for all units sold.

Answer:

a) Setting $MR = MC$ we obtain

$$20 - 2q = 2$$

which, rearranging, yields

$$18 = 2q \rightarrow q = \frac{18}{2} = 9 \text{ units}$$

Hence, monopoly price is

$$p = 20 - 9 = \$11$$

And monopoly profits are

$$\pi^{MONOP} = p \cdot q - MC \cdot q = 11 \cdot 9 - 2 \cdot 9 = \$81$$

b) The monopoly maximizes total profits from both segments

$$\max_{Q_1,Q_2} \pi = (20 - q_1)q_1 + (20 - q_2)(q_2 - q_1) - 2q_2$$

where $(20 - q_1)q_1$ represents the revenue from the first q_1 units, $(20 - q_2)(q_2 - q_1)$ is the revenue from the remaining $(q_2 - q_1)$ units, and $2q_2$ is the total cost (since the firm produces a total of q_2 units). The above profit simplifies to

$$= 20q_1 - q_1^2 + 20q_2 - 20q_1 - q_2^2 + q_2q_1 - 2q_2$$

c) Differentiating the above profit with respect to q_1 and q_2, we obtain

$$\frac{\partial \pi}{\partial q_1} = 20 - 2q_1 - 20 + q_1 = 0 \leftrightarrow q_2 = 2q_1$$
$$\frac{\partial \pi}{\partial q_2} = 20 - 2q_2 + q_1 - 2 = 0 \leftrightarrow 18 - 2q_2 + q_1 = 0$$

Substituting expression (1), i.e., $q_2 = 2q_1$, into (2), yields
$$18 - 2(2q_1) + q_1 = 0$$

which simplifies to
$$18 - 3q_1 = 0 \rightarrow q_1 = \frac{18}{3} = 6 \text{ units}$$

Therefore, $q_2 = 2 \cdot q_1 = 2 \cdot 6 = 12$ units. Hence, the first segment is 6 units, and the second segment is $q_2 - q_1 = 12 - 6 = 6$ units as well.

d) We can find prices for each segment as follows
$$p_1 = 20 - q_1 = 20 - 6 = \$14 \text{ per unit for the first six units}$$
and

$$p_2 = 20 - q_2 = 20 - 12 = \$8 \text{ per unit for the six units in the second segment.}$$

Intuitively, the consumer pays $14 per unit for the first $q_1 = 6$ units he purchases, but only $8 per unit for every unit after that.

e) Profits from block pricing (BP) are
$$\pi^{BP} = p_1 q_1 + p_2(q_2 - q_1) - 2q_2$$
$$= \$14 \cdot 6 + \$8 \cdot (12 - 6) - 2 \cdot 12$$
$$= \$108$$
which are larger than from charging the same price for all units purchased, as found in part (a) where profits are only $\pi = \$81$.

Exercise #2 – Battle of the sexes game

Consider the following game, describing the so-called "Battle of the sexes" game: A husband and wife must simultaneously and independently choose whether to go to the opera or the football game. (They talked about going to either place last night, but none of them can remember which place they agreed to attend. To make matters worse, there has been a major phone and internet breakdown and cannot communicate before heading to either event.) The husband would prefer that both attend the football game, the wife prefers that both attend to the opera, but both would be better off attending each other's preferred event rather than being alone at their most preferred event. For instance, the husband obtains a payoff of 1 when attending the opera with his wife, but zero if he is at the football game, or at the opera, alone.

	Wife		
Husband		Football	Opera
	Football	4, 2	0, 0
	Opera	0, 0	1, 5

a) Show that there are no strictly dominated strategies for the husband or the wife.
b) Show that there are two NE in pure strategies, and that your equilibrium results resemble those in coordination games.
c) Find the mixed strategy Nash equilibrium of the game, where the Husband goes to the football game with probability p and the Wife with probability q.

Answer:

a) *Husband.* First, consider the husband (row player). If his wife chooses football (in the left column), his payoff is higher choosing football (4) than opera (0). In contrast, if his wife chooses opera (right column), his payoff is higher choosing opera (1) than football (0). That is, the husband does not obtain a higher payoff with one of his strategies regardless of the strategy selected by his wife, which is what we need to identify a dominant strategy. Similarly, there is no strategy providing him with an unambiguously lower payoff regardless of the strategy chosen by his wife, what we need to find a dominated strategy. Therefore, the husband does not have dominant nor dominated strategies.
Wife. A similar argument applies to the wife (column player). If her husband chooses football (in the top row), her payoff is higher choosing football (2) than opera (0). In contrast, if her husband chooses opera (bottom row), her payoff is higher selecting opera (5) than football (0). That is, the wife does not obtain a higher payoff with one of her strategies regardless of the strategy selected by her husband, which is what we need to identify a dominant strategy. Similarly, there is no strategy providing her with an unambiguously lower payoff regardless of the strategy chosen by her husband, what we need to find a dominated strategy. Therefore, the wife does not have dominant nor dominated strategies.

b) The husband's best responses are

$$BR_{Husband}(F) = F \quad \text{and} \quad BR_{Husband}(O) = O$$

In words, when his wife goes to the football game, his best response is to go to the football game as well, whereas when she goes to the opera his best response is to go to the opera as well.
A similar best response applies to the wife

$$BR_{Wife}(F) = F \quad \text{and} \quad BR_{Wife}(O) = O$$

In words, when her husband goes to the football game, her best response is to go to the football game as well, whereas when he goes to the opera her best response is to go to the opera as well.
Thus our Nash Equilibria in this case are $(Football, Football)$ or $(Opera, Opera)$.
Since the best responses of every player dictate that they choose the same strategy as the opponent, this is a coordination game.

c) To find the mixed strategy, we must first find expected payoffs for each player. We start with the husband

$$EU_H(Football) = 4q + \ 0(1-q)$$
$$EU_H(Opera) = 0q + 1(1-q)$$

Since the husband is playing a mixed strategy (that is, randomizing between football and opera) it must be that his expected payoffs coincide. Otherwise, he would choose the strategy producing the highest expected payoff 100% of the time (thus playing pure strategies). Therefore, for the husband to play mixed strategies his expected payoff from football and opera must coincide,.

$$EU_H(Football) = EU_H(Opera)$$
$$4q + 0(1-q) = 0q + 1(1-q)$$

Rearranging, and solving for probability q, we obtain

$$4q = 1 - 1q$$
$$q = \frac{1}{5}$$

Thus, when $q = \frac{1}{5}$, the husband is indifferent between going to the football game and the opera.

Next, we look at the wife's choices

$$EU_W(Football) = 2p + 0(1-p)$$
$$EU_W(Opera) = 0p + 5(1-p)$$

Similarly as for the husband, if the wife is mixing (randomizing) between football and opera, it must be that her expected payoffs coincide; otherwise she would choose the pure strategy with the highest expected payoff. As a consequence, we must have that

$$EU_W(Football) = EU_W(Opera)$$
$$2p + 0(1-p) = 0p + 5(1-p)$$

Rearranging, and solving for probability p, we obtain

$$2p = 5 - 5p$$
$$p = \frac{5}{7}$$

Hence when $p = \frac{5}{7}$, the wife is indifferent between going to the football game or the opera

Based on what we found above, our mixed strategy Nash Equilibrium is

$$\left(\frac{5}{7}Football + \frac{2}{7}Opera, \frac{1}{5}Football + \frac{4}{5}Opera\right)$$

Exercise #3 – Choice under uncertainty

Consider an individual with utility function $u(x) = 5\sqrt{x}$, where $x \geq 0$ denotes money. He is considering to invest in two options: Option A provides him with a certain payoff of \$440 at the end of the month, while option B generates a payoff of \$800 with probability 0.4 or 200 with probability 0.6.

a) Find the expected value of each option.
b) Find the variance and standard deviation of each option.
c) Find the expected utility of option A, and option B. Which generates the highest expected utility?
d) Find the risk premium of option B. That is, by how much can the certain payoff of option A be reduced, and still induce the individual to choose option A?

Answer:

a) We define the expected value as the sum of the probabilities times their payoff. Since we only have two payoff options, our EV is defined as $EV = (Prob_A * Payoff_A) + (Prob_B * Payoff_B)$. We start by finding the expected value for option A

$$EV_A = 1 * 440 = 440$$

and for option B

$$EV_B = (.4 * 800) + (.6 * 200)$$
$$EV_B = 320 + 120 = 440$$

Thus, the expected value for both A and B are the same.

b) We define the variance as $Var = Probability * (Payoff - EV)^2$. Thus our variance for option A is

$$Var_A = P_A * (Payoff_A - EV)^2$$
$$Var_A = 1 * (440 - 440)^2 = 0$$

and for option B

$$Var_B = P_A * (Payoff_A - EV)^2 + P_B * (Payoff_B - EV)^2$$
$$Var_B = .4(800 - 440)^2 + .6(200 + 440)^2$$
$$Var_B = 51{,}840 + 34{,}560 = 86{,}400$$

The standard deviation is simply the square root of the variance. In this example, our standard deviations are

$$Std.Dev_A = \sqrt{0} = 0$$
$$Std.Dev_B = \sqrt{86{,}400} = 293.99$$

c) First, we find the expected utility for option A. In this setting,

$$EU_A = 5(440)^{\frac{1}{2}} = 104.881$$

Next, we find the expected utility for option B.

$$EU_B = .4\left(5(800)^{\frac{1}{2}}\right) + .6\left(5(200)^{\frac{1}{2}}\right)$$
$$EU_B = .4(141.42) + .6(70.71) = 98.994$$

Since, our expected utility is higher with option A, the consumer will choose option A.

d) To find our risk premium, we must find the point when $EU_A = EU_B$. In this setting,

$$.4\left(5(800)^{\frac{1}{2}}\right) + .6\left(5(200)^{\frac{1}{2}}\right) = 5(440 - RP)^{\frac{1}{2}}$$
$$98.994 = 5(440 - RP)^{\frac{1}{2}}$$
$$19.7988 = (440 - RP)^{\frac{1}{2}}$$
$$391.99248 = 440 - RP$$

RP = 48.01

Thus, we would need to give the consumer $48.01 to entice him to play the lottery.

Exercise #4 – Price competition with product differentiation.
Consider the following Bertrand game involving two firms (Coke and Pepsi) producing differentiated products. Firms have no costs of production. The demand for firm 1's product (Coke) is

$$q_1 = 1 - p_1 + 0.5p_2$$

Firm 2 (Pepsi) faces a demand

$$q_2 = 1 - p_2 + 0.3p_1$$

a. Find the best response function of Coke, i.e., the optimal price p_1 that Coke charges for every price charged by Pepsi, p_2.
b. Find the best response function of Pepsi, i.e., the optimal price p_2 that Pepsi charges for every price charged by Coke, p_1.
c. Solve for the Nash equilibrium of the simultaneous price-choice game, i.e., the price pair for which both best response functions cross each other.
d. Compute the firms' outputs and profits at the equilibrium prices you found in part (c).

***Answer*:**

A. Coke solves the following profit-maximizing problem

$$\max_{p_1} p_1(1 - p_1 + 0.5p_2)p_1$$

Differentiating with respect to p_1 yields

$$1 - 2p_1 + \frac{1}{2}p_2 = 0$$

Solving for p_1, we obtain

$$1 + \frac{1}{2}p_2 = 2p_1 \Leftrightarrow \frac{1}{2} + \frac{1}{4}p_2 = p_1$$

Hence, Coke's BRF originates at $\frac{1}{2}$, and increases in Pepsi's price, p_2, at a rate of $\frac{1}{4}$.

B. Pepsi solves

$$\max_{p_2} p_2 \cdot q_2 = p_2 \cdot (1 - p_2 + 0.3p_1)$$

Differentiating with respect to p_2 yields

$$1 - 2p_2 + 0.3p_1 = 0$$

Solving for p_2, we obtain

$$1 + 0.3p_1 = 2p_2 \Leftrightarrow \frac{1}{2} + 0.15p_1 = p_2$$

Therefore, Pepsi's BRF originates at $\frac{1}{2}$, and increases in Coke's price, p_1, at a rate of 0.15.

C. Plugging Coke's BRF into Pepsi's, we find

$$\frac{1}{2} + 0.15\underbrace{\left(\frac{1}{2} + \frac{1}{4}p_2\right)}_{p_1} = p_2$$

Solving for p_2, yields a price for Coke of

$$p_2^* = 0.59$$

which entails a price for Pepsi of

$$p_1^* = \frac{1}{2} + \frac{1}{4}p_2^* = \frac{1}{2} + \frac{1}{4}(0.59) = 0.649$$

D. Equilibrium output for Coke is

$$q_1^* = 1 - p_1^* + \frac{1}{2}p_2^* = 1 - 0.649 + \frac{1}{2}0.59 = 0.64$$

And for Pepsi is

$$q_2^* = 1 - p_2^* + 0.3p_1^* = 1 - 0.59 + (0.3 \times 0.649) = 0.6$$

Entailing that equilibrium profits for Coke are

$$\pi_1^* = p_1^* q_1^* = 0.649 \times 0.64 = 0.421$$

And profits for Pepsi are

$$\pi_2^* = p_2^* q_2^* = 0.59 \times 0.6 = 0.354.$$

Exercise #5 - Externalities and regulation

Consider an industry facing inverse demand function $p = 20 - Q$, and marginal cost of every firm is $MC = 7 + Q$. The industry produces an externality per unit of output (e.g., CO2 emissions). The negative effect of such externality are measured by the marginal external cost $MEC = 1 + 2Q$.

a. Find the output in the unregulated equilibrium (also referred to as "competitive equilibrium"). Which is the equilibrium price that emerges in this context?
b. Find the social optimal output, and compare your answer to that of part (a). Which is the price that emerges in this context?
c. Find the optimal tax or subsidy that induces firms to voluntarily produce the social optimum you found in part (b).

Answer:

a) The unregulated equilibrium occurs at the output level for which demand and supply (MC) equal each other,

$$20 - Q = 7 + Q$$

rearranging,

$$13 = 2Q$$

which, solving for Q, yields $Q^* = 6.5$ units

Hence, the price at the unregulated equilibrium is $p^* = 20 - \frac{13}{2} = \$13.5$

b) The socially optimal output requires

$$\text{Demand} = MC + MEC$$
$$20 - Q = (7 + Q) + (1 + 2Q)$$

Rearranging,

$$12 = 4Q$$

which, solving for Q, yields a socially optimal output of

$$Q^{SO} = \frac{12}{4} = 3 \text{ units}$$

and a price of

$$p^{SO} = 20 - Q^{SO} = 20 - 3 = \$17$$

Note that $Q^{SO} = 3 < 6.5 = Q^*$ and that $p^{SO} = \$17 > \$13.5 = p^*$. In words, the socially optimal output is lower than the equilibrium output, while the price at the social optimum is higher than at the competitive equilibrium.

c) We need to impose a tax that reduces Q^* to Q^{SO}. In particular, we need a tax t that satisfies

$$t = \$17 - (\text{Height of MC evaluated at } Q^{SO} = 3)$$
$$= \$17 - (7 + 3) = 17 - 10 = \$7$$