Exercise – Cournot competition with asymmetric costs

Consider a duopoly game in which firms compete a la Cournot, i.e., simultaneously and independently selecting their output levels. Assume that firm 1 and 2's constant marginal costs of production differ, i.e., $c_1 > c_2$. Assume also that the inverse demand function is $p(q) = a - bq$, with $a > c_i$. Aggregate output is $q = q_1 + q_2$

Find the pure strategy Nash equilibrium of this game. Under what conditions does it yield corner solutions (only one firm producing in equilibrium)?

Answer:

In a Nash equilibrium (q_1^*, q_2^*), firm 1 maximizes its profits by selecting the output level q_1 that solves

$$\max_{q_1 \geq 0, q_2^*} \pi_1(q_1, q_2^*) = \big(a - b(q_1 + q_2^*)\big)q_1 - c_1 q_1 = aq_1 - b{q_1}^2 - bq_2^* q_1 - c_1 q_1$$

where firm 1 takes the equilibrium output of firm 2, q_2^*, as given. Similarly for firm 2, which maximizes

$$\max_{q_2 \geq 0, q_1^*} \pi_2(q_1^*, q_2) = \big(a - b(q_1^* + q_2)\big)q_2 - c_2 q_2 = aq_2 - bq_1^* q_2 - b{q_2}^2 - c_2 q_2$$

which takes the equilibrium output of firm 1, q_1^*, as given. Taking first order conditions with respect to q_1 and q_2 in the above profit-maximization problem, we obtain:

$$\frac{\partial \pi_1(q_1, q_2^*)}{\partial q_1} = a - 2bq_1 - bq_2^* - c_1 \qquad (1)$$

$$\frac{\partial \pi_2(q_1^*, q_2)}{\partial q_2} = a - bq_1^* - 2bq_2 - c_2 \qquad (2)$$

Solving for q_1 in (1) we obtain firm 1's best response function:

$$q_1(q_2) = \frac{a - bq_2 - c_1}{2b} = \frac{a - c_1}{2b} - \frac{q_2}{2}$$

(Note that, as usual, we rearranged the expression of the best response function to obtain two parts: the vertical intercept, $\frac{a-c_1}{2b}$, and the slope, $-\frac{1}{2}$). Similarly, solving for q_2 in (2) we find firm 2's best response function

$$q_2(q_1) = \frac{a - bq_1 - c_2}{2b} = \frac{a - c_2}{2b} - \frac{q_1}{2}$$

Plugging firm 1's best response function into that of firm 2, we obtain:

$$q_2^* = \frac{a - c_2}{2b} - \frac{1}{2}\left(\frac{a - c_1}{2b} - \frac{q_2}{2}\right)$$

Since this expression only depends on q_2, we can now solve for q_2 to obtain the equilibrium output level for firm 2, $q_2^* = \frac{a + c_1 - 2c_2}{3b}$. Plugging the output we found q_2^* into firm 1's best response function $q_1 = \frac{a - 2bq_2 - c_2}{b}$, we obtain firm 1's equilibrium output level:

$$q_1^* = \frac{a - 2b\left[\frac{a + c_1 - 2c_2}{3b}\right] - c_2}{b} = \frac{a + c_2 - 2c_1}{3b}$$

Corner and interior solutions: Hence, for $q_1^* = \frac{a+c_2-2c_1}{3b} \leq 0$ we need firm 1's costs to be sufficiently high, i.e., solving for c_1 we obtain $\frac{a+c_2}{2} \leq c_1$. A symmetric condition holds for firm 2's output. In particular, $q_2^* = \frac{a+c_1-2c_2}{3b} \leq 0$ arises if $\frac{a+c_1}{2} \leq c_2$. Therefore, we can identify three different cases (two giving rise to corner solutions, and one providing an interior solution):

1. if $c_i \geq \frac{a+c_j}{2}$ for every firm $i = \{1,2\}$, then both firms' costs are so high that no firm produces a positive output level in equilibrium;
2. if $c_i \geq \frac{a+c_j}{2}$ but $c_j < \frac{a+c_i}{2}$, then only firm *j* produces positive output (intuitively, firm *j* would be the most efficient company in this setting, thus leading it to be the only producer); and
3. if $c_i \leq \frac{a+c_j}{2}$ for all firm $i = \{1,2\}$, then both firms produce positive output levels.

The next figure summarizes these equilibrium results as a function of c_1 and c_2. Note that, since $a > c_i$ for every firm *i* by assumption, we only need to look at the square region in the southwest corner of the figure (to the left of *a* on the horizontal axis, and below *a* on the vertical axis). In this square region, either only firm 1 produces (when c1 is low and c2 is high), both firms produce (when costs are relatively close to each other), or only firm 2 produces (when c1 is relatively high and c2 is low). In addition, the figure depicts what occurs when firms are cost symmetric, $c_1 = c_2 = c$ along the 45°-line, whereby either both firms produce positive output levels, i.e., if $c < \frac{a+c}{2}$ or $c < a$; or none of them does, i.e., if $c > a$.