Chapter 5

The Theory of Demand

Reading materials – Chapter 5

- Besanko and Braeutigam:
  - Chapter 5.
- Perloff:
  - pp. 111-116.
  - pp. 136-141.
  - pp. 152-164.
- I will post these pages on Blackboard.

Individual Demand

- **Price Changes:**
  - Using the figures developed in the previous chapters, we will be able to analyze the effect of a price change on the individual's optimal choice.
  - And the analysis of these effects will give us the demand curve.

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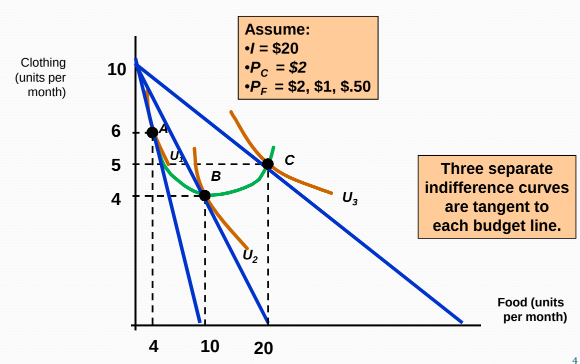
Individual Demand



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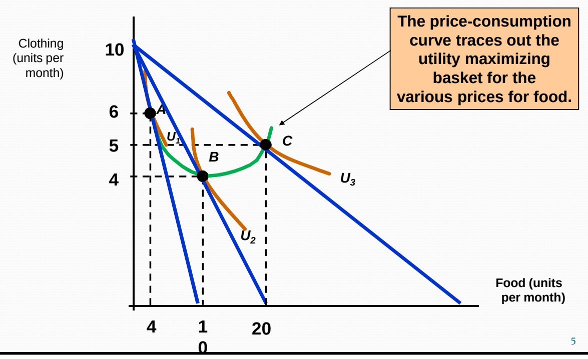
Individual Demand



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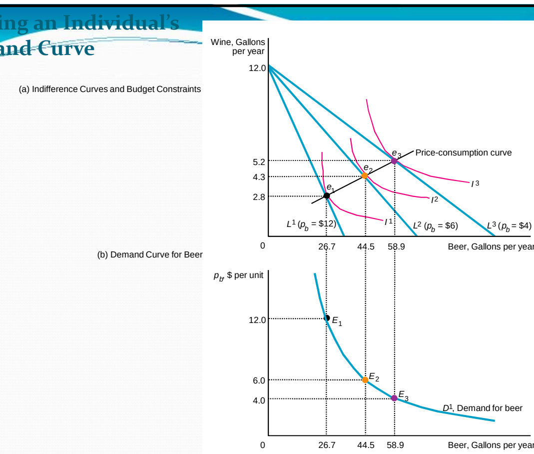
Deriving an Individual's Demand Curve



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Finding a demand curve - Example

- Consider the Cobb-Douglas utility function, u(x,y) = xy, with marginal utilities

$MU_x = y,\ MU_y = x$, and we leave prices and income levels as general as possible, i.e., $p_x,\ p_y,\ I$

- 1st) Budget constraint → On your own
- 2nd) MRSx,y=price ratio

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Finding a demand curve – Example

Consider now a utility function

$$u = xy + 10x, \text{ where } MU_x = y + 10,\ MU_y = x,$$

And assume I=$100, and p_x=$1 but we leave unspecified p_y since we seek to find the demand curve for good y (as a function of its price).

Otherwise, we would be obtaining an optimal consumption bundle (a number for the optimal consumption of good y, rather than a function of p_y).

- 1st) Budget constraint
  - 2nd) MRSx,y=price ratio → Next slide

- Learning by doing 5.2:

$U = xy + 10x \Rightarrow MU_x = y + 10$ and $MU_y = x \quad P_x X + P_y Y = I$

Suppose that $I = 100$ and $p_x = 1 \quad p_y = unknow$ then

1) $(p_x = 1)x + p_y y = 100 \Rightarrow x + p_y y = 100 \leftarrow$ B.L.

2) $MRS = \frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y+10}{x} = \frac{1}{p_y} \Rightarrow x = (y+10)p_y$ plug into 1

Plugging (2) into (1)

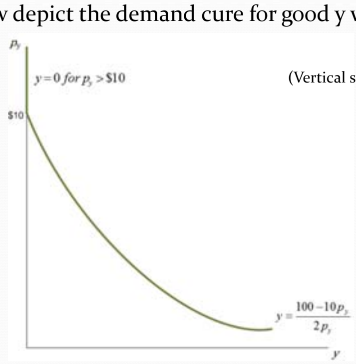
$(y+10)p_y + p_y y = 100 \Leftrightarrow 2p_y y + 10p_y = 100 \Leftrightarrow y = \frac{100 - 10p_y}{2p_y}$

- Learning by doing 5.2:
  - Hence, the demand for good y is positive,

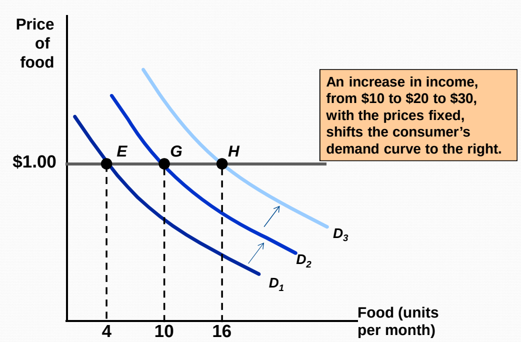
$\frac{100 - 10p_y}{2p_y} \geq 0$ only if $100\text{-}10p_y \geq 0 \Leftrightarrow 100 \geq 10p_y \Leftrightarrow 10 \geq p_y$

$$\Rightarrow y = \begin{cases} \frac{100 - 10p_y}{2p_y} & \text{if } p_y < 10 \\ 0 & \text{otherwise} \end{cases}$$

We can now depict the demand cure for good y we just found



Effects of Income Changes



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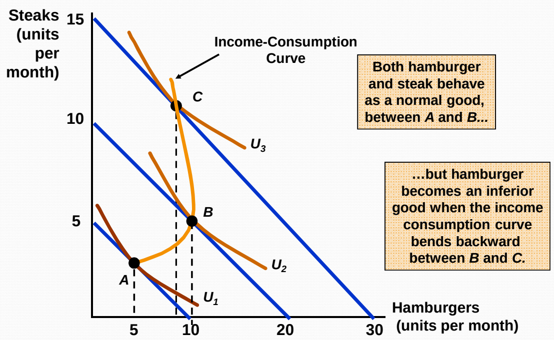
An Inferior Good



Individual Demand

Normal Good vs. Inferior Good

- Income Changes
  - When the income-consumption curve has a **positive** slope:
    - The quantity demanded increases with income.
    - The income elasticity of demand is positive.
    - The good is a normal good.

Individual Demand

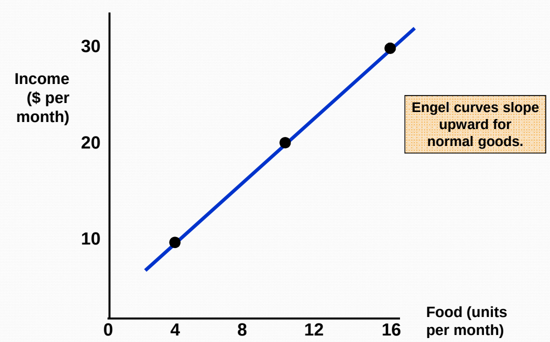
Normal Good vs. Inferior Good

- Income Changes
  - When the income-consumption curve has a **negative** slope:
    - The quantity demanded <u>decreases</u> with income.
    - The income elasticity of demand is <u>negative</u>.
    - The good is an <u>inferior</u> good.

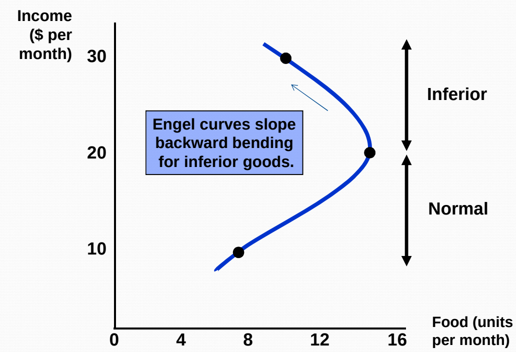
Effects of Income Changes: Engel Curves

- Engel curves relate the quantity of goods consumed to income.
  - If the good is a <u>normal</u> good, the Engel curve is upward sloping.
  - If the good is an <u>inferior</u> good, the Engel curve is downward sloping.
- Let's look at two examples:

Effects of Income Changes: Engel Curves



Engel Curves



Finding Engel Curves

Consider again the Cobb-Douglas utility function

$u(x, y) = x \cdot y$ with $MU_x = y$ and $MU_y = x$

$p_x x + p_y y = I$ is the budget line.

In order to find the Engel Curve for good x, we first need to find the demand curve of x :

Using the tangency condition, we obtain

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y}{x} = \frac{p_x}{p_y} \Rightarrow y = \frac{p_x \cdot x}{p_y}$$

Finding Engel Curves

Consider again the Cobb-Douglas utility function

Then, plugging the above result, $y = \frac{p_x \cdot x}{p_y}$, into the budget line yields

$$p_x x + p_y y = I \Rightarrow p_x x + p_y \left(\frac{p_x \cdot x}{p_y} \right) = I \Rightarrow 2 p_x x = I$$

Solving for x, we obtain the demand curve for x

$$x = \frac{I}{2 p_x}$$

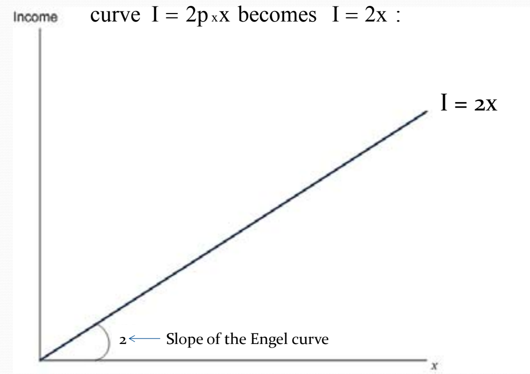
Finding Engel Curves

- Since the Engel curve has the amount of the good x on the horizontal axis and income (I) on the vertical axis…

we first need to solve for I in the demand curve $x = \frac{I}{2p_x}$, yielding an Engel curve of

$$I = 2p_x x$$

If, for instance $p_x = \$1$, then the Engel curve $I = 2p_x x$ becomes $I = 2x$:



Engel Curve in corner solutions

- What if, instead, we face a utility function for perfect substitutes, such as u(x,y) = x + y, which we know yields optimal consumption bundles at a corner?
- **Let's practice:**

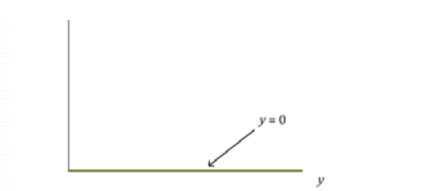
$U(x,y) = x + y$ its MRS $_{x,y}$ is $MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{1}{1} = 1$

$$if\ \ p_y > p_x, then \frac{P_y}{P_x} > 1 = MRS \rightarrow \frac{P_y}{P_x} > \frac{MU_y}{MU_x} \rightarrow \frac{MU_x}{P_x} > \frac{MU_x}{P_y}$$

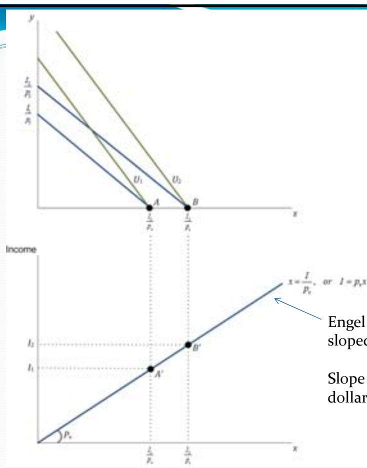
which implies that this consumer would be still willing to consume more units of x and less of y.

Graph on next slide.

- Of course, the Engel curve of good y implies $y = 0$ for all income levels.
  - That is, the equation of the Engel curve would be I = 0 for all y, as depicted in the figure.



What about the Engel curve for good x? →



Engel Curve, Positively sloped for all income levels.

Slope of 1: Every extra dollar is spent on good x.

Income and Substitution Effects

- A fall in the price of a good has two effects: *Substitution & Income*
  - Substitution Effect
    - Consumers will tend to buy more of the good that has become relatively cheaper, and less of the good that is now relatively more expensive.

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Income and Substitution Effects

- A fall in the price of a good has two effects: *Substitution & Income*
  - Income Effect
    - Consumers experience an increase in real purchasing power when the price of one good falls.

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Income and Substitution Effects

- Substitution Effect
  - The substitution effect is the change in an item's consumption associated with a change in its price, *holding the utility level constant.*
  - When the price of an item declines, the substitution effect always leads to an increase in the quantity of the item demanded.

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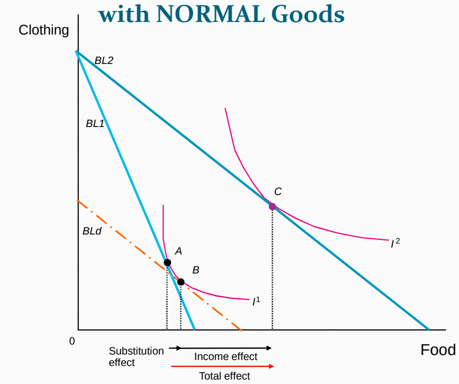
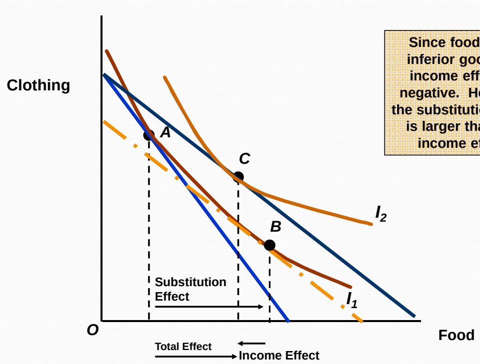
Income and Substitution Effects

- Income Effect
  - The income effect is the change in an item's consumption brought about by the increase in purchasing power, *with the price of the item held constant.*
  - When a person's purchasing power increases, the quantity demanded for the product may increase or decrease.

Substitution and Income Effects with NORMAL Goods

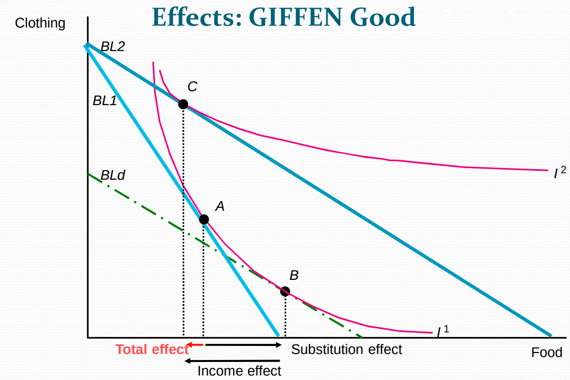


Income and Substitution Effects: INFERIOR Good



Since food is an inferior good, the income effect is negative. However, the substitution effect is larger than the income effect.

Income and Substitution Effects: GIFFEN Good



Income and Substitution effects

For a decrease in prices:

	SE	IE	TE	
Normal good	+	+	+	Demand curve is negatively sloped (as usual)
Inferior good	+	-	+	
Giffen good	+	-	-	Potatoes (in Ireland during the XIX century) and Tortillas (Mexico, 1990s). Demand curve is *positively* sloped

Please note that all signs would change if we analyze an increase in prices.

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Income and Substitution effects

- Note that if the total effect of a price decrease is positive (TE >0), the demand curve is negatively sloped.
  - Indeed, a decrease in p implies an *increase* in the quantity demanded.
  - This applies for both normal and inferior goods…
    - But NOT for Giffen goods: They would have a positively sloped demand curve, i.e., a more expensive good would lead to an increase in the quantity demanded (they are a rare species of goods!)

- Learning by doing 5. 4
- Consider again the Cobb-Douglas utility function

$U = xy$ with marginal utilities $MU_x = y$ and $MU_y = x$

Suppose that $I = \$72$ and $p_y = \$1$. In addition, assume that the initial $p_{x1} = 9$ then decreases to $p_{x2} = 4$

Basket A (at initial price $p_{x1} = 9$):

1) $9x + p_y y = 72$

2) $MRS = \frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y}{x} = \frac{9}{1} \Rightarrow y = 9x$ plug into 1

$9x + (9x) = 72 \Leftrightarrow 18x = 72 \Leftrightarrow x = 4$ plug into y

$y = 9(4) = 36$

$A = (4, 36)$

- Learning by doing 5. 4 (cont'd):

Basket C (at final prices $p_{x2} = \$4$):

1) $4x + p_y y = 72$

2) $MRS = \frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y}{x} = \frac{4}{1} \Rightarrow y = 4x$ plug into 1

$4x + (4x) = 72 \Leftrightarrow 8x = 72 \Leftrightarrow x = 9$ plug into y

$y = 9(4) = 36$

$C = (9, 36)$

A→C
4→9 of x ,
thus implying a total effect of TE=9-4

- Learning by doing 5.4 (cont.)
  - In order to disentangle the SE and IE from the TE = 5 we just found, we need to first find the decomposition Basket B.
  - We know that the decomposition bundle B satisfies two properties:
    - **First,** B must give the same utility level as the initial basket A. Hence, since basket A = (4, 36) yields a utility level of
      - u = xy = 4 (36) = 144, then basket B must also satisfy xy = 144.

Learning by doing 5.4 (cont.)

- **Second,** BL_d must be tangent to IC_1. Since the slope of BL_d is the same as BL_2, then

$$\frac{p_x}{p_y} = \frac{4}{1} \rightarrow \text{slope of } BL_d = \text{slope of } IC_1$$

$$\frac{4}{1} = \frac{MU_x}{MU_y}$$

$$4 = \frac{y}{x} \Rightarrow y = 4x$$

- Combining the information we found from both properties,

$$\left.\begin{array}{l} xy = 144 \\ y = 4x \end{array}\right\} \Rightarrow x(4x) = 144 \Rightarrow 4x^2 = 144 \Rightarrow x = 6$$

Hence, $y = 4(6) = 24$

$B = (6, 24) \leftarrow$ Decomposition Basket B

Learning by doing 5.4 (cont.)

- We can now summarize our results with a figure depicting the units consumed of good x (the good whose price changed):

----A----B----C→

4 → 6 → 9

SE IE

TE =5

$$SE = X_B - X_A = 6 - 4 = 2$$

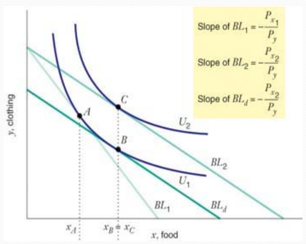
$$IE = X_C - X_B = 9 - 6 = 3 > 0 \; (normal\ good)$$

- **Summary of learning by doing 5.4:**
  - As price of good X decreases…
    - The *SE* leads us to an increase in consumption of good *X* from 4 to 6.
    - The *IE* will measure the consumption from decomposition basket *B* to basket *C (From 6 to 9)*

Note: When you examine the *SE* remember that at B you reach the same indifference curve as at A, but your BL has the same slope at bundle C.

Special Case:

What happens with the SE and IE when we deal with a quasilinear utility functions, such as $U(x,y)=\sqrt{x}+y$

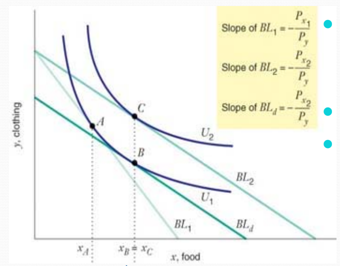


SE

IE=o

Hence, SE=E since IE =o

Special Case: SE and IE with quasilinear utility functions



- X is neither a normal good nor an inferior good.
- SE=X_B-X_A>o
- IE=X_C-X_B=o (no income effect)

SE

IE=o

Hence, SE=E since IE =o

Learning by doing 5.6, where IE = 0

Let's now consider a Quasi-linear utility function

$U=2\sqrt{x}+y \Rightarrow MU_x=\frac{1}{\sqrt{x}}$ and $MU_y=1$

$I=10,\ p_{x1}=0.5,\ p_{x2}=0.2,\ p_y=1$

(At initial prices)

Basket A:

1) $0.5x+y=10$

2) $MRS=\frac{MU_x}{MU_y}=\frac{p_x}{p_y}\Rightarrow\frac{\frac{1}{\sqrt{x}}}{1}=\frac{0.5}{1}\Rightarrow\frac{1}{\sqrt{x}}=\frac{1}{2}\Rightarrow x=4$

$0.5(4)+y=10\Rightarrow y=8$

$A=(4,8)$

Learning by doing 5.6, where IE = 0

(At final prices)

Basket C:

1) $0.2x+y=10$

2) $MRS=\frac{MU_x}{MU_y}=\frac{p_x}{p_y}\Rightarrow\frac{\frac{1}{\sqrt{x}}}{1}=\frac{0.2}{1}\Rightarrow\frac{1}{\sqrt{x}}=\frac{1}{5}\Rightarrow x=25$

$0.2(25)+y=10\Leftrightarrow y=5$

$C=(25,5)$

Learning by doing 5.6 (cont.)

Decomposition Basket B. It must satisfy two properties:

1) Same utility level as basket A=(4,8), so $U_1 = 2\sqrt{x} + y = 2\sqrt{4} + 8 = 12$

Thus we know, $2\sqrt{x} + y = 12$

2) BL_d must be tangent to IC_1 and parallel to BL_2

$$\underbrace{\frac{MU_x}{MU_y}}_{\text{slope of } IC_1} = \underbrace{\frac{p_x}{p_y}}_{\substack{\text{slope of } BL_2 \\ (\text{and } BL_d)}} \Rightarrow \frac{\frac{1}{\sqrt{x}}}{1} = \frac{0.2}{1} \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{5} \Rightarrow x = \boxed{25}$$

$$2\sqrt{25} + y = 12 \Rightarrow y = 2$$

Hence, the decomposition bundle B is $B = (25, 2)$

Learn by doing 5.6 (cont.)

- We can now summarize our results about SE and IE:

B coincides with *C*
↓

- $SE = x_B - x_A = 25 - 4 = 21$
- $IE = x_C - x_B = 25 - 25 = 0$
- Notice there is no *IE* because she consumes the same amount of good *x* at bundles *B* and *C*.

A more compact representation of the SE and IE using the compensated demand curve
From Perloff's textbook (posted on Blackboard)

- Let us use the Compensated Demand Curve to analyze SE and IE:
  - In our exercise from the expenditure minimization problem we obtain the compensated (or Hicksian) demand. (*See the last part of the Ch. 4 lectures*)

$$\left.\begin{array}{ll} \text{Min} & p_1 x_1 + p_2 x_2 \\ \text{subject to} & u(x) \geq \bar{u} \end{array}\right\} h(P_1, P_2, \bar{u})$$

- Using $h(p_1, p_2, \overline{U})$ we obtain the minimal expenditure that the consumer needs to incur in order to reach indifference curve $\overline{U}$ at prices p_1 and p_2.

$$E(p_1, p_2, \overline{U}) = p_1 \overbrace{h_1(p_1, p_2, \overline{U})}^{x_1} + p_2 \overbrace{h_2(p_1, p_2, \overline{U})}^{X_2}$$

Hence, $\dfrac{\partial E}{\partial p_1} = h_1(p_1, p_2, \overline{U})$

This is just an amount of good 1 bought by this consumer, i.e., equivalent to q_1.

A more compact representation of the SE and IE using the compensated demand curve

Plugging the minimal expenditure into the consumer's income level I we obtain

$$h(p_1, p_2, \bar{U}) = D\left(p_1, p_2, \underbrace{E(p_1, p_2, \bar{U})}_{\text{minimal expenditure to reach } \bar{U}}\right)$$

And differentiating with respect to p_1,

$$\frac{\partial h}{\partial p_1} = \frac{\partial D}{\partial p_1} + \frac{\partial D}{\partial E}\frac{\partial E}{\partial p_1}$$

$$\frac{\partial h}{\partial p_1} = \frac{\partial D}{\partial p_1} + \frac{\partial D}{\partial E} q_1$$

- This equation is often regarded on the "Slutsky equation" and describes the SE and IE as follows:

$$\underbrace{\frac{\partial h}{\partial p_1}}_{\text{SE}} = \underbrace{\frac{\partial D}{\partial p_1}}_{\text{TE}} + \underbrace{\frac{\partial D}{\partial E} q_1}_{\text{IE}}$$

  - That is, SE = TE + IE
- Note that $\frac{\partial h}{\partial p_1}$ captures the SE since the compensated demand operates on a "target" indifference curve $\bar{U}$ for different price levels.
- Second, $\frac{\partial D}{\partial E} q_1$ reflects the IE since $\frac{\partial D}{\partial E}$ indicates how an increase in expenditure increases the demand for good 1.

Multiplying all terms by $\frac{p_1}{q_1}$ and the last by $\frac{E}{E}$, we obtain

$$\underbrace{\frac{\partial h}{\partial p}\frac{p_1}{q_1}}_{\varepsilon^*} = \underbrace{\frac{\partial D}{\partial p_1}\frac{p_1}{q_1}}_{\varepsilon} + \underbrace{\frac{\partial D}{\partial E} q_1 \frac{p_1}{q_1}\frac{E}{E}}_{\underbrace{\frac{\partial D}{\partial E}\frac{E}{q_1}}_{\varepsilon_{q,I}} \underbrace{\frac{p_1 q_1}{E}}_{\theta}}$$

where:

1) ε is the standard definition of price elasticity of demand

2) ε^* is the definition of price-elasticity applied to the compensated demand curve

3) $\varepsilon_{q,I}$ is the income-elasticity of demand, and

4) θ is the budget share of that good.

Hence, we can summarize the above big expression more compactly as follows

$$\varepsilon^* = \varepsilon + \varepsilon_{Q,I}\theta$$

And rearranging, $\underbrace{\varepsilon}_{TE} = \underbrace{\varepsilon^*}_{SE} - \underbrace{\varepsilon_{Q,I}\theta}_{IE}$

Examples

- 1) Garlic, θ≅ 0. Hence IE≅ 0, and TE=SE
- 2) Housing, θ=0.4 (40% of monthly expenditures), $\varepsilon_{Q,I}$=1.38, ε=-0.6

Then we can find the value of ε^* by using the above expression $\varepsilon^* = \varepsilon + \theta \bullet \varepsilon_{Q,I} = -0.6 + (0.4*1.38) = -0.04$

- Intuitively, if the consumer is <u>not</u> compensated for a 1% price increase in housing, his demand drops by 0.6%.
- If, however, he is compensated (so he reaches his initial indifference curve at the new prices) his demand only decreases by 0.04%.

- For an *inferior* good, $\varepsilon_{Q,I} < 0$, implying

$$\underbrace{\varepsilon}_{-} = \underbrace{\underbrace{\varepsilon^*}_{-} - \underbrace{\underbrace{\theta}_{+} \bullet \underbrace{\varepsilon_{Q,I}}_{-}}_{+}}_{-}$$

Since $|SE| > |IE|$ for inferior (not Giffen) goods.

Hence, ε<0 which implies that a consumer's demand is negatively sloped (as usual).

- For a *Giffen* good, $\varepsilon_{Q,I} < 0$ (and extremely negative), implying that

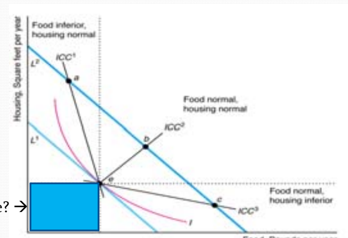
$$\underbrace{\varepsilon}_{+} = \underbrace{\underbrace{\varepsilon^*}_{-} - \underbrace{\theta \bullet \underbrace{\varepsilon_{Q,I}}_{--}}_{++}}_{+}$$

Because $|SE| < |IE|$ for Giffen goods.

Hence, ε> 0 which implies that demand is positively sloped! (only true for Giffen goods).

Can all goods be inferior? NO!!

- The figure represents an increase in income, shifting the consumer's budget line from BL_1 to BL_2. If his optimal consumption bundle under BL_1 is e, where does his optimal bundle under BL_2 lie?



Here? →

- **Some good must be normal (not all inferior):** If the consumer was <u>buying less of both x and y</u>, he would pick a bundle inside BL_1, thus not exhausting all his income.

Another way to show this result, by using elasticities:

Consider a list of optimal consumption bundles $x_1(p, I)$, $x_2(p, I)$, ..., $x_n(p, I)$. Hence, this consumer's budget is exhausted at this optimal consumption bundle, that is

Budget Line $P_1 X_1 + P_2 X_2 + \ldots P_n X_n = I$ for n goods

In order to examine what's the effect of an income change let's differentiate with respect to I,

$$P_1 \cdot \frac{dx_1}{dI} + P_2 \frac{dx_2}{dI} + \ldots P_n \frac{dx_n}{dI} = 1$$

We can now multiply and divide by x_i *I

$$P_1 \frac{X_1 I}{X_1 I} \cdot \frac{dx_1}{dI_1} + \ldots = 1$$

Rearranging,

$$\underbrace{\frac{P_1 X_1}{I}}_{\theta_1} \cdot \underbrace{\frac{dx_1}{dI} \cdot \frac{I}{X_1}}_{\varepsilon_{x_1, I}} + \ldots = 1$$

For a setting with only two goods (as in the above figure):

$$\theta_1 \cdot \overbrace{\varepsilon_{x_1,I}}^{+\,or\,-} + \theta_2 \cdot \overbrace{\varepsilon_{x_2,I}}^{+\,or\,-} = 1$$

$$\theta_1, \theta_1 \geq 0$$

Can all goods be inferior? Let's see what would happen if they were all inferior:

$$\varepsilon_{x_1,I} < 0 \; and \; \varepsilon_{x_2,I} < 0$$

If that was the case, we wouldn't be able to obtain a sum that adds up to 1:

$$\underbrace{\overbrace{\theta_1}^{+} \cdot \overbrace{\varepsilon_{x_1,I}}^{-}}_{-} + \underbrace{\overbrace{\theta_2}^{+} \cdot \overbrace{\varepsilon_{x_2,I}}^{-}}_{-} \neq 1$$

Example

Consider you work for a firm selling a good in a developing country.
For simplicity, assume that most households only spent money on food (f) and other goods (o), which is the goods you sell. In addition, consider you know the income elasticity of demand for food, $\varepsilon_{f,I}\, \varepsilon[0,1]$ but you don't know the income elasticity of other goods, $\varepsilon_{o,I}$.

However, from the above equation; we know that

$$\theta \cdot \varepsilon_{f,I} + (1-\theta) \cdot \varepsilon_{0,I} = 1. \text{Hence,} \varepsilon_{0,I} \, is \, \varepsilon_{0,I} = \frac{1-[\theta \cdot \varepsilon_{f,I}]}{1-\theta}$$ Don't know: $\varepsilon_{0,I}$

IF $\varepsilon_{f,I} \rightarrow 0 \Rightarrow \varepsilon_{0,I} = \frac{1}{1-\theta} > 1$ → Upper bound for $\varepsilon_{o,I}$ since $\theta \varepsilon(0,1)$

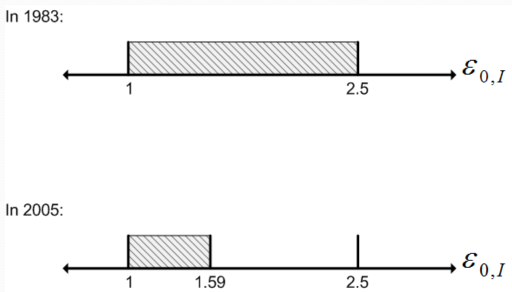
IF $\varepsilon_{f,I} \rightarrow 1 \Rightarrow \varepsilon_{0,I} = \frac{1-\theta}{1-\theta} = 1$ →lower bound for $\varepsilon_{o,I}$.

- Consider that you have information about the average budget share spent on food: as the country developed, it decreased from θ= 0.6 to θ = 0.37. We can then evaluate the upper bound for these values of θ, as follows:

IF $\theta = 0.6$ In 1983, $\theta = 0.37$ In 2005

Upper bound → $\frac{1}{1-\Theta} = \frac{1}{1-0.6} = 2.5$ In 1983

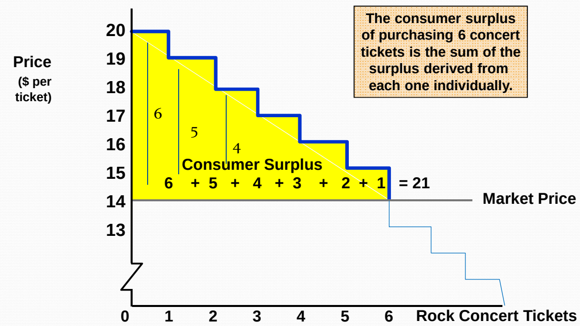
$= \frac{1}{1-0.37} = 1.59$ In 2005



While still imprecise, the range for $\mathcal{E}_{O,I}$ became more accurate as θ declined.
Business strategy: a 1% in GDP in the developing country will lead to a maximum increase in your sales of 1.59%.

Measuring Welfare Changes

Consumer Surplus



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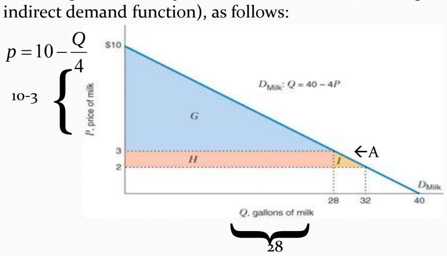
- Learning by doing (Consumer Surplus):

Q=# of gallons of milk purchased at P dollars

Q=40-4*p* —> But we want *p* to be in the y-axis, then we need to solve for p (obtaining the indirect demand function), as follows:

$$4p = 40 - Q \Rightarrow p = 10 - \frac{Q}{4}$$



- Learning by doing (cont.)

1) Point A is the crossing of the demand function $p=10-\frac{Q}{4}$ and market price *p*=3. Then,

$$3 = 10 - \frac{Q}{4} \Leftrightarrow \frac{Q}{4} = 7 \Leftrightarrow Q = 28$$ →point A ($3, 28 units)

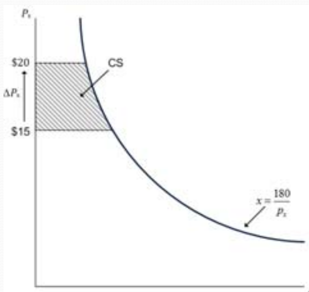
2) CS is the shaded blue triangle G in the previous slide

$$CS = \frac{1}{2}(28)\underbrace{(10-3)}_{\text{height}} = 98$$

-So far we measure CS using the area of the triangle below the Demand curve, because demand curve was linear (straight line)

What if it is not?

Example of a non-linear demand curve: $x = \frac{180}{p_x}$



What if the demand function is *not* a straight line?

- Consumer Surplus with a Cobb-Douglas Utility Function
  - Consider the following Cobb-Douglas utility function
    $U = x^{0.6}y^{0.4}$
  - And assume that, while income remains constant at I = 300, the price of good x increases from
    $p_x = 15$ to $p'_x = 20$

1) Find the consumer's demand curve for any P_x,
   As a practice, use the tangency condition to find that the market demand for good x is

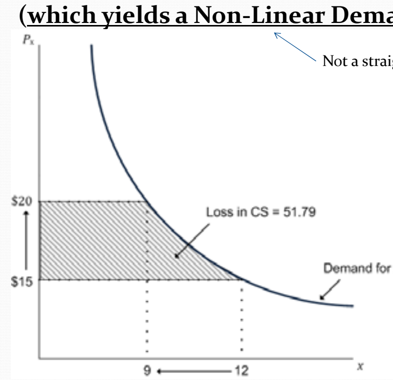
$$x = \frac{0.6I}{p_x} = \frac{0.6(300)}{p_x} = \frac{180}{p_x}$$

What if the demand function not a straight line?

- Hence, the decrease in CS when p_x increases from $15 to $20 is

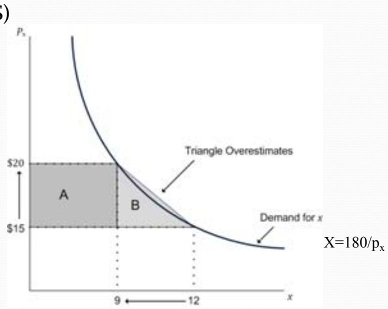
$$\nabla CS = -\int_{15}^{20} \frac{180}{p_x} dp_x = -180[\ln p_x]_{15}^{20} = -180[\ln 20 - \ln 15] = -51.79$$

Measuring CS of a Cobb-Douglas Utility Function
(which yields a Non-Linear Demand)



What if we incorrectly assume linear demand (i.e., so CS=rectangle+triangle):

1) Area of A=(20-15)9=45 (Rectangle)
2) Area of B=(20-15)*(1/2)*6=15 (Triangle)
3) Total loss in CS= 45+15= 60>51.79 (overestimation of the loss in CS)



Query #1

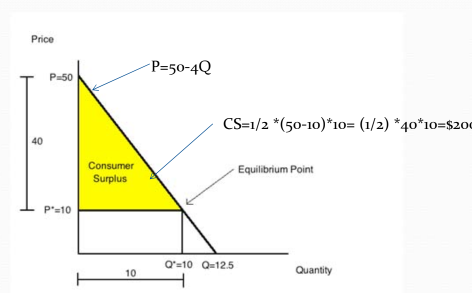
- Suppose the consumer's demand curve for a good can be expressed as $P=50-4Q^d$. Suppose that the market is initially in equilibrium at a price of $10.
- What is the consumer surplus at the original equilibrium price?

a) 100
b) 150
c) 200
d) 250

Query #1 - Answer

- Answer: C ($200)
- The consumer surplus is the difference between the maximum amount a consumer is willing to pay for a good (y-intercept) and the amount they must actually pay (equilibrium price) below the demand curve.
- From the demand curve, we see that when $Q^d = 0$, P=$50. When P=0, Q^d=12.5. Those are our y-and x-intercepts, respectively.
- From there, we can subtract the maximum price consumers are willing to pay, $50, form the equilibrium price, $10, which gives us a difference of $40.
- When we plug P*=$10, the equilibrium given to us, into the demand curce, we find that Q*=10
- Consumer surplus is simply the area of the triangle
- CS=(.5)(40)(10)=$200

Query #1 - Answer Graphically



Measuring Welfare Changes

- After a price decrease, the consumer is better off. But how can we measure by how much?
- **<u>Compensating Variation</u>**: A measure of how much money a consumer would be willing to give up <u>after</u> a reduction in the price of a good to be just as well off as <u>before</u> the price decrease.

Δp {
- If prices increase, then I need more income to maintain the same level of utility.
  - How much money would I need? CV

▼p {
- If prices decrease, then I don't need all my original income to remain at my original utility.
  - How much money would I be willing to give up in order to remain at the original utility level? CV

Measuring Welfare Changes

- **<u>Equivalent Variation</u>**: A measure of how much additional money a consumer would need <u>before</u> a price reduction to be as well off as <u>after</u> the price decrease.

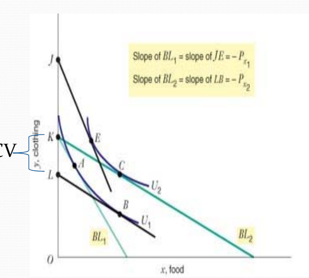
▼p {
- If prices are going to decrease, then consumers are going to be better off because they can purchase more.
  - How much income should we give them today, to make them just as well-off as they will be tomorrow after the price decrease? EV

Δp {
- If prices are going to increase, then consumers are going to be worse off.
  - How much money would we need to take from the consumer today, before the price increase, in order for him to be just as worse-off as he is going to be tomorrow? EV

- In order to provide a graphical representation of CV and EV, we normalize the price of good y, so that p_y = \$1.
- This allows us to identify the vertical intercept of any budget line as the individual income.
  - Why? Recall that the vertical intercept of BL is $\frac{I}{p_y}$, so if p_y = \$1, $\frac{I}{p_y}$ becomes I.

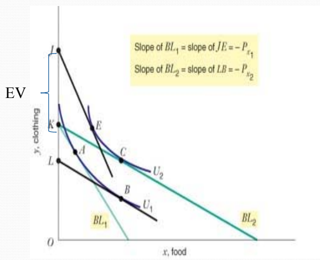
Compensating Variation of a price decrease

- CV



- 0K— consumer's income
- 0L— income needed to purchase basket B at the new prices of x
- 0K- 0L=KL is the CV
- Definition: The amount of money that the consumer will be willing to give up, <u>after</u> the price change, in order to maintain the original utility that he had <u>before</u> the price change.

Equivalent Variation of a price decrease



- 0J— income needed to buy basket E at the old prices of x.
- 0J - 0K=JK is the EV
- Definition: The amount of money that we need to give to the consumer, before the price change, in order to make him just as well-off as he will be after the price change.
- Generally, CV ≠ EV, except with quasilinear utilities (where we will find that CV = EV).

Query #2

Compensating variation is:

a) the change in income necessary to hold the consumer at the final level of utility as price changes
b) always the area under the demand curve and above the price paid
c) the change in income necessary to restore the consumer to the initial level of utility
d) the difference in the consumer's income between the purchase of the original basket and the new basket at the old prices

Query #2 - Answer

- Answer *C*
- Compensation variation is a measure of how much money a consumer would be willing to give up *after* a reduction in the price of a good, just to be as well off *before* the price decrease.
- Answer A describes the Equivalent variation.
- The income effect causes the magnitude of the compensation and equivalent variations to be different.

Example – CV and EV with quasi linear utility

- **CV and EV with no Income effect**:
  - Consider the following quasi linear utility function

$$U = 2\sqrt{x} + y$$

- Assume that I = 10 and $p_y = 1$, but the price of good x decreases from $p_{x1} = 0.5$ to $p_{x2} = 0.2$.
- Compensating Variation (CV)
  - CV= Cost of initial basket A (\$10) - Cost of the decomposition basket B (0.2(25)+2=\$7)
  - CV= 10-7=\$3

Recall that basket B=(25,2)

Example – CV and EV with quasi linear utility

- Equivalent Variation (EV)
  - EV= Cost of buying basket E at initial prices – Cost of buying initial basket A ($10)

    But, where is basket E?

In search of basket E…

- We know that basket E must reach the same utility level as basket C.
- Since basket C is C = (25, 5), its associated utility is u = $2\sqrt{25} + 5 = (2*5) + 5 = 15$.
- Hence, we need that basket E satisfies $2\sqrt{x} + y = 15$.

In search of basket E…

- We also know that at E, the slope of U_2 = slope of BL_1 (old price ratio)

$$\frac{MU_x}{MU_y} = \frac{\frac{1}{\sqrt{x}}}{1} = \frac{0.5}{1} \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$$

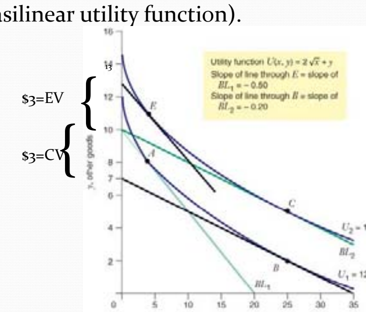
- Plugging this result, x = 4, into the condition we found above $2\sqrt{x} + y = 15$, yields

$$2\sqrt{4} + y = 15 \Rightarrow y = 11$$

Therefore Basket E is E = (4, 11).

- Hence, the cost of buying E is (0.5)4+11=13
- As a consequence
  - EV = Cost of basket E – Cost of basket A = 13 – 10 = $3

- In summary, the CV and EV coincide, i.e., CV=EV= $3, when the IE is absent (which occurs when we have a quasilinear utility function).



- What if we use CS to measure change in welfare that arises from the decrease in the price of good x? If we did…

$$U = 2\sqrt{x} + y$$

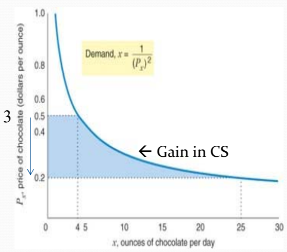
$$I = 10,\ p_{x1} = 0.5,\ p_{x2} = 0.2,\ p_y = 1$$

The demand for x is $x = \frac{1}{p_x^2}$.

- Hence, since the demand function is a curve, the increase in CS is given by the integral:

$$\Delta CS = \int_{0.5}^{0.2} \frac{1}{p_x^2}\, d p_x = \left[\frac{1}{p_x} \right]_{0.5}^{0.2} = \frac{1}{0.2} - \frac{1}{0.5} = 3$$

- which is exactly the same as the CV and EV since we have a quasilinear utility function.



Note: Using CS only yields the same welfare change as CV and EV if income effects are absent, i.e, IE = 0. Otherwise CV≠EV≠CS.

What if IE > 0?

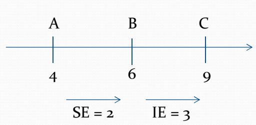
- Consider the following Cobb-Douglas utility function
- U(x,y)= xy

I = \$72

P_y= \$1

P_x= \$9➔P_x' = \$4

From L.D. 5.4 we found A= (4,36)
C= (9, 36)
B= (6, 24)



Since IE ≠ 0 we can then expect that CV ≠ EC ≠ CS.

What if IE > 0?

a) CV

CV= Cost of initial bundle A – Cost of decomp. basket B at the final prices =

= 72 – (4•6)+(1•24)= 72-48= \$24

Cost of bundle A
\$9*4+\$1*36=\$72

Cost of bundle B(6,24) at the final prices

- **B)** EV

EV= Cost of basket E of final prices – Cost of initial basket A

We know that the cost of buying basket A is \$72, but…

Where is basket E?

1) On one hand, we know that E reaches the same utility level U2 as basket C.

Since basket C is C = (9, 36), its utility level is U2 = 9*36 = 324.

Hence, basket E must satisfy x*y = 324.

On the other hand, we know that at E,

slope of U_2= slope of BL_1 $\frac{y}{x} = \frac{9}{1} \rightarrow y = 9x$

Combining y = 9x with the condition we found above, xy = 324, yields x (9x) = 324, or x = 6 units.

This entails that y = 9x = 9*6 = 54 units.

Therefore, basket E is E = (6, 54).

Hence, the cost of buying basket E at initial prices (recall initial prices were P_x =\$9 and P_Y =\$1) is (9•6)+(1•54)= \$108

Then, EV= 108-72= 36

c) Comparing CV and EV in this exercise, CV= 24 ≠ EV= 36

What if we measure the change in consumer welfare using CS?

- First, we find the demand curve for good x when utility function is

U(x,y)=xy

I=\$72

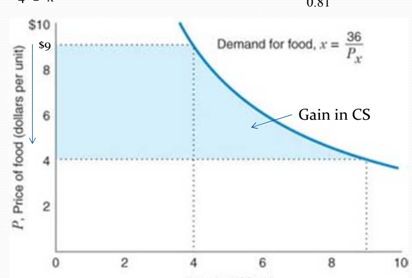
P_y=\$1

$$x = \frac{I}{2p_x} = \frac{72}{2p_x} = \frac{36}{p_x}$$

(non-linear in p_x)

Hence, the gain in CS resulting from the decrease in the price of good x from \$9 to \$4 is given by integral:

$$\Delta CS = \int_4^9 \frac{36}{p_x} dp_x = [36\ln(P_x)]_4^9 = 36\underbrace{[\ln 9 - \ln 4]}_{0.81} = \$29.20$$



- Summarizing:

$$\begin{pmatrix} \Delta CS = 29 \\ CV = 24 \\ EV = 36 \end{pmatrix} -$$

These measures of welfare change are different (Since IE>0), but… is this usual?

Not so much since θ *and* $\varepsilon_{Q,I}$ are low (making income effects low for many goods).

$$\underbrace{\varepsilon_{Q,p}}_{\text{TE}} = \underbrace{\varepsilon^{C}_{Q,p}}_{\text{SE}} + \underbrace{\theta \cdot \varepsilon_{Q,I}}_{\text{IE}}$$

Application: Automobile export restrictions for Japanese cars in 1984

- Prices when up about 20% for JPN cars sold in U.S.

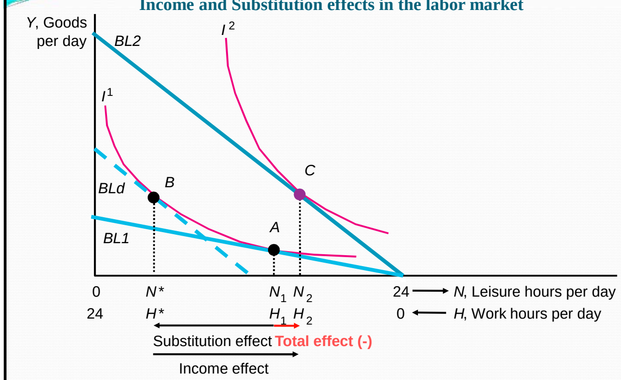
  CV=$14 billion: additional income needed after the prices change (due to the export restrictions) is $14 billion.

  Since in 1984 there were 11 million new cars bought, the CV per car buyer is

$$\frac{14{,}000}{11} = \$1{,}272$$

Application: *Should I really pay my workers more?*

Income and Substitution effects in the labor market



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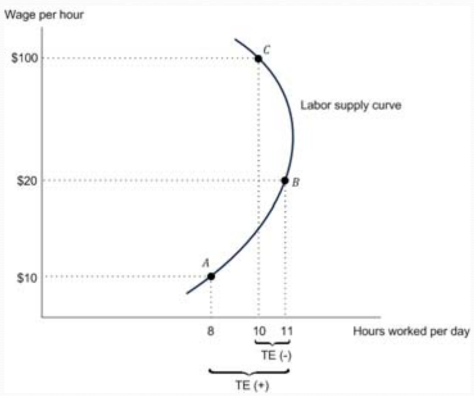
- If the income effect is sufficiently small, total effect of an increase in the wage rate would be still positive. (more working hours applied)
- However, if IE is really large, TE would be negative (workers would supply less working hours).

Backward-bending Labor Supply Curve



Analysis of labor supply

Consider a worker with utility function $u = (wH)^{\alpha} \cdot (24 - H)^{1-\alpha}$

Where H : # hours worked per day
w : Wage per hour

The first term represents utility function from the goods that the worker can purchase with their total salary,
while the second term represents the utility from leisure, i.e., remaining hours in a day that he/ she doesn't work.

Taking F.O.C. in utility function $u = (wH)^{\alpha}(24-H)^{1-\alpha}$ with respect to the number of hours worked, H;

$$\frac{\partial u}{\partial H} = \alpha(wH)^{\alpha-1} \cdot w \cdot (24-H)^{1-\alpha} - (wH)^{\alpha}(1-\alpha)(24-H) = 0$$

$$\alpha(wH)^{\alpha-1}(24-H)^{1-\alpha} = (wH)^{\alpha}(1-\alpha)(24-H)^{-\alpha}$$

$$\alpha w(wH)^{-1}(24-H) = 1-\alpha$$

$$\frac{1}{H}(24-H) = 1-\alpha$$

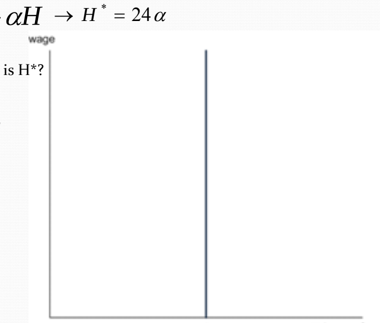
Rearranging,

$$24\alpha - \alpha H = (1-\alpha)H$$

$$24\alpha - \alpha H = H - \alpha H \rightarrow H^* = 24\alpha$$

b) Consider $\alpha = \frac{1}{3}$, What is H*?

$$H^* = 24\frac{1}{3} = 8 \quad hours$$



- Since labor supply is insensitive to variations in the wage rate, *w*, then the substitution and income effects must completely offset each other, i.e., |SE| = |IE|, implying a null total effect, TE = 0.

Tax Revenue and Labor Supply

The above analysis about SE and IE in labor markets can be used to analyze tax policy.

$\text{TaxRevenue} = \tau w h(\omega)$

where $h(\omega)$ is the number of hours worked when wage net of taxes is $\omega = (1-\tau)w$

Some people argue that increasing tax rates would reduce incentives to work. We all agree with that, but the question is by how much.

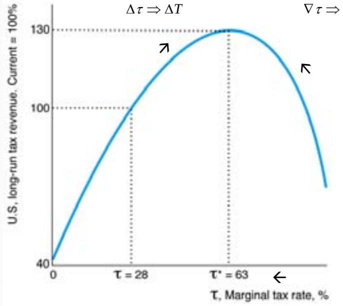
- If the reduction in working hours is sufficiently large, total tax revenue will actually decrease.
- If, in contrast, such a reduction is only minor, total tax revenue will increase.

This relationship is graphically represented with the **Laffer curve**, depicting tax rates on the horizontal axis and total tax collection on the vertical axis.

The Laffer curve

$\frac{\partial T}{\partial \tau} > 0$ $\qquad$ $\frac{\partial T}{\partial \tau} < 0$

$\Delta\tau \Rightarrow \Delta T$ $\qquad$ $\nabla\tau \Rightarrow \Delta T$



Tax Revenue and Labor Supply

Let us analyze under which conditions we are in the increasing or decreasing portion of the Laffer curve.

$\text{TaxRevenue} = \tau w h(\omega)$

where $h(\omega)$ is the number of hours worked when wage net of taxes is $\omega = (1-\tau)w$

$$\frac{\partial T}{\partial \tau} = \underbrace{\overbrace{wh(w)}^{+}}_{\substack{\text{positive effect on } T \\ \text{from higher rate}}} - \underbrace{\overbrace{\tau w^2 \frac{dh}{dw}}^{-}}_{\substack{\text{negative effect on } T \\ \text{from fewer hours worked}}}$$

For $\frac{\partial T}{\partial \tau} < 0$ (a decrease in tax rates to cause an increase in revenue), we need

$$wh(\omega) < \tau w^2 \frac{dh}{d\omega} \Leftrightarrow \frac{1}{\tau} < \frac{dh}{d\omega}\frac{w}{h(\omega)}$$

Tax Revenue and Labor Supply

$$\frac{1}{\tau} < \frac{dh}{d\omega}\frac{w}{h(\omega)}$$

Multiplying both terms by $(1-\tau)$, we obtain

$$\frac{1-\tau}{\tau} < \frac{dh}{d\omega}\frac{\overbrace{w(1-\tau)}^{\omega}}{h(\omega)}$$

$$\frac{1-\tau}{\tau} < \underbrace{\frac{dh}{d\omega}\frac{\omega}{h(\omega)}}_{\text{elasticity of supply of labor}}$$

More compactly:

$$\frac{1-\tau}{\tau} < \varepsilon_{\text{supply, }\omega}$$

A 1% increase in net wages, w, produces an $\varepsilon_{supply,w}$ percentage increase in working hours.

- Hence, for total tax collection ,*T*, to raise from a small fall in the tax rate, we need

$$\varepsilon_{\text{supply, }\omega} > \frac{1-\tau}{\tau}$$

- *Example 1*: $\tau = 25\%$ if your income is about average $35,000 a year

$$\varepsilon_{\text{supply, }\omega} > \frac{1-0.25}{0.25} = 3$$

Unlikely: Bush admin. or nowadays.

- *Example 2*: τ=80%

$$\varepsilon_{Supply,w} > \frac{1-0.8}{0.8} = 0.25$$

Likely: Japan, and Sweden 90s, US during Kennedy

	τ	τ*	Maximum Additional Tax Revenue
United States	28 →	63(!)	30
EU-14	41	62	8
Ireland	27 →	68	30
United Kingdom	28	59	17
Portugal	31	59	14
Spain	36 →	62	13
Germany	41	64	10
Netherlands	44	67	9
Greece	41	60	7
France	46	63	5
Italy	47	62	4
Belgium	49	61	3
Finland	49	62	3
Austria	50	61	2
Sweden	56	63	1
Denmark	57	55	1

Only recommended decrease

Source: Trabandt and Uhlig (2009)

Market Demand

- We now seek to aggregate the individual demands of all consumers in a given industry to obtain the market (or aggregate) demand.
- We will need to horizontally sum their individual demands. That is, for a given price, we add up the units demanded by each consumer.
- Let us look at one example

Market Demand

- We horizontally sum the individual demand of casual consumers and health-conscious consumers, obtaining the aggregate demand.

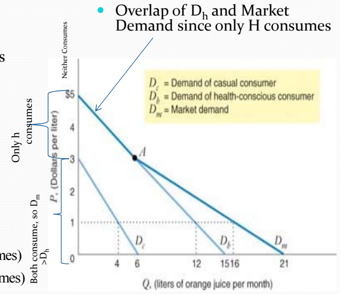
$$Q_h(p) = \begin{cases} 15 - 3p & \text{if } p < 5 \\ 0 & \text{if } p \geq 5 \end{cases}$$

$$Q_c(p) = \begin{cases} 6 - 2p & \text{if } p < 3 \\ 0 & \text{if } p \geq 3 \end{cases}$$

The market demand will be

$$Q_M(p) = \begin{cases} 21 - 5p & \text{if } p < 3 & \text{(Both)} \\ 15 - 3p & \text{if } 3 \leq p \leq 5 & \text{(Only h consumes)} \\ 0 & \text{if } p \geq 5 & \text{(Neither consumes)} \end{cases}$$

- Overlap of D_h and Market Demand since only H consumes



Network Externalities - Example

- Bandwagon effect: A **positive** network externality that refers to the **increase** in aggregate demand for a good as more consumers buy the good. (e.g., online games)
- Snob effect: A **negative** network externality that refers to the **decrease** in aggregate demand for a good as more consumers buy the good. (e.g., gym memberships)