Consumer Choice

Chapter 4

Preferences

- At this point, we know a lot about preferences and their representation with utility functions
- **Preferences** tell us how a consumer ranks a given bundle compared to another bundle, and the utility function tells us how much utility a consumer derives from different bundles.

Preferences

- However…
  - Different bundles do not always cost the same amount, and therefore, all of the available bundles may not be reasonable given a consumer's budget.
- Therefore, in order to determine how a consumer chooses amongst different bundles, we need to take into account not only consumer preferences but also the *budget constraint.*

Budget Constraint and Line

- The *budget constraint* is the set of bundles that a consumer can purchase with a limited amount of income.
- The *budget line* is the set of bundles that a consumer can purchase when spending **all** of his or her available income.
- Example….

Budget Constraint

- Suppose a consumer purchases only two types of goods, food and clothing. Let…
  - x = food   y = clothing   I = income
- To construct the budget line, we must mathematically represent that the total expenditure of x and y coincides with the consumer's income I.

Budget Line:

$p_x x + p_y y = I$

where

$p_x x$ – total expenditure on food

$p_y y$ – total expenditure on clothing

Example

$$P_x X + P_y Y = I$$

$$20x + 40y = 800$$

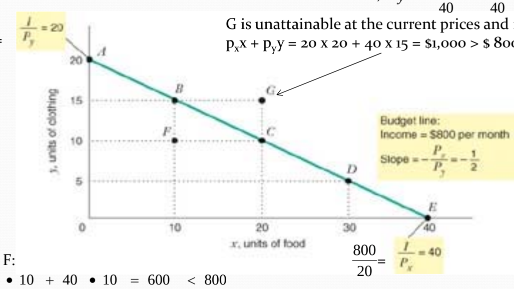
$$\Rightarrow y = \frac{800}{40} - \frac{20}{40}x$$

Let's assume that $p_x = 20$, $p_y = 40$, $I = 800$

$\frac{800}{40} =$

G is unattainable at the current prices and income

$p_x x + p_y y = 20 \times 20 + 40 \times 15 = \$1{,}000 > \$800$



At F:

$20 \bullet 10 + 40 \bullet 10 = 600 < 800$

$\frac{800}{20} =$

$p_x x + p_y y \le I$ ← illustrates the *budget constraint* because it states that you can only spend as much money as you have

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To find the slope of the budget line, we must solve for y… (the good in the vertical axis)

- $p_x x + p_y y = I$
- $\Leftrightarrow p_y y = I - p_x x$
- $\Leftrightarrow \quad y = \frac{I}{P_y} - \left(\frac{P_x}{P_y}\right)x$
  - Notice that this equation has the linear format of y=b+mx, and thus, the slope of the budget line is $-\frac{P_x}{P_y}$
- Note that if x=0, then the consumer can afford $y = \frac{I}{P_x}$ (vertical intercept), while if y=0, the consumer can afford $x = \frac{I}{P_x}$ (horizontal intercept)

The slope of the Budget Line tells us how much of the good on the y axis we must give up to buy one more unit of good on the x axis, as we move from left to right on the figure.

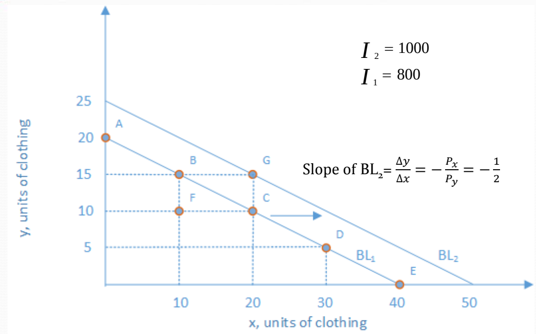
- From our example with food and clothing…

$$-\left(\frac{P_x}{P_y}\right) = -\frac{20}{40} = -\frac{1}{2}$$

  - Thus, the slope indicates that our consumer must give up ½ a unit of clothing (y) to acquire one more unit of food (x). Intuitively this makes sense because food is half as expensive.

- But how does the budget line change when income changes?
  - Given constant prices, the budget line will make a *parallel shift outward* with a raise in income, or inward with a reduction in income.
  - The budget line remains parallel because the slope, $-(p_x/p_y)$, remains unchanged, assuming only income is changed.

Income Change



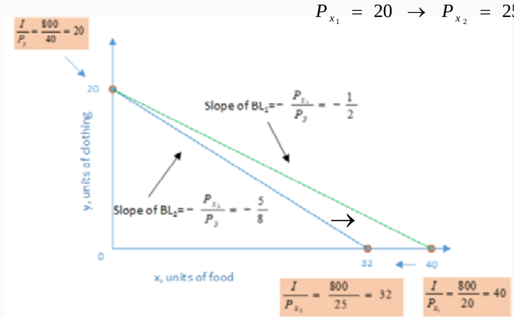
Parallel Shift in the Budget line
Slope of budget line unchanged at $-\frac{P_x}{P_y} = -\frac{1}{2}$

Price Changes

- What about a change in prices?
  - A change in the price of either good changes the slope of $-(p_x/p_y)$.
  - For instance, a change in the price of food (x) from 20 to 25 changes the slope to $-\frac{Px}{Py} = -\frac{25}{40} = -\frac{5}{8}$
  - Thus, the consumer must give up more clothing to obtain more food…

Price Change

$P_{x_1} = 20 \rightarrow P_{x_2} = 25$



Steeper budget line $\rightarrow$ BL Pivots Inward
Reduction in the set of affordable bundles

- What if both income and all prices are doubled? Is this good/bad news for the consumer?
  - A proportionate change in income and prices (let's say by doubling each) causes no change in the budget line…

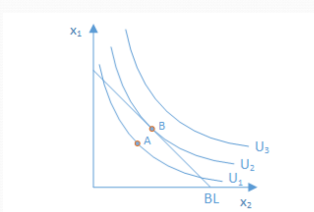
Vertical intercept $2I/2p_y = I/p_y$
Horizontal intercept $2I/2p_x = I/p_x$
Slope $= -\ 2p_x/2p_y = -\ p_x/p_y$

  - The 2's cancel out and leave B.L. unchanged
  - Hence, the consumer's purchasing power is unaffected.

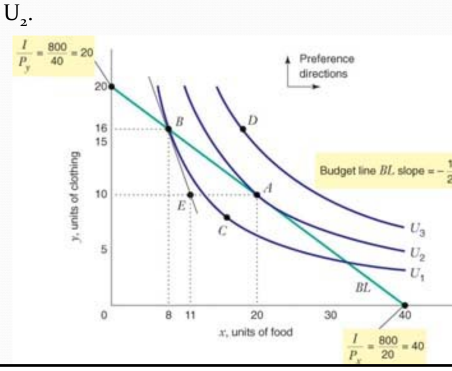
Putting Preferences and B.L. Together

- Given preferences and budget constraints, we can find a consumer's ***optimal choice***, the optimal amount of each good to purchase.
- His or her ***optimal choice*** bundle will be a bundle that fulfills two conditions
  - (1) Maximize Utility (preference)
  - (2) Is affordable (lies on the budget line)

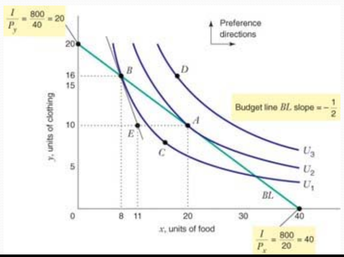
- Immediately we can see that the optimal choice bundle must be located *on* the budget line. Why?
  - Because any bundle inside the B.L. would leave the consumer with extra money that could be spent to further increase utility
  - And any bundle outside the B.L. wouldn't be affordable



- In this continuing example, A is the optimal bundle. It is the only affordable bundle that reaches a utility level of U_2.



- Why other baskets are **<u>not</u>** optimal?
  - D → unaffordable
  - C → money unspent could increase our utility by spending it (generally, it could be future consumption of the good, or savings)
  - B → lower I.C. than at A



- Notice that at the optimal bundle A, the budget line and the utility curve U_2 are tangent.
  - That is, they have the *same slope* (This is crucial!)
- From chapter 3, we know that the slope of an indifference curve $-\left({MU_x}/{MU_y}\right)$ is the marginal rate of substitution,
- Slope of I.C.= Slope of B.L.
- Therefore, $$-\left(\frac{MU_x}{MU_y}\right) = -\left(\frac{P_x}{P_y}\right)$$

or

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

This is called the ***Tangency Condition*** and it reveals ***Interior Optimums*** to us, where the optimum bundle is at a tangency between B.L. and I.C.

- By rearranging the terms we get…

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

(The marginal utilities per dollar of both good x and y at bundle A are equal, so you get the same BANG FOR YOUR BUCK for each good)
Extra utility per dollar spent on x=extra utility per dollar spent on y

- If this were not the case, the bundle would not be optimal because you could take some dollars away from the good with less marginal utility per dollar and put it into buying more of the good with a higher marginal utility

Example

U(x,y) = xy

Hence, $MU_x = y$, $MU_y = x$

$I = 800$, $p_x = 20$, $p_y = 40$

- **Tangency condition…**
  - 1) $p_x x + p_y y = I$ → $20x + 40y = 800$
  - 2) $\frac{MU_x}{MU_y} = \frac{P_x}{P_y} \rightarrow \frac{y}{x} = \frac{20}{40} \rightarrow 2y = x$

$$\downarrow$$

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

$20(2y) + 40y = 800$
$80y = 800$
$y = 10$
$x = 2y = 2 \times 10 = 20$

$$A \rightarrow (x, y) = (20,10).$$

- Point B=(8,16) was not optimal because…

$$\frac{MU_y}{P_y} \neq \frac{MU_x}{P_x} \qquad \frac{x}{40} \neq \frac{y}{20} \rightarrow \frac{8}{40} \neq \frac{16}{20}$$

$$\downarrow$$

$$0.2 > 0.8$$

$$\frac{MU_y}{P_y} < \frac{MU_x}{P_x}$$

- Thus, at point B

$$\frac{MU_y}{P_y} < \frac{MU_x}{P_x} \Rightarrow \uparrow x \text{ and } \downarrow y$$

(At bundle B, the marginal utilities per dollar show us that spending more money on good y and less on good x will increase overall utility)

Utility Maximization Problem

- So, mathematically we represent the constrained maximization problem as

Max *U(x,y)*

subject to $p_x x + p_y y = I$

- In words: Maximize the consumer's utility level subject to his budget constraint

- We can hence use the ***Lagrange Multiplier Method of*** Constraint ***Maximization*** to solve the utility maximization problem.

$$£(x,y;\lambda) = U(x,y) + \lambda\, [I - p_x x - p_y y]$$

- By taking F.O.C.s, we come to the same two conditions as before for a bundle to be optimal (max U subject to B.L.)

- Let's try to show this using Cobb-Douglas Utility Function…

Example: Cobb Douglas Utility function: $U(x,y) = x^{\alpha}y^{1-\alpha}$

MAX $U(x,y) = x^{\alpha}y^{1-\alpha}$

$S.T.\ P_x X + P_y Y = I$

$$\ell(x,y,\lambda) = x^{\alpha}y^{1-\alpha} + \lambda[I - p_x x - p_y y]$$

Taking F.O.Cs with respect to x $u(x,y)$

$$\frac{\partial \ell}{\partial x} = \alpha x^{\alpha-1}y^{1-\alpha} - \lambda px = 0 \rightarrow \alpha \cdot x^{\alpha}y^{1-\alpha} \cdot \frac{1}{x} = \lambda px$$

$$\rightarrow \frac{\alpha \cdot u(x,y)}{P_x \cdot x} = \lambda \qquad (1)$$

And with respect to y,

$$\frac{\partial \ell}{\partial y} = (1-x)x^{\alpha}y^{1-\alpha-1} - \lambda py \rightarrow (1-\alpha)x^{\alpha}y^{1-\alpha}\frac{1}{y} = \lambda py$$

$$\rightarrow \frac{(1-\alpha)u(x,y)}{P_y y} = \lambda \qquad (2)$$

And taking FOCs with respect to

$$\frac{\partial \ell}{\partial \lambda} = I - P_x x - P_y y = 0 \rightarrow P_x x + P_y y = I \qquad (3)$$

From (1) and (2) ,

$$\frac{\alpha \bullet u(x,y)}{P_x x} = \frac{(1-\alpha) \bullet u(x,y)}{P_y y} \qquad \frac{\alpha}{P_x \cdot x} = \frac{(1-\alpha)}{P_y y}$$

$$\rightarrow \frac{\alpha}{1-\alpha} = \frac{P_x x}{P_y y} \rightarrow \frac{\alpha}{1-\alpha} \frac{y}{x} = \frac{px}{py}$$

Which is exactly the tangency condition $\frac{MUx}{MUy} = \frac{px}{py}$ since MUx = $\alpha x^{\alpha-1} y^{1-\alpha}$

And MUy = $(1-\alpha) x^{\alpha} y^{-\alpha}$

Using the tangency condition $\frac{\alpha}{1-\alpha} \frac{y}{x} = \frac{px}{py}$ and

rearranging it, we obtain $\frac{\alpha}{1-\alpha}$ $P_y y = p_x x$

Plugging this into (3), yields

$$\frac{\alpha}{1-\alpha} P_y y + P_y y = I \rightarrow y = (1-\alpha) \frac{I}{P_y}$$

Plugging $y = (1-\alpha)\frac{I}{py}$ into the tangency condition yields $x = \alpha \frac{I}{px}$

Hence, demands are $y = (1-\alpha)\frac{I}{py}$ and $x = \alpha \frac{I}{px}$

- Budget Shares
- After finding the demand for x and y, we can find their budget shares (dividing by the income level, *I*), as follows
- Budget share for x, $\frac{P_x \cdot X}{I} = \frac{I \cdot \alpha}{I} = \alpha$
- Budget share for y, $\frac{P_y Y}{I} = \frac{P_y \bullet \left[(1-\alpha) \bullet \frac{I}{P_y} \right]}{I} = \frac{(1-\alpha)\frac{I}{P_y}}{I} = 1-\alpha$

Budget shares are, hence, constant in income
e.g. if α = 1/3, the budget share of good x is 33% while that of good y is 66%

Examples of Utility Maximization Problems

Approach #1: Using the Lagrange Method
Approach #2: Using the tangency condition

Approach #1: Using the Lagrange method

$$u(x,y) = x^{\frac{1}{3}} y^{\frac{2}{3}} \text{ s.t. } 10x + 20y = 100$$

$$\ell(x,y;\lambda) = x^{\frac{1}{3}} y^{\frac{2}{3}} + \lambda[100 - 10x + 20y]$$

$$FOC_x \quad \frac{1}{3} x^{-\frac{2}{3}} y^{\frac{2}{3}} - \lambda 10 = 0 \Rightarrow \frac{\frac{1}{3} y^{\frac{2}{3}}}{10 x^{\frac{2}{3}}} = \lambda$$

$$FOC_y \quad \frac{2}{3} x^{\frac{1}{3}} y^{-\frac{1}{3}} - \lambda 20 = 0 \Rightarrow \frac{\frac{2}{3} x^{\frac{1}{3}}}{20 y^{\frac{1}{3}}} = \lambda$$

$$FOC_\lambda \quad 100 - 10x - 20y = 0$$

We now set FOCx=FOCy

$$FOC_x = FOC_y$$

$$\lambda = \lambda$$

$$\frac{\frac{1}{3} y^{\frac{2}{3}}}{10 x^{\frac{2}{3}}} = \frac{\frac{2}{3} x^{\frac{1}{3}}}{20 y^{\frac{1}{3}}}$$

- Which simplifies to

$$\frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}} = \frac{x^{\frac{1}{3}}}{y^{\frac{1}{3}}}$$

- Further rearranging, yields x=y

Use the FOCλ

$$FOC_\lambda \quad 10x + 20y = 100$$

$$x = y$$

- Therefore,

$$10x + 20x = 100 \Rightarrow 30x = 100 \Rightarrow x = \frac{100}{30} = 3.33$$

- And the optimal consumption of good y is...

$$y = x = 3.33$$

Approach #2: Using the Tangency Condition

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \rightarrow \frac{\frac{1}{3}x^{-\frac{2}{3}}y^{\frac{2}{3}}}{\frac{2}{3}x^{\frac{1}{3}}y^{-\frac{1}{3}}} = \frac{10}{20}$$

Rearranging the tangency condition yields

$$\frac{y^{\frac{1}{3}}y^{\frac{2}{3}}}{2x^{\frac{1}{3}}x^{\frac{2}{3}}} = \frac{10}{20}$$

$$\frac{y}{2x} = \frac{1}{2}$$

$$y = x$$

- We can now plug our result from the tangency condition, y=x, in the budget line $10x + 20y = 100$

$$10x + 20x = 100$$

$$30x = 100$$

$$x = \frac{100}{30} = 3.33$$

- And good y is y=x=3.33 units.

Budget Shares

Let us evaluate the budget shares in each good…

$$Good x, \quad \frac{px \bullet x}{I} = \frac{10 \bullet 3.33}{100} = \frac{3.33}{10} = \frac{1}{3}$$

$$Good y, \quad \frac{py \bullet y}{I} = \frac{20 \bullet 3.33}{100} = \frac{6.66}{10} = \frac{2}{3}$$

Recall that these budget shares coincide with the exponents of the Cobb Douglas utility function.

- After going over this numerical example of utility maximization problems (UMP), let us now continue with our discussion of UMPs:
  - **Interior solutions.** We have analyzed interior solutions (positive amounts of both goods), but…
  - **Corner solutions.** What if the solution happens to be in a corner (implying a zero consumption of one of the two goods?)

Corner Solutions

- What do we do in optimization cases where the B.L. is never tangent to an indifference curve?
- That is, if $\frac{MU_x}{P_x} \neq \frac{MU_y}{P_y}$
- Example…

Numerical Example of Corner solutions

U(x,y) = xy + 10x

MU_x = y + 10

MU_y = x

I = 10, p_x = 1, p_y = 2

- Notice that utility can be positive where zero units of good y are consumed. This is because the I.C. crosses the x-axis.
- This didn't happen in the Cobb-Douglas utility function where, if x=0 or if y=0, utility was zero.

- Let's approach this like an *interior optimum*…
  - (1) x+2y=10 (B.L.) $P_x \cdot X + P_y Y = I$
  - (2) $\frac{MU_x}{MU_y} = \frac{P_x}{P_y} \Leftrightarrow \frac{y+10}{x} = \frac{1}{2} \Leftrightarrow 2y + 20 = x$

(2y+20) + 2y = 10

4y = -10

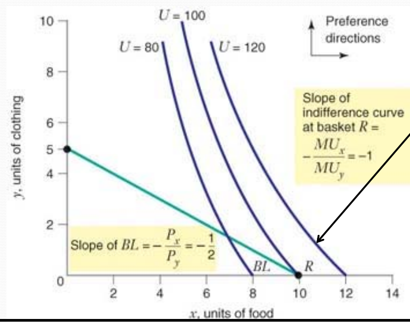
y = -2.5 (NO!! y cannot be negative, thus we have a corner solution where y = 0)

$B.L. \rightarrow x + 2y = 10$

$Since\ y = 0,\ then\ x = 10$

  - The tangency approach gives us a negative amount of clothing which shows us that the solution is not an interior optimum. The solution will be a corner point, where y=0.
  - Negative amounts indicate that the solution must be in a corner.

- Then, a consumer wants to increase his consumption of x as much as possible. That is, the consumer spends all his income in x, $\frac{I}{P_x}=\frac{10}{1}=10$, and no income in y.



Bundle R is optimal

At R,

$\frac{MU_x}{MU_Y}>\frac{P_x}{P_y}$

$\frac{MU_x}{P_x}>\frac{MU_y}{P_y}$

FURTHER INCREASE x.

Corner Point Solution with Perfect Substitutes $U(x,y)=ax+by$

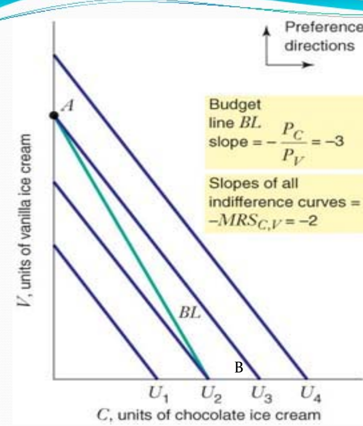
- Let's consider the case where a consumer is perfectly willing to substitute one good for another at a constant ratio.
  - For instance, Sara likes both chocolate and vanilla ice cream, and is always willing to substitute one scoop of chocolate for two scoops of vanilla.
- $MRS_{choc.,vanilla}=\frac{MU_c}{MU_v}=2,\ \ and\ \frac{P_c}{P_v}=3$
  - (constant MRS because of Perfect substitutes)
  - Hence,

$$\frac{P_c}{P_v}>\frac{MU_c}{MU_v}\Leftrightarrow\frac{MU_v}{P_v}>\frac{MU_c}{P_c}\qquad \text{MU per dollar spent on v} > \text{MU per dollar spent on c}$$

- $MRS_{choc.,vanilla}=\frac{MU_c}{MU_v}=2$
  - (constant MRS because of Perfect substitutes)

$$\frac{P_c}{P_v}>\frac{MU_c}{MU_v}\Leftrightarrow\frac{MU_v}{P_v}>\frac{MU_c}{P_c}$$

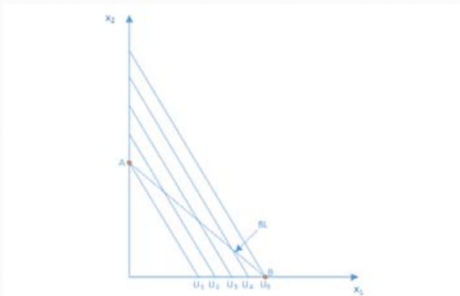
- Consequently, since the marginal utility per dollar of vanilla ice cream is greater than that of chocolate ice cream, Sara should only consume vanilla ice cream.



[What if price ratio was lower than MRS? Then BL becomes flatter than I.C., and the consumer buys only chocolate]

Figure in next slide

When $MRS_{1,2} > \frac{P_1}{P_2}$, we have:

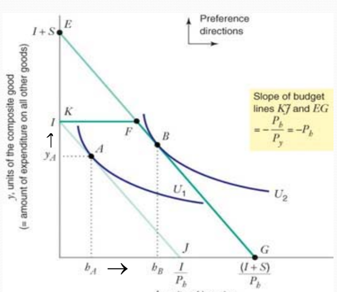


At Point B, $\frac{MU_1}{MU_2} > \frac{P_1}{P_2} \Rightarrow \frac{MU_1}{P_1} > \frac{MU_2}{P_2}$ so the consumer spends all his money on good 1 alone.

Application I: Coupons vs Cash Subsidies

- Before we start with applications, one note:
- ***Composite Goods*** are goods that represent the composite expenditures on every other good besides the commodity being considered.
  - That is, we use composite goods when we wish to isolate how a consumer selects one specific good.
- The price of composite goods is $p_y = 1$ so that it measures not only units of y consumed, but also total expenditure on y ($p_y y$=total expenditure on y)

- The government might consider that the amount of housing individuals select, h_A, is insufficient, and seeks to promote a larger amount of housing, h_B.
- It can achieve this goal by providing a **subsidy** S (which shifts individuals' budget line to EG), or with a **coupon** that can only be used to purchase housing (which yields the new budget line KFG).



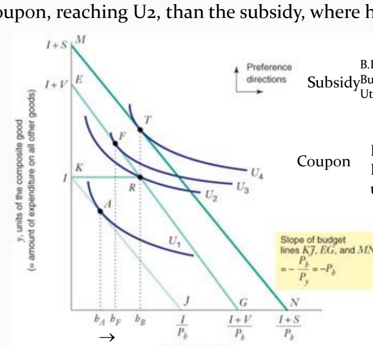
Cash subsidy of S shifts budget line out by S, to B.L. EG

Coupon for housing of S units only increases the budget with respect to housing , so the new budget line with the coupon is KFG

Cash: B.L. is EG. Bundle B with Utility U_2

Coupon: B.L. is KFG. Bundle B with Utility U_2

- In the previous case, the consumer was equally happy with either program because both achieve I.C. of U_2.
- However, in the situation below the consumer is worse off with the coupon, reaching U2, than the subsidy, where he reaches U4.

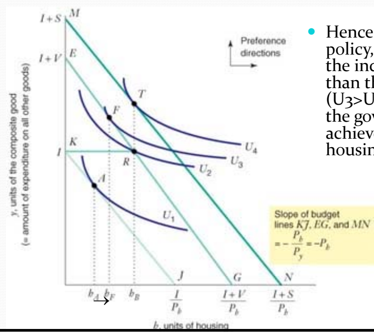


Subsidy: B.L. is MN Bundle T, with Utility U_4

Coupon: B.L. is KRG. Bundle R with utility U_2

Consider instead that the government, still seeking to induce h_B units of housing, does not provide a coupon, but rather provides a voucher *V*, which gives an amount of cash to the individual so that his budget line shifts until the point in which he can afford to buy h_B units of housing at point R. That is, budget line EG emerges from the voucher policy.

- However, will the individual buy bundle R? ***No!*** While R and F are both affordable, he reaches a higher utility level, U3, with bundle F than with bundle R (where he only reaches U2).



- Hence, the voucher policy, despite making the individual happier than the coupon policy (U3>U2), does not help the government achieve a minimal housing level of h_B

Application II: Joining a Club

- Many consumers can join clubs to receive member discounts. Yet, they must also pay a membership fee.
  - So is it worth joining? Let's look at an example…

<u>Before joining</u>: I= 300, p_x = $20

$\frac{I}{P_x} = \frac{300}{20} = 15 units$ Horizontal Intercept of B.L1

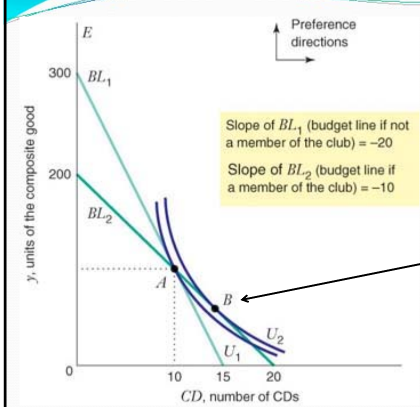
At BL_1 He chooses basket A

$\rightarrow P_y = \$1 \rightarrow \frac{I}{P_y} = \frac{300}{1} = 300 units$ Vertical Intercept of B.L1

<u>After joining:</u> I = 300 – 100 (fee) = $200,

p_x = $10 → $\frac{I}{P_x} = \frac{200}{1} = 20 units$ Horizontal intercept of BL_2

p_y = $1 → $\frac{I}{P_y} = \frac{200}{1} = 200 units$ Vertical intercept of BL_2



Consumer chooses bundle A before joining club

After joining, consumer chooses bundle B and is better off than before because B is on I.C. U_2

- A similar analysis is applicable to:
  - Cell phone service: you pay a monthly subscription fee in order to enjoy calls at lower prices per minute
  - Gym, golf clubs, etc.

Example: AT&T plans for its Apple iPhone 3g (Chicago, 2009)

Plan A: $40 fee, 450 minutes
each additional minute $.40

Plan B: $60 fee, 900 minutes
each additional minute $.40

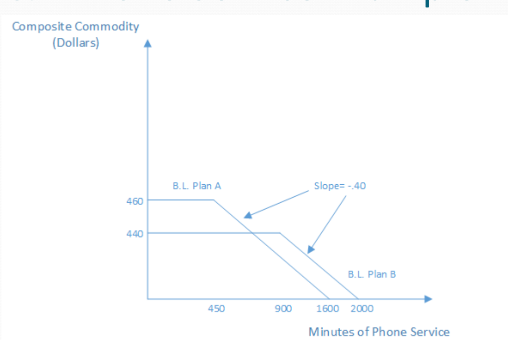
Drawing the B.L. for Plan A:

Income: $500
After fee, Income = $500-40=460 (vertical intercept)
Flat for the first 450 minutes, then slope of .40

Drawing B.L. for Plan B:

After the $60 fee, income is $500-60=440 (vertical intercept)
Flat for B.L. for the first 900 minutes, then slope of .40

AT&T Phone Service Example



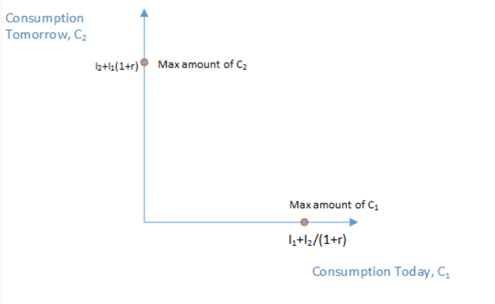
Application III: Borrowing and Lending

- Without borrowing or lending, a consumer consumes I_1 today and I_2 tomorrow, where I_1 is the consumer's salary today, and I_2 is his salary tomorrow.
- *However*, introducing borrowing and lending allows a consumer to…
  - put off consuming today so that today's income can be lent, so tomorrow he or she can consume $I_2 + I_1(1+r)$, with $(1+r)$ being the interest earned from lending income.
  - Consume $I_1 + \frac{I_2}{1+r}$ today by borrowing, and not consuming anything tomorrow.

e.g. $I_2 = 100, r = 0.05$

$$\frac{I_2}{1+r} = \frac{100}{1.05} < 100$$

We want to depict B.L in following axis



- How do we find the slope of the BL in this context?
  - Intuitively, the price of giving up one consumption unit tomorrow (y axis) in order to gain one more unit of consumption today (x axis) is measured by the opportunity cost of every dollar borrowed, (1+r). We can state this more formally as...

We know that the B.L. is $y = a + m \times x$

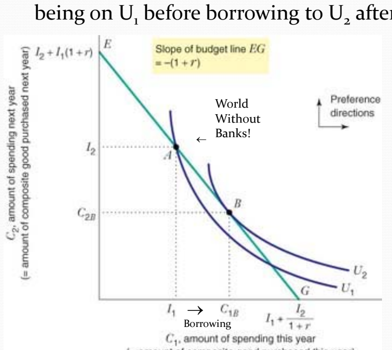
$$y = \underbrace{I_2 + I_1(1+r)}_{a} + m \times x$$

and, in addition, at y = 0 we have x = $I_1 + I_2/(1+r)$, then...

$$0 = I_2 + I_1(1+r) + m\left(\frac{I_1+I_2}{1+r}\right)$$

$$\rightarrow m = \left[\frac{I_2 + I_1(1+r)}{-\left[\frac{I_1+I_2}{1+r}\right]}\right] = -(1+r)\left[\frac{I_2+I_1(1+r)}{I_2+I_1(1+r)}\right] = -(1+r)$$

- Here it is better to borrow some money this year to be paid back next year, because the optimum bundle went from being on U_1 before borrowing to U_2 after borrowing.



In this example both the lending interest rate and borrowing interest rate are 10 percent.

- So far we assumed that the lending and borrowing interest rates coincide, but....
- What if the lending and borrowing interest rates are different?

Lending, r_L=.05

Next year he will have 13,200 from tomorrow's income = 13,200
10,000 (1+0.05) from savings = 10,500

23,700

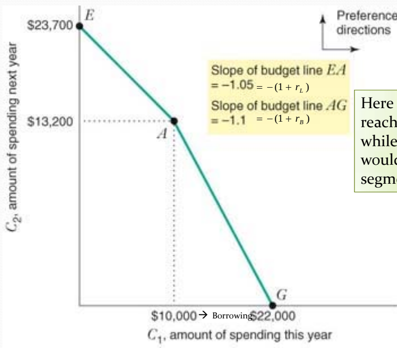
- Borrowing, r_B=.10

Today I can have : 10,000 from today's income= 10,000

$\frac{13,200}{1+0.10}$ → = 12,000

22,000

- Here the interest rates for borrowing and lending are different.



Here the I.C. of a saver would reach tangency on segment AE, while the I.C. of a borrower would reach tangency on segment AG

Application IV: Quantity Discounts

- Sometimes consumers are offered ***quantity discounts***, so that the price of a good is not constant but changes after some defined quantity (i.e., a discount on electricity after x number of units).
  - Notice this will mean the budget line has a kink where the slope changes because of the new price ratio.
  - Let's consider the case of electricity where the prices are as follows…

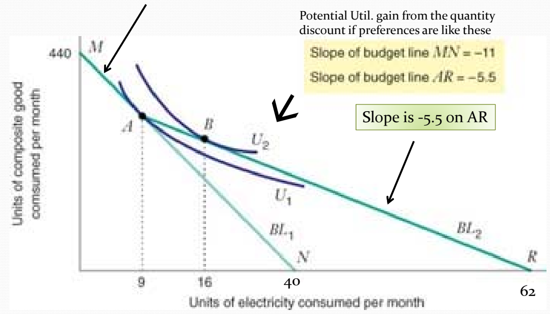
p_y = \$1
p_x = \$11 for the first 9 units (e.g., kw/h)
\$5.5 for all additional units

- How to find the vertical and horizontal intercepts in this case?
  - Vertical Intercept = $\frac{I}{P_y} = \frac{\$440}{\$1} = 440$
  - Horizontal intercept of BL_1 (no discounts) = $\frac{I}{P_x} = \frac{440}{11} = 40$
  - Horizontal intercept of BL_2 (with discounts) = $\frac{440 - 9 \bullet 11}{5.5} = 62$

    where we subtract the amount of money already spent on good x before experiencing the price discount, 9 units x \$11 per unit, to the consumer's initial income.

Slope is -11 on segment MA

Potential Util. gain from the quantity discount if preferences are like these

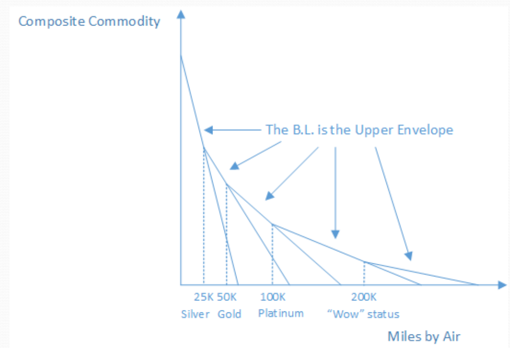


Slope is -5.5 on AR

At point A, we can use the BL in order to find its height: 1y+(9*11)=440, which yields a value of y=440-99=341

- Another Example: Frequent flyer programs, e.g, after 100k miles the price of another ticket decreases.
  - For this example, we can actually have many quantity discounts, as the next figure illustrates.

BL for Frequent Flyer Programs



Examples about optimal consumption decisions

Utility maximization problems and Expenditure minimization problems (Appendix to Chapter 4)

Example 1: Utility Maximization Problem (UMP) with the Cobb-Douglas utility function.

- Let us first consider an example of a utility maximization problem. Take the Cobb-Douglas function expressed by $u(x_1, x_2) = x_1^{1/2} \cdot x_2^{1/2}$
- The budget constraint : $2x_1 + x_2 = 100$
- So we know that:
  - price of good 1 is \$2,
  - that of good 2 is \$1, and
  - income is \$100.

- Setting up the Lagrangian:

$$L(x_1, x_2; \lambda) = x_1^{1/2} \cdot x_2^{1/2} + \lambda(100 - 2x_1 - x_2)$$

- We can take first order conditions:

$$\frac{\partial L}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^{1/2} - 2\lambda = 0$$

$$\frac{\partial L}{\partial x_2} = \frac{1}{2} x_1^{1/2} x_2^{-1/2} - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 100 - 2x_1 - x_2 = 0$$

- Taking the ratio of the first two terms shows that:

$$\frac{x_2}{x_1} = \frac{2}{1} \text{ or rearranging, } x_2 = 2x_1$$

  - (Note that this exactly coincides with the tangency condition between the I.C. and the budget line, that's why the tangency condition and the Lagrangian method yield the same results.)
  - Indeed, the MUx1 is $\frac{\partial u}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^{1/2}$
  - while the MUx2 is... $\frac{\partial u}{\partial x_2} = \frac{1}{2} x_1^{1/2} x_2^{-1/2}$
- Hence, the MRS is $MRS_{x_1,x_2} = \frac{\frac{1}{2} x_1^{-1/2} x_2^{1/2}}{\frac{1}{2} x_1^{1/2} x_2^{-1/2}} = \frac{x_2}{x_1}$

- Substituting the tangency condition $x_2 = 2x_1$ into the budget set $2x_1 + x_2 = 100$ gives

$$2x_1 + 2x_1 = 100$$

$$\Rightarrow x_1 = 25 \text{ units}$$

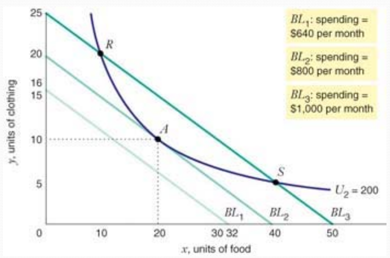
- And about good 2, we have $x_2 = 2x_1 = 2 \times 25 = 50$ units

- Therefore, the utility level that the consumer reaches is given by plugging $x_1 = 25$ and $x_2 = 50$ into the utility function

$$u(x_1, x_2) = x_1^{1/2} x_2^{1/2} = \sqrt{25} \times \sqrt{50} = 35.35$$

- This utility level from selecting utility maximizing bundles is often referred to as "indirect utility"

- *Example 2: Expenditure minimization problem (EMP) with a Cobb-Douglas utility function.*
- Another way to think about optimum bundles is as ***Expenditure Minimization Problems***: the bundle that will give you a given level of utility at the lowest expenditure.
  - Min Expenditure $p_x x + p_y y$
  - Subject to U(x,y) = $\overline{u}$ (reaching a given level of utility)



Example 2: Expenditure minimization problem (EMP) with a Cobb-Douglas utility function.

- Consider a consumer with Cobb-Douglas utility function, who tries to reach a utility level of u=35.35. That is,

$$u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \text{ and } p_1 = 2,\ p_2 = 1, u = 35.35.$$

- To find the compensated demands for goods 1 and 2, let us solve the expenditure minimization problem (EMP)

$$\min \quad (2x_1 + x_2) \qquad \text{s.t.} \quad x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} \geq 35.35 = \overline{u}$$

- Setting up the Lagrangian in this case, yields

$$L(x_1, x_2; \mu) = 2x_1 + x_2 + \mu\left(x_1^{1/2} x_2^{1/2} - 35.35\right)$$

where μ denotes the Lagrange multiplier of this expenditure minimization problem (we use μ rather than λ, which is the Greek letter we used in the UMP).

- In the case of interior solutions, the above first order conditions become

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = 2 - \dfrac{1}{2} \cdot \mu x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} = 0 \cdots\cdots ① \\ \dfrac{\partial L}{\partial x_2} = 1 - \dfrac{1}{2} \cdot \mu x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}} = 0 \cdots\cdots ② \\ \dfrac{\partial L}{\partial \mu} = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} - 35.35 = 0 \cdots\cdots ③ \end{cases}$$

- Let's solve the previous system of equations,

> Our old friend!! The same exact tangency condition we found when finding the optimal consumption bundle using the UMP approach.

From ①, $\mu x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}} = 4 \Rightarrow \frac{\sqrt{x_2}}{\sqrt{x_1}} = \frac{4}{\mu} \Rightarrow \mu = \frac{4\sqrt{x_1}}{\sqrt{x_2}}$

From ②, $\mu x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}} = 2 \Rightarrow \frac{\sqrt{x_1}}{\sqrt{x_2}} = \frac{2}{\mu} \Rightarrow \mu = \frac{2\sqrt{x_2}}{\sqrt{x_1}}$

Since $\mu = \mu \Rightarrow \frac{4\sqrt{x_1}}{\sqrt{x_2}} = \frac{2\sqrt{x_2}}{\sqrt{x_1}} \Rightarrow 4x_1 = 2x_2 \Rightarrow x_2 = 2x_1 \cdots\cdots$ ④

From ③, $x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} = 35.35 \Rightarrow \sqrt{x_1} \cdot \sqrt{x_2} = 35.35 \cdots\cdots$ ⑤

Plugging ④ into ⑤

$\sqrt{x_1} \cdot \sqrt{2x_1} = 35.35 \Rightarrow \sqrt{2} \cdot x_1 = 35.35 \Rightarrow x_1 = \frac{35.35}{\sqrt{2}} \approx 25$

Using ④, $x_2 = 2x_1 \approx 2 \times 25 = 50$

- For the Cobb-Douglas utility function $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$ considered above, we just found that the compensated demands for goods 1 and 2 are

$$x_1^c(p,u) = 25,\ x_2^c(p,u) = 50$$

which coincide with the optimal consumption bundles that we found when solving the UMP!!

- Therefore, the total expenditure required to purchase these compensated demands is

$$e(p,u) = p_1 \cdot x_1^c(p,u) + p_2 \cdot x_2^c(p,u) = 2 \times 25 + 1 \times 50 = 100$$

which coincides with the budget constraint of the consumer in the UMP (where we started saying the consumer had an income of $100 to spend).

Revealed Preferences

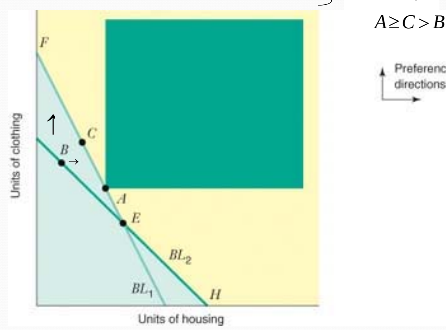
Revealed Preferences

- We have now learned how to find the optimal consumption bundle when we know both a consumer's preferences and his or her budget lines.
- But what if we do not know a consumer's preferences? *We can infer her preferences by analyzing her actual choices in different situations.*

(1) When facing BL_1, consumer chose A instead of any point inside BL_1, such as B. Then A ≥B
(2) A and C were equally costly, but he chose A. Hence A ≥C
(3) Since C lies to the northeast of B, then C >B

A >B (By Transitivity)

$$A \geq C > B \rightarrow A > B$$



Alternative Procedure to check if a consumer's choice maximizes utility

Initial Bundle: (X_1, Y_1)
Final Bundle: (X_2, Y_2)

1) At initial prices, Basket 1 costs $P_x X_1 + P_y Y_1$
At initial prices, Basket 2 costs $P_x X_2 + P_y Y_2$

Let's suppose that, at initial prices:

$$P_x X_1 + P_y Y_1 \geq P_x X_2 + P_y Y_2$$

Since at initial prices he chose Basket 1 (despite Basket 2 was affordable), it must be that he prefers Basket 1 to basket 2

2) At final prices $\tilde{P}_x$ and $\tilde{P}_y$, he chooses Basket 2

Since he already revealed a preference for Basket 1, Basket 2 must be cheaper than Basket 1 at the new prices. Otherwise, he would have selected Basket 1

$$\tilde{P}_x X_2 + \tilde{P}_y Y_2 \leq \tilde{P}_x X_1 + \tilde{P}_y Y_1$$

If this condition wasn't true, the consumer would be revealing a strong preference for Basket 2 over Basket 1, contradicting his revealed preference at the initial prices.

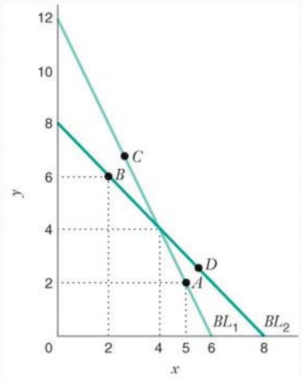
- Here is an example where a consumer fails to maximize utility…

Income I= $24

$(p_x, p_y) = (\$4, \$2)$
$(x_1, y_1) = (5, 2) \rightarrow A$

$\tilde{P}_x, \tilde{P}_y = (\$3, \$3)$
$(x_2, y_2) = (2, 6) \rightarrow B$

Let us see why he fails to maximize utility….



- BL_1:
  - (1) when facing BL_1, the consumer chose A instead of C, in spite of being available (affordable), A≥C
  - (2) Since C is to the northeast of B, C>B

  } A >B $\quad A \geq C > B \rightarrow A > B$

- BL_2:
  - when facing BL_2, he chose B in spite of D being available, B≥D
  - Since D is to the northeast of A, D>A

  } B >A $\quad B \geq D > A \rightarrow B > A$

- *Contradiction! Bundle A cannot be strongly preferred to B, and B strongly preferred to A in a case where both are affordable.*

Alternative Approach, using total expenditure

BL_1 : Consumer prefers basket A.
Cost of basket A, \$4×5 + \$2×2 = \$24 $\rightarrow P_x X_1 + P_y Y_1$
Cost of basket B, \$4×2 + \$2×6 = \$20 $\rightarrow P_x X_2 + P_y Y_2$

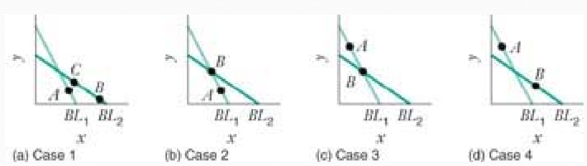
BL_2 : Consumer prefers basket B. Thus B >A
Cost of basket B, \$3×2 + \$3×6 = \$24 $\rightarrow \tilde{P}_x X_2 + \tilde{P}_y Y_2$
Cost of basket A, \$3×5 + \$3×2 = \$21 $\rightarrow \tilde{P}_x X_1 + \tilde{P}_y Y_1$

We need $P_x X_1 + P_y Y_1 \geq P_x X_2 + P_y Y_2$ (\$24 > \$20), and…

$\tilde{P}_x X_2 + \tilde{P}_y Y_2 \leq \tilde{P}_x X_1 + \tilde{P}_y Y_1$ (but in this case we obtain $(\$24 \not\leq \$21)$.

Hence, he is NOT maximizing utility

- Let us try a few more…



- Case 1:
  - (1) C>A since it is to the northeast of A
  - (2) B ≥C since B was chosen when C was affordable

  By Transitivity $B \geq C > A \rightarrow B > A$ — B >A
- Case 2:
  - (1) BL_2 : From above, we know that B >A since A is inside BL_2 (Affordable Under BL_2)
  - (2) BL_1: Additionally, A ≥B since both were affordable when facing BL_1
    - Contradiction, not utility maximizing behavior
- Case 3:
  - (1) BL_1 : the consumer chose A when both A and B were affordable, A ≥B
  - (2) BL_2 : the consumer chose B when A wasn't affordable

  Ranking $A \geq B$ is all we can say
- Case 4:
  - (1) BL_1 : chose A but B wasn't affordable
  - (2) BL_2 : chose B but A wasn't affordable

  We can't infer anything from these choices because we don't know which bundle the consumer would choose if both were affordable