

Chapter 3

Consumer Preference and The Concept Of Utility

Representation of Preferences

- Bundle/basket**- a combination of goods and services that an individual might consume.
Example: Bundle A = (60, 30) contains 60 units of food and 30 units of clothing.
- Consumer preferences:** an indication of how a consumer would rank the desirability of baskets.

Consumer Preference-Assumptions

- 1) Preferences are complete:**
 - Ability to compare, either $A > B$ or $B > A$, or $A \sim B$ ("A is indifferent to B")
 - Preferences must state that either the consumer prefers one basket to another or he is indifferent.
- 2) Preferences are transitive:**
 - For three bundles A, B, and C, if $A > B$ and $B > C$, then $A > C$.
 - If we allow for non-transitive preferences, we can show that they must cycle, thus being prone to exploitation.

Consumer Preference - Assumptions

- Example of non-transitive preferences:**
 - "I would rather have an orange than a banana. And I would rather have a banana than an apple. However, (puzzlingly) I would rather have an apple than an orange."

1 st Orange > Banana	2 nd Banana > Apple	3 rd Apple > Orange
Orange - \$1	Banana - \$1	Apple - \$1

- But now the intransitive consumer is at the original position (owning an orange), and we can start the money pump again.
- Individuals with intransitive preferences would then be wiped out of the market.

Consumer Preferences-Assumptions

- **3) More is better (also referred to as “nonsatiation”)**
 - A consumer should prefer two hot dogs to one, and three hot dogs to two.
 - This assumption helps rank some of the bundles in the previous figure.

$$A > E > H$$

$$A > B > J \quad \text{And} \quad A > G > D$$

- But it doesn't help us rank other bundles:
 $G > E$? We don't know.

Ranking Systems

- **Ordinal Ranking**
 - allows us to gather information about the *order* in which a consumer ranks a bundle. We can know that a consumer prefers basket A to B but not *how much more* we prefers A to B.
 - We only need ordinal ranking in order to develop all our analysis of consumer theory.
- **Cardinal Ranking**
 - allows us to answer the *intensity* at which a consumer prefers one bundle to another. For example, I like basket A twice as much as basket B. It is usually hard for consumers to articulate their intensity.

Utility Functions

- It measures the level of satisfaction from consuming different bundles. It represents the consumer's preferences. The unit of measurement is Utils.
- We can think of Utils as a measure of a consumer's happiness in consuming a good, or a bundle of goods.

Utility Functions – Example

- **Example:** A utility function for a single good.
 - $u(y) = y^{1/2} = \sqrt{y}$
 - Does it meet the above 3 assumptions?

Utility Functions - Example

- Completeness**

For any two bundles A and B, we must have that either $u(A) \geq u(B)$, or $u(A) \leq u(B)$, or $u(A) = u(B)$, which indicates that either $A > B$, $A < B$, or $A \sim B$.

- Hence, Completeness is satisfied since for any two bundles x and z, either
 - $u(x) = \sqrt{x} > \sqrt{z} = u(z)$, or
 - $u(x) = \sqrt{x} < \sqrt{z} = u(z)$, or
 - $u(x) = \sqrt{x} = \sqrt{z} = u(z)$.

Example:

A	$y_a = \$4$	$u(4) = \sqrt{4} = 2$
B	$y_b = \$9$	$u(9) = \sqrt{9} = 3$

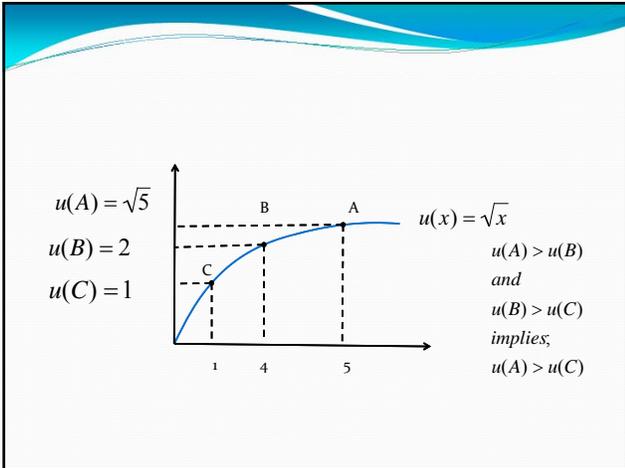
Utility Functions – Example

- Transitivity**

For example, for three bundles
 $C=1, B=4, A=5$, then $u(C) = (1)^{1/2} = 1$
 $u(B) = (4)^{1/2} = 2$
 $u(A) = (5)^{1/2} = 2.23$

Thus if $u(A) > u(B)$ and $u(B) > u(C)$, then we have that $u(A) > u(C)$.
 Therefore the preferences are transitive because:
 if $A > B$ and $B > C$, then $A > C$.

Note that this property holds for any three bundles (numbers, since we are talking about quantities of a single good) a, b, and c. In particular, if $\sqrt{a} > \sqrt{b}$ and $\sqrt{b} > \sqrt{c}$ then it must be that $\sqrt{a} > \sqrt{c}$.



Utility Functions - Example

- Non-satiation (more is better)**

Indeed it is satisfied since $u(B) > u(A)$ for any two bundles in which B is larger than A

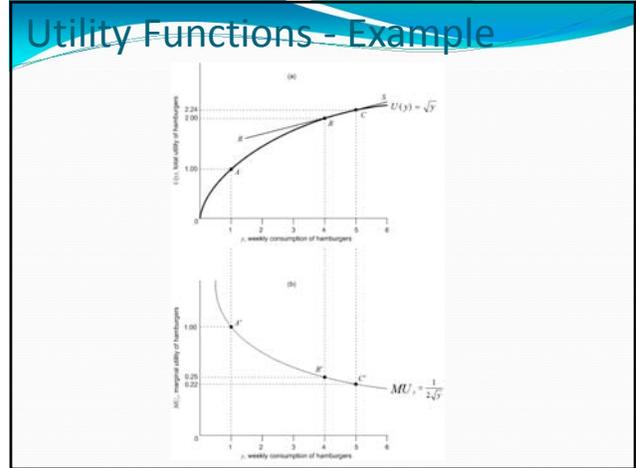
Example:
 $A = 1 \text{ unit} \rightarrow B = 2 \text{ units}$
 $U(1) = \sqrt{1} = 1 \rightarrow u(2) = \sqrt{2} > 1$.

Utility Functions

- **Marginal Utility**- the rate at which utility changes as consumption of a good increases.
 - *Interpretation*: how much better off we would be if we received one more unit of good y?
 - In mathematical terms, it is the first derivative of the Utility Function with respect to good y.

$$MU_y = \frac{\Delta u}{\Delta y} = \frac{\partial u(y)}{\partial y}$$

- That is, the slope of the utility function, i.e., its rate of growth as we consume more units of the good.



Utility Functions-Example

- Mathematical Example

$$\begin{aligned} \text{If } u(y) &= \sqrt{y}, \\ \text{then } \frac{\partial u}{\partial y} &= \frac{1}{2}y^{\frac{1}{2}-1} = \frac{1}{2}y^{-\frac{1}{2}} \\ &= \frac{1}{2}\left(\frac{1}{\sqrt{y}}\right) \\ &= \frac{1}{2\sqrt{y}} \end{aligned}$$
- As illustrated in the figure $u(y) = \sqrt{y}$ is increasing in y, but its marginal utility, $\frac{\partial u}{\partial y} = \frac{1}{2\sqrt{y}}$ is decreasing in y.

Utility Functions

- Three Important Points when drawing Total and Marginal Utility Graphs:
 - 1) Total Utility and marginal utility cannot be plotted on the same graph, because the variable in the vertical axis differs on each graph.
 - 2) The marginal utility function is the slope of the total utility function.
 - 3) The relationship between total and marginal utility holds for other measures in economics that will be discussed later on in the semester (such as total and marginal product for a firm).

Utility Functions

- *Diminishing Marginal Utility*: additional units add less utility (ΔU smaller and smaller)
- Indeed, marginal utility $\frac{\partial U}{\partial y} = \frac{1}{2\sqrt{y}}$ is decreasing in y .
- Nonetheless, we will assume that $\Delta U > 0$ always.
 - That is, more is always better, although the last unit consumed is not as good as the first unit consumed (Think about your first glass of water on a hot day and your fourth glass of water).

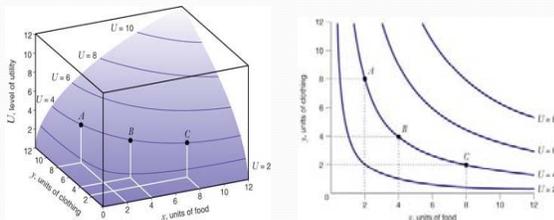
Utility Functions

- Utility Functions with only one good are very restrictive.
 - We seek to generalize our analysis to more than one product.
- Utility functions with more than one good, e.g.

$$u(x, y) = \sqrt{xy}$$

Utility Functions

- Utility functions with more than one good, e.g. $u(x, y) = \sqrt{xy}$



- Point A, $(x,y)=(2,8)$ provides the same utility as Point B $(x,y)=(4,4)$, since they both reach the same height.

Utility Functions

- Utility functions with more than one good.
 - Mathematical Example:

$$U(x, y) = \sqrt{xy} = \sqrt{x} * \sqrt{y}$$

- Let us now find the marginal utilities

Amount of y is fixed \rightarrow $MU_x = \frac{\partial U(x, y)}{\partial x} = \frac{1}{2\sqrt{x}} \sqrt{y}$

Amount of x is fixed \rightarrow $MU_y = \frac{\partial U(x, y)}{\partial y} = \frac{1}{2\sqrt{y}} \sqrt{x}$

Utility Functions-Exercise

- **Learning by doing 3.1**
 - $U(x,y) = \sqrt{xy}$
 - Satisfies more is better?
 - Satisfies diminishing marginal utility?
- **More is better** must be checked in two different ways:
 - Increase in $x \rightarrow$ increase in U
 - Increase in $y \rightarrow$ increase in U
- Alternatively, we can check if $MU_x > 0$ and $MU_y > 0$ for all $x,y > 0$. This property is satisfied since:
 - $MU_x = \frac{1}{2\sqrt{x}}\sqrt{y}$ is positive for any amount of good x and y
 - Similarly, $MU_y = \frac{1}{2\sqrt{y}}\sqrt{x}$ is positive for any amount of good x and y

Utility Functions – Exercise

- **Diminishing marginal utility** just tells us that the additional utility we get from consuming further amounts of goods is smaller and smaller.
- The additional utility we get from consuming additional goods is MU_x and MU_y . So we need to show:
 - MU_x is decreasing in x (it turns out to be true)
 - MU_y is decreasing in y (it turns out to be true)
 - Mathematically, in order to see whether MU_x is decreasing in x , we need to check the second derivative of the utility function with respect to x . That is:
 - $MU_{xx} < 0$ and $MU_{yy} < 0$ for all $x,y > 0$

$$MU_{xx} = \frac{\partial}{\partial x} \left(\frac{1}{2\sqrt{y}}\sqrt{x} \right)$$

$$= \frac{-\sqrt{y}}{4x^{3/2}} < 0 \quad \text{for all } x \text{ and } y$$

$$MU_{yy} = \frac{\partial}{\partial y} \left(\frac{1}{2\sqrt{x}}\sqrt{y} \right)$$

$$= \frac{-\sqrt{x}}{4y^{3/2}} < 0 \quad \text{for all } x \text{ and } y$$

Query #1

- Consider the utility function $U = \min (3x^2 + 7y^3)$.
- Does good x satisfy the diminishing marginal utility property?

A. Yes
 B. No
 C. Depending on the number of units of good y that the individual consumes.

Query #1- Answer

- MU_x is $\frac{dU}{dx} = 6x$ which increases in x
- Indeed, $\frac{dMU_x}{dx} = \frac{d(6x)}{dx} = 6 > 0$
- Hence, $u(x,y) = 3x^2 + 7y^3$ does not satisfy the diminishing marginal utility property for good x .
- As a practice, show that $u(x,y)$ doesn't satisfy it for good y either.

- What about a different utility function, such as $u(x,y) = 3x^2 + 7y^3$?
- Then, $MU_x = 6x5y^3$, which is still increasing in x .
- Therefore, diminishing marginal utility doesn't hold in this case either.

Indifference Curves

- **Indifference Curves:** curves connecting consumption bundles that yield the same level of utility.

Indifference Curves

- For utility function $u(x, y) = \sqrt{xy}$ bundles A, B and C yield the same utility level (and thus lie on the same indifference curve).
- Graphically, all three bundles help a consumer reach the same height in his “utility mountain” (i.e., the 3D representation of his utility function we depicted a few slides ago).
- In particular,
 - $u(8,2) = \sqrt{8 * 2} = \sqrt{16} = 4$ for bundle A,
 - $u(4,4) = \sqrt{4 * 4} = \sqrt{16} = 4$ for bundle B,
 - $u(2,8) = \sqrt{2 * 8} = \sqrt{16} = 4$ for bundle C.

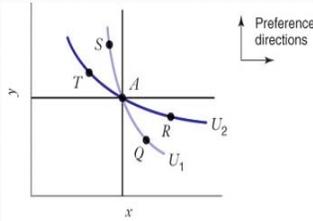
Indifference Curves – Properties

- **4 properties of indifference curves (1 of 4):**
 1. When the consumer likes both goods ($MU_x > 0, MU_y > 0$), indifference curves are negatively sloped

- Indeed, by the “move is better” assumption (i.e., $MU_x > 0$ and $MU_y > 0$), points to the northeast of A must be strictly better, while points to the southwest of A must be strictly worse. Hence, none of the shaded areas can lie on the same indifference curve as Point A
- As a consequence, the indifference curve of bundles yielding the same utility as bundle A must lie on the unshaded areas. Thus, indifference curves must be negatively sloped.

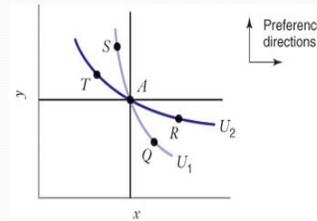
Indifference Curves – Properties

- 4 properties of indifference curves (2 of 4):
 1. Indifference curves cannot intersect.
 2. If they did (as in the figure) then $R > Q$ (implying that $U_2 > U_1$), but $S > T$ (implying that $U_1 > U_2$), which leads to a contradiction.



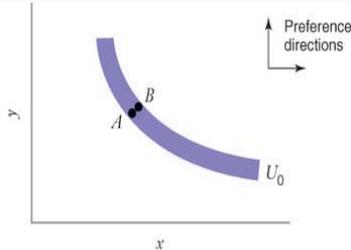
Indifference Curves – Properties

- 4 properties of indifference curves (3 of 4):
 1. Every consumption bundle lies on one and only one IC.
 2. This property follows from the previous. (We cannot have points like A, which lies on two different I.C.s and thus would have two different utility values)



Indifference Curves – Properties

- 4 properties of indifference curves (4 of 4):
 1. ICs are not thick.
 2. Otherwise, bundle B would yield the same utility level as A, which cannot be since B is to the northeast, containing more units of both x and y, thus it must be strictly preferred to A (by the “more is better” assumption).

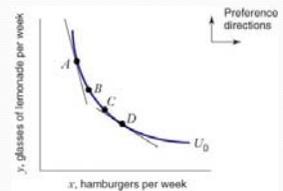


Indifference Curves

- **Marginal Rate of Substitution:** The rate at which a consumer will give up one good (y) to get one additional units of another good (x), keeping his or her utility level constant.

- $MRS_{xy} = -\text{slope of IC} = \frac{MU_x}{MU_y}$

- Where is this expression of $MRS_{x,y}$ coming from?
 - (Proof on the next slide)



MRS of $u(x,y)$

- First, note that we need to totally differentiate with respect to x and y (indeed, when moving rightward along an indifference curve, we are simultaneously varying x and y , i.e., lowering y in order to get more of x).
- Totally, differentiating, we obtain

$$\Delta u = MU_x \Delta x + MU_y \Delta y \rightarrow \Delta u = \partial u / \partial x \cdot \Delta x + \partial u / \partial y \cdot \Delta y$$

Since we move along the same indifference utility is unaffected, i.e., $\Delta u = 0$

$$0 = MU_x \Delta x + MU_y \Delta y$$

Rearranging,

$$-MU_x \cdot \Delta x = MU_y \cdot \Delta y$$

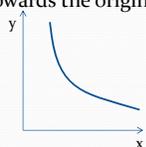
$$-\frac{MU_x}{MU_y} \frac{\Delta y}{\Delta x} = \text{slope of IC}$$

$$MRS_{xy} = -\text{slope of IC}$$

MRS of $u(x,y)$

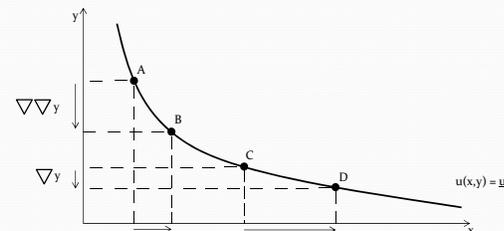
- Diminishing MRS:**
- Initially you are willing to give up many glasses of lemonade (y) for an additional hamburger (x).
- However, when you have few glasses of lemonade and many hamburgers you will not be willing to give up as many (or none) glasses of lemonade in order to get an additional hamburger.

That is, IC's are bowed in towards the origin.



Decreasing slope of I.C.
Decreasing $MRS_{x,y}$

MRS of $u(x,y)$



- The same increase in one hamburger (good x) leads the consumer to give up many glasses of lemonade when he is at bundle A (consuming a lot of lemonade), but to give up only a few glasses of lemonade when he is at point C (when he has few glasses of lemonade).

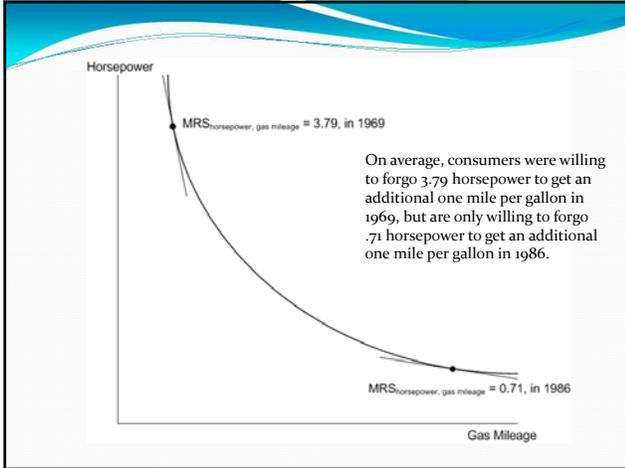
Indifference Curve - Application

- Demand for attributes in cars**

$$U(x,y) = \sqrt{xy}$$
 where x is gas mileage and y is horse power

$MRS_{x,y}$ in this context represents how a typical consumer is willing to forgo horsepower (y) in order to get one more unit of gas mileage (x).

In 1969, $MRS_{x,y} = 3.79$, while in 1986, such $MRS_{x,y}$ dropped to 0.71 showing that people are less willing to give up horsepower (y) in order to get additional units of miles per gallon (x)



Indifference Curves - Exercise

- Learning by doing 3.3**

$$U(x, y) = xy$$

$$MU_x = \frac{\partial U(x, y)}{\partial x} = y$$

$$MU_y = \frac{\partial U(x, y)}{\partial y} = x$$

a) Draw the IC corresponding to utility level of $U_1 = 128$
 $xy = 128$ in many of its combinations

For example,
 G: $x = 8, y = 16$
 H: $x = 16, y = 8$
 I: $x = 32, y = 4$

Generally, the trick is

$x \cdot y = 128 \rightarrow y = 128/x \rightarrow y = 128/8 = 16$	Point G
$y = 128/16 = 8$	Point H
$y = 128/32 = 4$	Point I

Indifference Curves – Exercise

$u(x, y) = x * y$

b) Does I.C. intersect either axis?

No, if IC intersected x-axis, then at the crossing point, $y = 0$, utility level would be $U_1 = 0 \neq 128$. Similarly, if IC intersected y-axis, then at the crossing point, $x = 0$, utility level would be $U_1 = 0 \neq 128$

Indifference Curves – Exercise

$u(x, y) = x * y$

c) Does IC indicate that $MRS_{x,y}$ is diminishing?

Yes, since IC is bowed in toward the origin.

Note that: $MRS_{xy} = \frac{MU_y}{MU_x} = \frac{y}{x}$
 so as we move from left to the right (Δx), we get smaller ratios, \rightarrow smaller $MRS_{x,y}$ (diminishing MRS), i.e., the indifference curve becomes flatter.

Query #2

Consider the utility function $U = 5x + 3y$, which has the following marginal utilities $MU_x = 5$ and $MU_y = 3$. The indifference curves for this utility function:

- A. Will have a *diminishing* marginal rate of substitution of x for y as x increases.
- B. Will have a *constant* marginal rate of substitution of x for y as x increases.
- C. Will have an *increasing* marginal rate of substitution of x for y as x increases.

Query #2 - Answer

- Answer B
- Taking utility function $U(x,y) = 5x + 3y$, and finding the $MU_x = 5$ and $MU_y = 3$, we obtain the MRS between goods x and y is $5/3$.
- Hence, the MRS is constant in x
- Hence, answer B is correct.

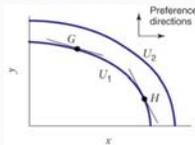
Example of an Increasing MRS_{x,y}

- $U(x,y) = Ax^2 + By^2$

In this case, $MU_x = 2Ax$ and $MU_y = 2By$

Hence,

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{2Ax}{2By} = \frac{Ax}{By}, \text{ which increases in } x$$



Unlike in previous examples, these indifference curves are bowed away from the origin

Special Utility Functions-Perfect Substitutes

- **Perfect Substitutes:** Aquafina versus Dasani bottled water; or Kingston versus Scandisk memory sticks.
 - Close substitutes: butter versus margarine; or coffee versus black tea.
- $MRS_{B,M} = MRS_{M,B} = \text{constant}$
- **Example: Butter (B) and Margarine (M)**
- $U = aB + bM$, then $MU_B = a$ and $MU_M = b$
- Hence, $MRS_{B,M} = \frac{MU_B}{MU_M} = \frac{a}{b}$
- Then, slope of IC = $-\frac{a}{b}$ (constant, i.e., independent in x)
- Note that if $u = aB + aM$, so $MU_b = a$ and $MU_m = a$, Then $MRS_{b,m} = a/a = 1$

Special Utility Functions - Perfect Substitutes

- Example:** Pancakes and Waffles
 - $U = P + 2W$, which implies $MU_P = 1$ and $MU_W = 2$.
 - A waffle gives him double utility of a pancake.
 - The consumer is always willing to trade 2 pancakes for 1 waffle.

$$MRS_{P,W} = \frac{MU_P}{MU_W} = \frac{1}{2}$$

- slope of IC = -0.5 (It doesn't change in the amount of pancakes)
- The slope of the indifference curve is constant \rightarrow I.C. are straight lines
- Hence, the MRS is also constant

Special Utility Functions - Perfect Complements

- Perfect Complements:** left shoe and right shoe (consumer wants them in fixed proportions), cars and gasoline, or two scoops of peanut butter to one scoop of jelly for the perfect PB&J sandwich.

- $U = 10 \times \min(R,L)$
- At G, we have that $R=2$ and $L=2$, then $U_2 = 10 \times \min(2,2) = 10 \times 2 = 20$
- At H, we have that $R=3$ and $L=2$, then $U_2 = 10 \times \min(3,2) = 10 \times 2 = 20$
- Consumption choices should be allocated at the kink (Why spend money on additional units of R if they won't increase your utility at all?)

Special Utility Functions - Perfect Complements

- More generally, utility functions for goods that are perfect complements look like:

$$u(x_1, x_2) = A \cdot \min(a x_1, b x_2),$$

where $A, a, b > 0$ are all positive constants.

Special Utility Functions - Perfect Complements

- Slope of IC (MRS) is:
 - Zero at the flat segment of the IC
 - Infinity at the vertical segment of the IC
 - Undefined at the kink of the IC (infinitely many slopes)

Query #3

Identify the truthfulness of the following statements:

- I. Diminishing marginal utility and increasing total utility are incompatible with each other.
 - II. When Marginal utility is negative, total utility is decreasing.
- a) Both I and II are true
 - b) Both I and II are false
 - c) I is true; II is false
 - d) I is false; II is true

Query #3 – Answer

- Answer D
- Statement I is *false*, because *total utility can increase* and yet experience diminishing marginal utility, i.e., in the increasing, but concave, portion of the total utility, additional increments of utility become smaller and smaller (diminishing MU), but total utility is still increasing.
- Statement II is *true*, since for marginal utility to become negative, it must be that additional consumption actually reduces your total utility, i.e., total utility is decreasing.
- For a graphical representation of total and marginal utility, see slide 13 of this chapter.

Special Utility Functions – Cobb Douglas

- **Cobb-Douglas Utility function**
 - $U(x,y) = \sqrt{xy}$ and $U = xy$ are examples of Cobb-Douglas utility functions
 - Generally, Cobb-Douglas utility functions look like
 - $U(x,y) = Ax^\alpha y^\beta$ where parameters $A, \alpha, \beta > 0$ are all positive constants.
 - if $\alpha = \beta = 1/2$, then $U = Ax^{1/2}y^{1/2} = A(xy)^{1/2}$
 - if $\alpha = \beta = 1$ and $A = 1$, then $U = xy$

Special Utility Functions – Cobb Douglas

- The Douglas utility function $u(x,y) = Ax^\alpha y^\beta$ satisfies 3 properties:

1) $MU_x > 0$ and $MU_y > 0$. That is, “more is better” property holds

$$MU_x = \frac{\partial U(x,y)}{\partial x} = A\alpha x^{\alpha-1}y^\beta > 0 \text{ for any } x \text{ and } y$$

$$MU_y = \frac{\partial U(x,y)}{\partial y} = A\beta x^\alpha y^{\beta-1} > 0 \text{ for any } y \text{ and } x$$

Special Utility Functions - Cobb-Douglas

2. Since $MU_x > 0$ and $MU_y > 0$, then IC is downward sloping (recall property 1 of ICs)
 [what about the case in which the consumer dislikes some goods, $MU_x < 0$? You will see that in the Review Session]

3. Diminishing $MRS_{x,y}$:

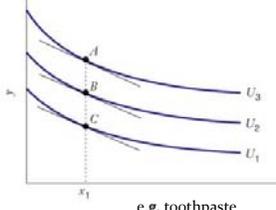
$$MRS_{x,y} = \frac{A\alpha x^{\alpha-1} y^\beta}{A\beta x^\alpha y^{\beta-1}} = \frac{\alpha}{\beta} * x^{\alpha-1-\alpha} * y^{\beta-(\beta-1)}$$

$$= \frac{\alpha}{\beta} * \frac{1}{x} * y = \frac{\alpha}{\beta} * \frac{y}{x}$$

Then movements from left to right (Δx) will shrink this ratio, and reduce $MRS_{x,y} \rightarrow$ Diminishing $MRS_{x,y}$ (Flatter I.C.)

Special Utility Functions - Quasilinear Utility

- **Quasi-linear:** Used in economic applications for situations in which the amount of a commodity (such as toothpaste or garlic) doesn't change very much with income.
- ICs are parallel displacements to each other
- Same slope, same MRS between goods, even if income changes.



e.g. toothpaste
garlic etc.

Special Utility Functions - Quasilinear Utility

- **General Form of Quasilinear utility:** $U(x,y) = v(x) + by$
 - (linear in y, but not linear in x)
 - where $v(x)$ is a function that increases in x,
 - such as $v(x) = (x)^{1/2}$ or $v(x) = x^2$.
 - and where $b > 0$ is a positive constant.
- If we find the $MRS_{x,y}$, we obtain

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{v'(x)}{b}$$

- That is, for a given value of x, the $MRS_{x,y}$ is constant in y.

Special Utility Functions - Quasilinear Utility

- **General Form of Quasilinear utility:** $U(x,y) = v(x) + by$
 - (Example: $U(x,y) = x^2 + 3y$. In this case, $MRS_{x,y}$ is

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{2x}{3}$$

- For a given $x=5$, $MRS_{x,y} = 10/3$, which is constant in y (as depicted in the previous figure).

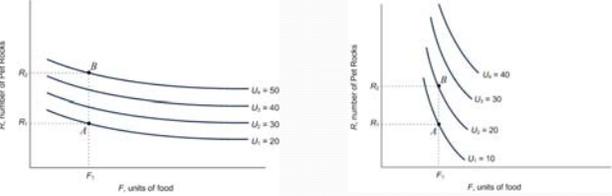
Special Utility Functions - Quasilinear Utility

Pet Rock Fad as an example of quasilinear utility.
 In case you didn't remember what *Pet Rocks* are...
 surprisingly, they were famous a few years ago.



Special Utility Functions - Quasilinear Utility

Pet Rock Fad Application: observe the differences in the graphs.



Pet Rock Fad: all additional income is spent on pet rocks.

The fad is almost over: additional income is spent on both goods.