Chapter 2

Demand and Supply

Overview

- In chapter 2, we deal with demand and supply analysis in perfectly competitive markets.
  - Perfectly competitive markets consist of a large number of buyers and sellers.
  - The transactions of any individual buyer or seller are so small, in comparison to the overall volume of the good or the service traded in the market, that the buyer or seller has no choice but to takes the price set by the market.

Demand Curves

- **Market Demand Curve:** A curve that shows us the quantity of good that consumers are willing to buy at different prices.
- **Law of Demand**: The negative relationship between the price of a good and the quantity demanded, when all other factors that influence demand are held fixed.

As illustrated in the following graph…

- The direct demand curve will generally take the linear form…

Q = a – bP

where 'a' = vertical intercept, and 'b' = slope

Rearranging and solving for P, we get…

$$P = \frac{a}{b} - \frac{Q}{b}$$

Vertical intercept: $\frac{a}{b}$; slope: $-\frac{1}{b} * Q$

This is the ***Inverse Demand Curve***

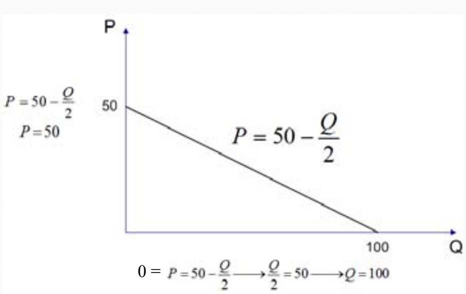
Example

- Direct Demand Curve: **Q = 100 -2P**

  Q -100 = -2p
  2P =100 - Q

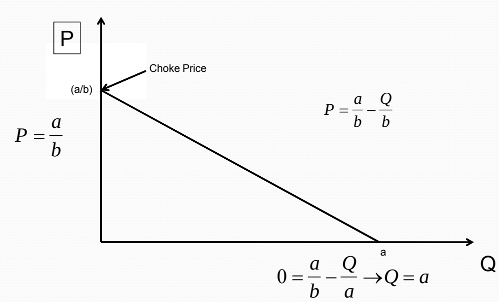
- Inverse Demand hence becomes: $P = 50 - \frac{Q}{2}$
  - The "Choke Price" is the price at which Q=0, or simply put, at what price consumers demand 0 units of the good. Setting Q=0, the "**Choke Price**" = **50**

And so, the graph looks like a straight line . . .



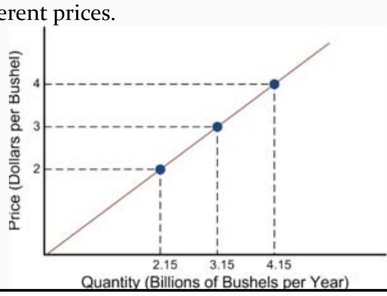
- **Or more generally . . .**

Demand: Q = a – bP, then Inverse Demand : $P = \frac{a}{b} - \frac{Q}{b}$



Supply Curve

- **Market Supply Curve:** A curve that shows us the total quantity of goods that their suppliers are willing to sell at different prices.



Example

Linear Supply Curve: $\mathbf{Q^S = 0.15 + P}$

- Find the quantity of wheat supplied if…

$P = \$2 \rightarrow Q^S = .15 + 2 = 2.15$

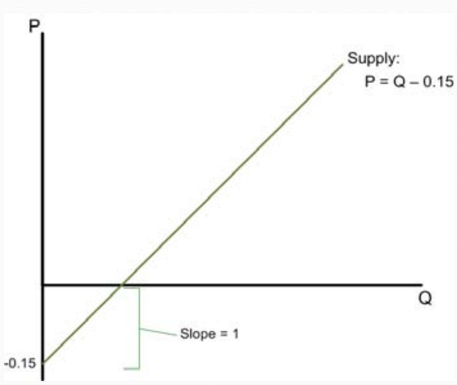
$P = \$3 \rightarrow Q^S = .15 + 3 = 3.15$

- Let us sketch this supply curve…

$Q^S = 0.15 + P$ and solving for P, we get the inverse supply curve $\mathbf{P = Q^S - 0.15}$

  - So the slope = 1 (coefficient of Q^S)
  - Intercept = -0.15

The following figure illustrates this supply curve…



Equilibrium

- In equilibrium, a perfectly competitive market will set a price and quantity such that there is no excess supply and no excess demand, hence demand equals supply.

$$Q^s = Q^d$$

- If there is excess supply Qs > Qd → prices should go down
- If there is excess demand Qs< Qd → prices should go up

Example

Demand Curve: $Q^d = 500 - 4P$

Solving for P:
$4p = 500 - Q^d$
$p = 500/4 - Q^d/4$

Supply Curve: $Q^S = -100 + 2P$

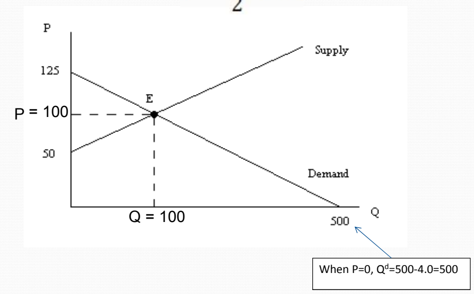
Solving for P:
$2p = Q^s + 100$
$p = Q^s/2 + 50$

- Before finding the equilibrium output and price level…

Let us sketch these curves on the same graph with quantity on the horizontal axis and price on the vertical axis.

Inverse Demand Curve $\rightarrow P = \frac{500}{4} - \frac{Q^d}{4}$

Inverse Supply Curve $\rightarrow P = \frac{Q^S}{2} + 50$



- At what price and quantity do you reach equilibrium?

$$Q^S = Q^d$$
$$500 - 4P = -100 + 2P$$
$$600 = 6P$$
$$100 = P$$

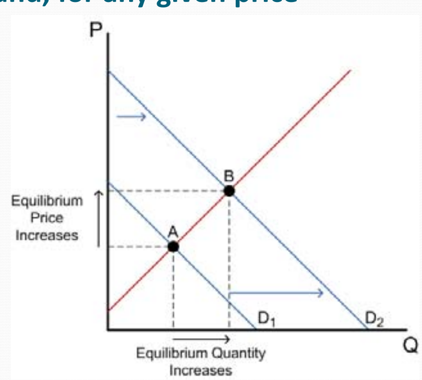
- And then take this p=100 and plug it into either the demand or supply curve to find the equilibrium quantity…

P=100

$$Q^S = 500 - 4(100) = 100$$

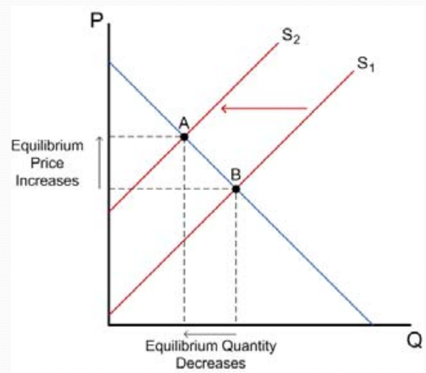
And so, equilibrium occurs at P=100 and Q=100

Comparative Statics: An Increase in Demand, for any given price



- An increase in demand as the one depicted above can originate from an increase in income, or in the consumer's preference for the good. For any given price, the quantity that consumers demand has now gone up.
  - You can visually see that by extending a long horizontal dotted line which maintains your focus on a given (fixed price). The point where the dotted line crosses each demand curve represents the quantity demanded.

A decrease in supply, for any given price



- A decrease in supply might originate from an increase in production costs, which lead producers of the good to supply lower amounts of the good at any given price. [Follow similar graphical representation as above]
  - Extend a horizontal dotted line at the price p=$10. The quantity supplied is lower after the increase in production costs (S_2) than before the increase (S_1).

Example: Market for Aluminum

Demand: $Q^d = 500 - 50P + 10I$ where P=price and I=income

Supply: $Q^S = -400 + 50P$

- Let's analyze the equilibrium when income is I=10 and how it is affected when income decreases to I=5
- **First, Equilibrium when I = 10…**

Plug in I=10 into Q^d to get $Q^d = 500 - 50P + 10(10) =$ **$600 - 50P$**

- Equating Q^d=Q^S we obtain…

$600\text{-}5p = -400\text{-}50p$

$1000 = 100p$

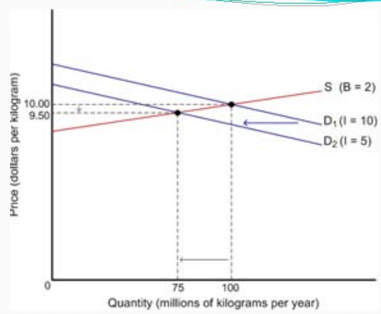
Or $p = \$10 \rightarrow Q^S = -400 + 50(\$10) = 100$

- **Second, Equilibrium when I = 5…**

Plug I = 5 into Q^d to get
$Q^d = 500 - 50P + 10(5) = 550 - 50P$

- *Now equate Q^d to Q^S,*

Qd Qs
$550 - 50P = -400 + 50P$
$950 + 100P$
$\mathbf{9.5 = P \rightarrow Q^S} = -400 + 50(9.5) = 75$



- (1) Demand shifts inward,
- (2) equilibrium output decreases from 100 to 75,
- (3) equilibrium price decreases from \$10 to \$9.50

Query #1

- Consider the demand Curve

$Q^d = 1000 - 20P - 6r$

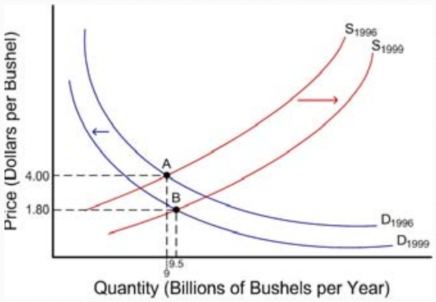
- If the value of r falls, the demand curve will:

a) Shift to the left
b) Shift to the right
c) Remain unchanged
d) Rotate along the quantity axis

Answer

- B, shift to the right
- In particular, differentiating the demand function with respect to r,
- $\partial Q^d/\partial r = -6 < 0$
  - Hence Δr produces a reduction in Q^d and vice versa, a decrease in r produces an increase in Q^d

A change in both the demand and supply curve



Interpretation…

- A *decrease* in demand and an *increase* in supply:
  - Unambiguously produce a reduction in the equilibrium price; but…
  - the effect on the equilibrium quantity is more ambiguous. In the figure, the outward shift in the supply curve dominates the inward shift in the demand curve, producing an overall increase in the equilibrium quantity. Otherwise, the equilibrium quantity would decrease.

Can you draw a figure in which Demand increases and Supply decreases, but equilibrium output decreases?

Price Elasticity of Demand

Now that we know how to construct a demand curve, to what extent does a change in one variable affect quantity demanded?

- **Price Elasticity of Demand:** A measure of the percentage change of quantity demanded as a result of a 1% increase in price, holding all other determinants of demand constant.

- We can express the elasticity of demand as…

$$\varepsilon_{Q,P} = \frac{\%\ change\ in\ Q}{\%\ change\ in\ P} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

$\frac{\Delta Q}{\Delta P} \rightarrow \frac{\partial Q}{\partial P}$

where $\frac{\Delta Q}{Q} = \frac{Q_1 - Q_0}{Q_0}$ and $\frac{\Delta P}{P} = \frac{P_1 - P_0}{P_0}$

- Notice that the first term $\frac{\Delta Q}{\Delta P}$ is simply the derivative of quantity demanded with respect to P.

Let's look at an example…

Example

- Suppose that we have a price change that results in the following changes in quantity demanded…
  - When P=10, quantity is Q=50
  - When P=12, quantity falls to Q=45 (So ΔQ = -5)

What is the elasticity of demand?

$$\varepsilon_{Q,P} = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} = \frac{-5}{2} \times \frac{10}{50} = -0.5$$

Δ1% in P→ Δ0.5% in Q (less-than-proportional decrease)

Why is $\varepsilon_{Q,P}$ always negative?

$$\varepsilon_{Q,P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

- Where $\frac{\Delta Q}{\Delta P}$ represents the slope of the demand curve, which is negative by the *Law of Demand*.
- Where $\frac{P}{Q} > 0$ since both p>o and Q>o.

Hence, the product (-)•(+) must be negative, making the elasticity $\varepsilon_{Q,p}$ a negative number.

Example

- Let's look at another example of elasticity… Demand curve **Q = 100 – 2(P)**

Let's find the elasticities when . . .

- A) P=40, so Q= 20
- B) P=25, so Q= 50
- C) P=10, so Q= 80

- Remember that our demand curve is in the form **Q = a – bP**, and that the first term of our elasticity equation is simply the derivative with respect to P (i.e., -b) so our elasticity equation is…

$$Ed = \frac{\Delta Q}{\Delta P} * \frac{P}{Q} \quad \frac{\partial Q}{\partial P} = -2 \qquad \varepsilon_d = -b\frac{P}{Q}$$

- So now, we simply plug in the particular values of P and Q to find the elasticity for each price.

A) When P=40…

$$\varepsilon_d = -2 \cdot \frac{40}{20} = -4$$

Constant slope (-2), but not constant elasticity ε_d

B) When P=25…

$$\varepsilon_d = -2 \cdot \frac{25}{50} = -1$$

C) When P=10…

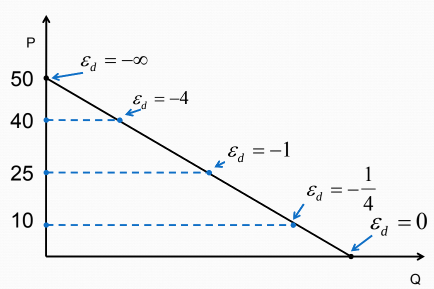
$$\varepsilon_d = -2 \cdot \frac{10}{80} = -\frac{1}{4}$$

- What about the elasticities for this demand curve at the vertical and horizontal intercepts?
  - **Vertical Intercept**

$$\varepsilon_d = -2 \cdot \frac{50}{0} = -\infty$$

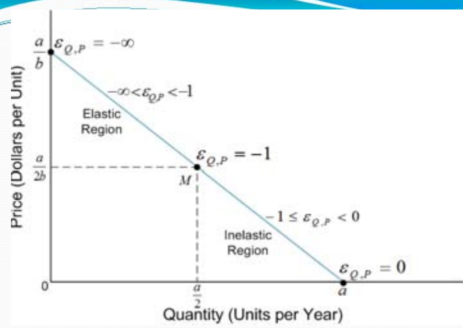
  - **Horizontal Intercept**

$$\varepsilon_d = -2 \cdot \frac{0}{100} = 0$$



Elasticity of demand in the linear demand curve, Q = a-bP

- We can actually summarize where a linear demand will be…
  - (1) Elastic, when |ε|>1
  - (2) Unit Elastic, when |ε|=1
  - (3) Inelastic, when |ε|<1



Remember, *elasticity of demand does not equal the slope. That is, a linear demand curve (constant slope) can have different elasticities, as we just showed.*

Why not use slope of the demand function, rather than $E_{Q, p}$?

- The reason we use price elasticity of demand (and not simply the slope of the demand curve) is because by using the former we can produce a unit-free measure of how sensitive is the demand curve to changes in prices.

- Indeed, note that units cancel out when you use the formula of price-elasticity of demand, but they wouldn't if you were simply using the slope of the demand curve.

$$\frac{\frac{\Delta Qd \text{ in tons}}{Q \text{ in tons}}}{\frac{\Delta P \text{ in US\$}}{P \text{ in US\$}}} = \frac{\frac{\Delta Qd}{Q}}{\frac{\Delta P}{P}}$$

A number

  - We need a unit-free measure of Q^d and Price to be able to compare the changes in one with the changes in the other.

- In this way we can also compare the sensitivity of demand to price changes across different goods:
  - Measured in different units (uranium versus steel)
  - Consumed in different countries (prices expressed in different currencies)
  - Etc…

Examples of price-elasticities of demand:

Cigarettes	-0.107
Pet food	-0.061
Air travel, leisure	-1.52
Air travel, business	-0.70

- **Intuition**: a 10% increase in the price of cigarettes produces a 1.07% reduction in the quantity demanded. Hence, we can claim that demand for cigarettes is relatively *insensitive* to price changes.

Constant Elasticity Demand Curve

- **Constant Elasticity Demand Curve:**
  - A demand curve of the form **Q = aP^{-b}** where 'a' and 'b' are positive constants.
  - The term –b is the price elasticity of demand along all points in this curve.

- Let us see why…
- First, recall the formula of price elasticity;

$$\varepsilon_d = \frac{dQ}{dP} \bullet \frac{P}{Q}$$

Constant Elasticity Demand Curve

We will just find $\frac{dQ}{dP}$, and then plug into the ε_d formula;

$$\frac{dQ}{dP} = -b \cdot ap^{-b-1} = -b \cdot ap^{-b} \cdot \frac{1}{p} = \frac{-b}{p} \cdot ap^{-b}$$

- Therefore,

$$\varepsilon_d = \frac{dQ}{dP} \cdot \frac{p}{Q} = \frac{-b}{p} \cdot ap^{-b} \cdot \frac{p}{ap^{-b}} = -b$$

$Q = a * p^{-b}$

Constant in P

Query #2

Consider the demand Curve

$Q^d = 5P^{-1}$

The elasticity of demand along this demand curve:

A. Is inelastic
B. Is elastic
C. Is unitary elastic
D. Falls as the price falls

Answer

- C, it is unitary elastic
- The elasticity of demand of a demand curve $Q^d = A\ p^{-b}$ is –b
- In this case, $Q^d = 5p^{-1}$, so the exponent -1 is the elasticity, which is constant in prices.

Different Types of Elasticities of Demand

- So far we have only compared how quantity demanded varies with price, in order to see how price sensitive certain commodities are.
- This analysis, however, can be extended to compare more than just quantity with price.

Let's extend this elasticity analysis to compare Q^d with other factors…

What about changes in income? How do they affect demand?

- **Income Elasticity of Demand:** The percentage change in quantity demanded resulting from a 1% increase in income, holding price and all other determinants of demand constant.

$$\varepsilon_{Q,I} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta I}{I}} \text{ and rearranging } \varepsilon_{Q,I} = \frac{\Delta Q}{\Delta I}\frac{I}{Q}$$

Examples of Income-elasticity of demand

- Apples = 1.32

  Δ1% in the one produces a 1.32% increase in the quantity of apples demanded

- Flour = -0.36

  Δ1% in the one produces a 0.36% decrease in the quantity of flour demanded

What about changes in the price of other goods?

For instance, how the change in the price of Pepsi change the quantity of Coke demanded?

- **Cross-Price Elasticity of Demand:** The percentage change of the quantity of one good demanded (good *i*) that results from a 1% increase in the price of another good (good *j*).

$$\varepsilon_{Q_i P_j} = \frac{\frac{\Delta Q_i}{Q_i}}{\frac{\Delta P_j}{P_j}} \text{ and rearranging } = \frac{\Delta Q_i}{\Delta P_j}\frac{P_j}{Q_i}$$

Example of Cross-Price elasticity of demand (Application 2.4 in textbook)

- Δ1% in the price of Nissan Sentra

produces a change in the demand of…

Ford Escort

BMW 735

Example of Cross-Price elasticity of demand (Application 2.4 in textbook)

- Δ1% in the price of Nissan Sentra produces a…
  - Δ0.454% in the quantity demanded of Ford Escort
  - Δ0% in the quantity demanded of Lexus or BMW 735
- Generally…
  - If $\varepsilon_{Qi,Pj}$ >0, goods i and j are regared as *substitutes*.
    - (i.e. two brands of mineral water, Nissan Sentra and Ford Escort)
  - If $\varepsilon_{Qi,Pj}$ <0, goods i and j are regarded as *complements.*
    - (i.e. left and right shoe, cars and gasoline, etc)

Application: Coke vs. Pepsi

Application: Coke vs. Pepsi

	Coke	Pepsi
Price Elasticity of Demand $\epsilon_{Q,P}$	-1.47	-1.55
Cross-Price Elasticity of Demand	0.52	0.64
Income Elasticity of Demand	0.58	1.38

Intuition…

- Therefore, a Δ1% in the P_{coke} produces…
  - ∇1.47% in the quantity demanded of Coke, but…
  - Δ0.64% in the quantity demanded of Pepsi

    (So people, on average, regard Coke and Pepsi as substitutes!)
- On the other hand, a Δ1% in the income level produces…
  - Δ0.58% in the quantity demanded of coke

(You can repeat a similar analysis starting with Δ1% in the price of Pepsi)

- As you might have guessed, we can even extend this analysis to how changes in prices affect the quantity supplied.
  - This is called the **Price Elasticity of Supply**.
  - It shows us how a 1% change in price affects the amount of the good supplied. Or, how price sensitive is supply?

$$\varepsilon_{Q^S,P} = \frac{\frac{\Delta Q^S}{Q^S}}{\frac{\Delta P}{P}} = \frac{\Delta Q^S}{\Delta P} \cdot \frac{P}{Q^S}$$

- ***Back of Envelope calculations about linear demand curves:***
- Now that we know how a demand curve is constructed and how to use its various parts to analyze the relationship between quantity demanded, quantity supplied, and price, *we can work backwards* to actually construct a demand curve.
- Consider that we know the prevailing Q, the P, and $\varepsilon_{Q,P}$. We want to find the value of *a* and *b* in ***Q=a-bp***, where…
  - 'a' (vertical intercept or choke price)
  - 'b' (slope of demand which we can derive from elasticity)

Q^d =a-bP

We know that, from $Q = a - bP, \frac{\partial Q}{\partial P} = -b$

- Let's first find parameter b...

$$\varepsilon_{Q,P} = -b \cdot \frac{P}{Q} \rightarrow So \ldots b = -\varepsilon_{Q,P} \frac{Q}{P}$$

- Now, let's find a...

$$Q = a - bp \rightarrow So\ldots$$

$$a = Q + bp$$

- And using the value of b we found above,

$$a = Q + bp \rightarrow a = Q + \left(-\varepsilon_{Q,P} \frac{Q}{P}\right) \cdot P$$

$$= Q + (-\varepsilon_{Q,P} Q) \rightarrow a = Q(1 - \varepsilon_{Q,P})$$

Example:

[Recall that Q= a –bP]

- Q=70, p=$0.70, $\varepsilon_{Q,P}$=-0.55
- We to find parameters a and b, so we find the demand function

$$b = \varepsilon_{Q,P} \cdot \frac{Q}{P} = -(0.55) \cdot \frac{70}{0.70} = -55$$

- (1)

$$a = [1 - \varepsilon_{Q,P}] \cdot Q = [1 - (-0.55)] \cdot 70 = 108.5$$

- (2)
- Hence, the demand curve is $Q^d = 108.5 - 55p$

a b