Chapter 14

Game Theory and Strategic Behavior

Game Theory

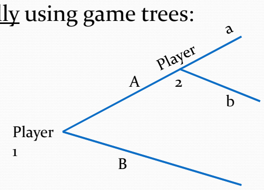
- Game Theory: The branch of microeconomics concerned with the analysis of optimal decision making in competitive situations.
- Strategy: a detailed plan of action under any possible situaton that the player might face (known as a complete contingent plan).
- Nash Equilibrium (NE): a situation in which each player is choosing his Best Response given the strategy chosen by the other player.

- We represent games in which players choose their actions simultaneously using matrices:

Player 2 (columns)

Player 1 (rows)	Left	Right
Up		
Down		

- We represent games in which players choose their actions sequentially using game trees:



Game Theory

Example 1: Toyota and Honda simultaneously decide to build or not a build a plant

Toyota

Honda	Build	Not Build
Build	16, 16	20 , 15
Not Build	15, 20	18, 18

Honda chooses a row.
Toyota chooses a column.

Let's first assume we are in the shoes of Honda…

If Toyota chooses to Build, in the left column, we can focus only on that column.

Toyota

Honda		Build	Not Build
	Build	<u>16,</u> 16	20 , 15
	Not Build	15, 20	18, 18

↑

- If Toyota builds, we are in this column. His rival, Honda, obtains the largest payoff by building as well since 16 > 15.
- We underline that payoff, 16
- We say that Honda's best response to Toyota building, $BR_H(B)$, is to build, i.e., $BR_{Honda}(B) = B$

Let's first assume we are in the shoes of Honda…

If Toyota chooses to Not Build, in the right column, we can focus only on that column.

Toyota

Honda		Build	Not Build
	Build	16, 16	<u>20</u> , 15
	Not Build	15, 20	18, 18

↑ Toyota doesn't build

- Hence, Honda's best response is $BR_{Honda}(NB)=B$, since 20 > 18

Let's first assume we are in the shoes of Toyota…

If Honda chooses to Build, in the top row, we can focus only on that row.

Toyota

Honda		Build	Not Build
Honda Builds →	Build	16, <u>16</u>	20 , 15
	Not Build	15, 20	18, 18

- Hence, the best response of its rival, Toyota, is $BR_{Toyota}(B)=B$ since 16 > 15 (Second element of every payoff pair).

Let's first assume we are in the shoes of Toyota…

If Honda chooses to Not Build, in the bottom row, we can focus only on that row.

Toyota

Honda		Build	Not Build
	Build	16, 16	20 , 15
Honda does not build →	Not Build	15, <u>20</u>	18, 18

- Hence, Toyota's BR is $BR_{Toyota}(NB)=B$, since 20 > 18.

Game Theory

Example 1: Toyota and Honda simultaneously decide to build or not a build a plant

Toyota

Honda		Build	Not Build
	Build	16, 16	20 , 15
	Not Build	15, 20	18, 18

Summary of Best Responses: $BR_H(B)=B$ $BR_T(B)=B$
$BR_H(NB)=B$ $BR_T(NB)=B$

Unique Nash equilibrium: (Build (Honda), Build (Toyota))

Game Theory-Example 1 (cont.)

Recall the Procedure:

Honda Column player: BR_{Honda}	Toyota Row player: BR_{Toyota}
Fix column in Build (meaning consider only the upper left box and the lower left box) Will Honda Build? Yes, because 16>15 $BR_H(B)=B$ (meaning Honda's best response, given that Toyota Builds, is to Build also)	Fix row in Build Will Toyota Build? Yes, because 16>15 $BR_T(B)=B$ (meaning that Toyota's best response, given that Honda builds, is to build as well.)
Fix column in Not Build Will Honda Build? Yes, because 20>18 $BR_H(NB)=B$	Fix row in Not Build Will Toyota Build? Yes, because 20>18 $BR_T(NB)=B$

Interesting…

- The NE is (Build,Build) with payoff $16 for each firm.
- However, if both firms coordinated in not bulding the outcome would be (not build, not build) yielding a higher payoff ($18) for each firm.
- This result illustrates an important feature of the NE:
  - The NE does not necessarily correspond to the outcome that maximizes aggregate payoffs.
  - In fact, this conflict between individual and group payoffs is relatively common in social sciences and business.
  - It also emerges in the so-called "prisoner's dilemma" game.

Prisoner's Dilemma

- A game where there exists a tension between the self-interests of the player and the collective interest. The players' strategies do not result in the outcome that is best for everyone.
- Story:
  - Two people in different cells, being arrested by police.
  - Police has only minor evidence against them, yielding to a minor sentence.
  - However, police suspects these two people were criminals, and offer them this deal:
    - If you confess on the crime, and your partner doesn't, we will let you go for free, while your partner will get 10 years in jail.
    - If your partner confesses, but you don't, you will spend 10 years in jail, while he/she doesn't serve a day!
    - If you confess on the crime, but your partner also confesses, you will both get 5 years in jail.
    - However, if none of you confesses, the police only has minor evidence against you, implying you will only serve a year in jail.

Prisoner's Dilemma:

Two prisoners being interrogated in different cells

Prisoner's Dilemma

David

Ron

	Confess	Not Confess
Confess	-5, -5	0 , -10
Not Confess	-10, 0	-1, -1

Prisoner's Dilemma

Let's start with the row player (Ron). If David confesses (in the left column) what is Ron's best response?

David

Ron

	Confess	Not Confess
Confess	-5, -5	0 , -10
Not Confess	-10, 0	-1, -1

If David confesses...

Ron's BR is $BR_R(C)=C$

Prisoner's Dilemma

What if David does not confess (in the right column)? Then Ron's best response is...

David

Ron

	Confess	Not Confess
Confess	-5, -5	0 , -10
Not Confess	-10, 0	-1, -1

If David doesn't confess...

Ron's BR to NC, $BR_R(NC)=C$

Prisoner's Dilemma

Let us now move on to David (the column player). If Ron confesses (in the top row), what is David's best response?

David

Ron

If Ron confesses... →

	Confess	Not Confess
Confess	-5, -5	0 , -10
Not Confess	-10, 0	-1, -1

David's BR is $BR_D(C)=C$

Prisoner's Dilemma

What if Ron does not confess (in the bottom row)? Then David's best response is...

David

Ron

If Ron doesn't confesses... →

	Confess	Not Confess
Confess	-5, -5	0 , -10
Not Confess	-10, 0	-1, -1

David's BR is $BR_D(NC)=C$

Prisoner's Dilemma

Collective Interest calls for (NC,NC) where -1 each.
Self-interested N.E. result is (C,C) where -5 each.

Summarizing... David

Ron

	Confess	Not Confess
Confess	-5, -5	0 , -10
Not Confess	-10, 0	-1, -1

NE= (confess, confess)since Ron: $BR_{Ron}(C)=C$, $BR_{Ron}(NC)=C$

David: $BR_{David}(C)=C$, $BR_{David}(NC)=C$

Application of the PD game to other fields in social sciences:

- Price wars
- Negative campaigns in politics
- Tariff setting by countries.

Game Theory

- Dominant Strategy: A strategy that yields a higher payoff than any other strategy, no matter what strategy the other player follows.
- Dominated Strategy: A strategy such that the player has another strategy that gives a higher payoff, no matter what the other player does.

Prisoners Dilemma:

Ron: Confess for Ron is a Dominant strategy:

-5>-10 when David confesses

0>-1 when David does not confess

As a consequence, Don't Confess is a dominated strategy

David: Confess for David is also a Dominant strategy:

-5>-10 when Ron confesses

0>-1 when Ron does not confess

As a consequence, Don't Confess is a dominated strategy

Application in the Textbook

- Hiring a lawyer as a prisoner's dilemma game.
- Hiring a lawyer is a *dominant strategy* in a litigation.
  - Indeed, if the other party hires a lawyer you'd better do that as well.
  - If the other party doesn't, your chances of winning increase if you hire a lawyer
    - *Example:* In public employees wage disputes in NJ during 1980's the chances of successfully persuading the arbitrator to accept its wage proposals went up from 50% to 75% when you hired a lawyer and the other party did not.
    - If both parties hired a lawyer, odds of winning remained at 50%
    - Hence, regardless of what the other party does. hiring a lawyer is beneficial.
- Both parties end up hiring a lawyer, their chances of winning remain the same, but society as a whole loses (except lawyers, of course).

Dominant and Dominated strategies

- When a player has a dominant strategy he/she will **always** use it, regardless of what his/her opponent does.
  - *Example:* "Confess" in the Prisoner's Dilemma game.
- When a player has a dominated strategy he/she will **never** use it, regardless of what his/her opponent does.
  - *Example:* "Don't Confess" in the Prisoner's Dilemma game.

Dominant and Dominated strategies

- This helps us delete all Dominated strategies for a player, since he/she will never use them.
- Graphically,
  - we extend a line on the top of a row associated to the strategy that the row player regards as a dominated strategy.
  - we extend a line on the top of a column associated to the strategy that the column player regards as a dominated strategy.
- This is great!! It substantially reduces the strategies we have to focus on.
  - Let's see one example:

Deleting dominated strategies…

- Marutti *doesn't have* a dominated strategy:
  - Not building is better if Ambassador builds (i.e., 15>12 in the first column), but
  - Building is better if Ambassador Does not build (i.e., 20>18 in the second column).
- In contrast, Ambassador *has* a dominated strategy:
  - Building yields a higher payoff both
    - when Marutti also builds (i.e., 4>3 in the first row), and
    - when Marutti does not build (i.e., 6>5 in the second row).

TABLE 14.3 Capacity Expansion Game between Marutti and Ambassador*

		Ambassador	
		Build a New Plant	**Do Not Build**
Marutti	**Build a New Plant**	12, 4	20, 3
	Do Not Build	15, 6	18, 5

*Payoffs are in millions of rupees.

Table 14.3

Deleting dominated strategies…

- Since Do Not Build is a dominated strategy for Ambassador, this firm will never use it, and we can delete it from the matrix.
- Marutti can anticipate this deletion, which makes his decision-making process really easy:
  - "Given that Ambassador builds a new plant (first column), my payoff is higher if I don't' build a plant (15) than if I do (12)."

TABLE 14.3 Capacity Expansion Game between Marutti and Ambassador*

		Ambassador	
		Build a New Plant	**Do Not Build**
Marutti	**Build a New Plant**	12, 4	20, 3
	Do Not Build	15, 6	18, 5

*Payoffs are in millions of rupees.

Nash Equilibrium

Table 14.3

Table 14.4- plant construction game with *three* strategies

		toyota		
		Build large	**Build Small**	**Do Not Build**
	Build Large	0,0	12,8	18,9
Honda	**Build Small**	8,12	16,16	20,15
	Do Not Build	9,18	15,20	18,18

We first put ourselves in the shoes of **Honda, the row player,** obtaining:

- If Toyota builds large (left column), Honda's BR is Don't build, since 9>8>0.
- If Toyota builds small (middle column), Honda's BR is to Build Small, since 16>15>12.
- If Toyota doesn't build (right column), Honda's BR is to Build Small, since 20>18>18.

Table 14.4- plant construction game with *three* strategies

		toyota		
		Build large	**Build Small**	**Do Not Build**
	Build Large	0,0	12,8	18,9
Honda	**Build Small**	8,12	16,16	20,15
	Do Not Build	9,18	15,20	18,18

We can now put ourselves in the shoes of **Toyota, the column player,** obtaining:

- If Honda builds large (top row), Toyota's BR is Don't build, since 9>8>0.
- If Honda builds small (middle row), Toyota's BR is to Build Small, since 16>15>12.
- If Honda doesn't build (bottom row), Toyota's BR is to Build Small, since 20>18>18.

Table 14.4- plant construction game with three strategies

Summarizing both players' best responses:

		toyota		
		Build large	Build Small	Do Not Build
Honda	Build Large	0,0	12,8	18,9
	Build Small	8,12	16,16	20,15
	Do Not Build	9,18	15,20	18,18

Honda: $BR_H(BL)=DB$, $BR_H(BS)=BS$, $BR_H(DB)=BS$

Toyota: $BR_T(BL)=DB$, $BR_T(BS)=BS$, $BR_T(DB)=BS$

Hence, the NE is: (Build Small, Build Small)

Alternative approach…

- Rather than underlying payoffs in order to find players' best responses, we can start by deleting dominated strategies for each player.
- This will yield a smaller matrix.
  - Build Large is dominated for Honda.
  - Build Large is also dominated for Toyota.

TABLE 14.4 Modified Capacity Expansion Game between Toyota and Honda*

		Toyota		
		Build Large	Build Small	Do Not Build
Honda	~~Build Large~~	~~0, 0~~	~~12, 8~~	~~18, 9~~
	Build Small	8, 12	16, 16	20, 15
	Do Not Build	9, 18	15, 20	18, 18

*Payoffs are in millions of dollars.

[8]B. J. Nalebuff and A. M. Brandenberger, *Coopetition* (New York: Currency Doubleday, 1996).
[9]This is the same logic that we employed in Chapter 13 when we argued that the Cournot equilibrium was the natural outcome of the one-shot quantity game between Samsung and LG.

Table 14.4

Doing this we obtain…

TABLE 14.5 Modified Capacity Expansion Game between Toyota and Honda after Eliminating Dominated Strategies*

		Toyota	
		Build Small	Do Not Build
Honda	Build Small	16, 16	20, 15
	Do Not Build	15, 20	18, 18

*Payoffs are in millions of dollars.

Table 14.5

- A matrix that *exactly coincides* with the one we used before to find the NE of this game:
  - (Build Small, Build Small).
- Of course, both approaches yield the same NE prediction.

Table 14.6 - Pricing between Coke and Pepsi.

First, note that this game can be solved by underlining payoffs (BRs), but it is so painful…

		Coke			
		$10.50	$11.50	$12.50	$11.50
Pepsi	~~$6.25~~	~~66.190~~	~~68.199~~	~~70.198~~	~~73.191~~
	~~$7.25~~	~~79.201~~	~~82.211~~	~~85.214~~	~~89.205~~
	$8.25	82.212	86.224	90.229	95.225
	~~$9.25~~	~~75.223~~	~~80.237~~	~~85.244~~	~~91.245~~

Alternative: Deleting dominated strategies:

1) For Pepsi, p= $8.25 is a dominant strategy
2) This allows us to delete prices (rows) p=$6.25, $7.25, and $9.25 for Pepsi, since they are all dominated by p=$8.25.

After deleting dominated strategies for Pepsi:

TABLE 14.6 Price Competition between Coke and Pepsi*

		Coke			
		$10.50	$11.50	$12.50	$13.50
Pepsi	$6.25	~~66, 190~~	~~68, 199~~	~~70, 198~~	~~73, 191~~
	$7.25	~~79, 201~~	~~82, 211~~	~~85, 214~~	~~89, 208~~
	$8.25	82, 212	86, 224	90, 229	95, 225
	$9.25	~~75, 223~~	~~80, 237~~	~~85, 244~~	~~91, 245~~

*Payoffs are in millions of dollars.

Table 14.6

- Given that the only undominated price of Pepsi is $8.25, Coke best responds with a price of $12.50,
  - which yields a NE of: (8.25,12.50)
- At this NE, Pepsi profits are $90, and Coke profits are $229.

Query #1

	Player B	
Player A	**B1**	**B2**
A1	30,30	40,20
A2	20,40	35,35

Which of the following is true?

a) Player A has a dominant strategy.
b) Player B has a dominant strategy.
c) Both players have dominant strategies.
d) Neither player have dominant strategies.

Query #1 - Answer

- Answer C
- Both players have dominant strategies.
- Let us first analyze player A:

	Player B	
Player A	**B1**	**B2**
A1	30,30	40,20
A2	20,40	35,35

- When player B selects B1, player A obtains a larger payoff with A1, 30, than with A2, 20.
- Similarly, when player B selects B2, player A obtains a larger payoff with A1, 40, than with A2, 35.
- Hence, regardless of what player B does, player A is better off choosing A1 than A2.
- A1 is player A's dominant strategy.

Query #1 - Answer

- Let us next analyze player B:

	Player B	
Player A	**B1**	**B2**
A1	30,30	40,20
A2	20,40	35,35

- When player A selects A1, player B obtains a larger payoff with B1, 30, than with B2, 20.
- Similarly, when player A selects A2, player B obtains a larger payoff with B1, 40, than with B2, 35.
- Hence, regardless of what player A does, player B is better off choosing B1 than B2.
- B1 is player B's dominant strategy.
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Let's recap…

- So far all games…
  - Had an equilibrium, and
  - That equilibrium was unique!
- Are there games with more than one equilibrium?
  - YES!!!

    Remember James Dean?

Game Theory

- More than one Nash Equilibrium in the "game of chicken" or "rebel without a cause"
- Game of Chicken: Two players drive their cars toward each other:
  - If they both swerve to avoid an accident they walk away unharmed and with no good story to tell.
  - If one player doesn't swerve while the other does, the swerving player suffers embarrassment while the other player gets to tell a good story.
  - If neither player swerves the cars crash and both players suffer greatly.

The Game of "Chicken" (or Rebel without a cause).

Slick

Luke		Swerve	Stay
	Swerve	0, 0	-10, 10
	Stay	10, -10	-100, -100

We first fix our attention on **Luke, the row player**:
- If Slick chooses to Swerve (left column), Luke's BR is to Stay, since 10>0.
- If Slick chooses to Stay (right column), Luke's BR is to Swerve, since -10>-100.

The Game of "Chicken" (or Rebel without a cause).

Slick

Luke		Swerve	Stay
	Swerve	0, 0	-10, 10
	Stay	10, -10	-100, -100

We first fix our attention on **Slick, the column player**:
- If Luke chooses to Swerve (top row), Slick's BR is to Stay, since 10>0.
- If Slick chooses to Stay (bottom row), Slick's BR is to Swerve, since -10>-100.

The Game of "Chicken" (or Rebel without a cause).

Summarizing the best responses we found:

Slick

Luke		Swerve	Stay
	Swerve	0, 0	-10, 10
	Stay	10, -10	-100, -100

- Luke's Best Responses:
  - BR_S (Swerve)=Stay
  - BR_S (Stay)=Swerve
- Slick's Best Responses:
  - BR_L (Swerve)= Stay
  - BR_L (Stay)= Swerve

- Two NE (Stay, Swerve) and (Swerve, Stay)
  - The players seek to miscoordinate.
  - They essentially want to end up choosing the opposite strategy of his opponent:
    - Stay when his opponent Swerves ("look how tough I am"), or
    - Swerve when his opponent Stays ("I don't want to die!!")

Anti-coordination Game.

The game of chicken between XM and Sirius satellite radio. (application of the game of chicken)

SIRIUS

XM		Stay	Exit
	Stay	-200, -200	300, 0
	Exit	0, 300	0, 0

BR_{XM}(stay)= Exit
BR_{XM}(Exit)= Stay

BR_{sirius}(stay)= Exit
BR_{sirius}(Exit)= Stay

2 NE: (Exit, Stay) and (Stay, Exit)

- In 2008, XM "swerved", i.e., was acquired by its rival (SIRIUS) to form SIRIUS XM radio
- Intuitively, the satellite radio market was not large enough to sustain two firms. But who will leave?

The “game of chicken” at the movies

Mode	Description
Tractors	*Footloose* (1984, movie)
Bulldozers	Buster and Gob in *Arrested Development* (2004, TV)
Wheelchairs	Two old ladies with motorized wheelchairs in *Banzai* (2003, TV)
Snowmobiles	"[Two adult males] died in a head-on collision, earning a tie in the game of chicken they were playing with their snowmobiles." <www.seriouslyinternet.com/278.0.html>
Film release dates	Dreamworks and Disney-Pixar (2004)
Nuclear weapons	Cuban Missile Crisis (1963): "Since the nuclear stalemate became apparent, the Governments of East and West have adopted the policy which Mr. Dulles calls 'brinksmanship.' This is a policy adapted from a sport which, I am told, is practised by some youthful degenerates. This sport is called 'Chicken!'" (Bertrand Russell, *Common Sense and Nuclear Warfare*, 1959)

- We have seen examples of anti-coordination games.
- What about coordination games?

Example of Coordination game

Bank Run Game (another game with 2 NE)

Depositor 2

Depositor 1		Withdraw	Don't Withdraw
	Withdraw	25, 25	50, 0
	Don't Withdraw	0, 50	110, 110

Depositor 1:
BR_1(withdraw)= withdraw
BR_1(don't withdraw)= Don't withdraw

Depositor 2:
BR_2(withdraw)= withdraw
BR_2(don't withdraw)= Don't withdraw

2 NEs: (withdraw, withdraw), (DW, DW)

- Intuition: if you don't withdraw, the bank will be solid, so I won't need to withdraw my money (as in a bank run). However, if everyone withdraws, I'd better withdraw as well before my deposits are at risk.
- *Recent examples of Bank Runs*: Countrywide financial in 2008, and Landsbanki (Iceland) also in 2008
- FDIC avoids bank runs (unless your checking account exceeds the FDIC guarantee).

Query #2

	Player B	
Player A	B1	B2
A1	5,6	7,2
A2	4,5	9,1

Which of the following is true?

a) (A1,B1) is a Nash equilibrium.
b) (A2,B2) is a Nash equilibrium.
c) There is no Nash equilibrium in pure strategies.
d) There are multiple Nash equilibria in pure strategies.

Query #2 - Answer

- Answer A
- First, notice that Player B finds that B1 strictly dominates B2.
  - Indeed, B1 yields a strictly larger payoff:
    - Both when player A chooses A1, i.e., 6 > 2, and when player A chooses A2, i.e., 5 > 1.
    - That is, regardless of what player A does, player B is better off choosing B2 than B1.
    - As a consequence, we can delete B1 from ever being used by player B, as we do in the following matrix.

Query #2 - Answer

	Player B	
Player A	B1	B2
A1	5,6	7,2
A2	4,5	9,1

- After deleting B1, Player A just needs to choose their higher Payoff from column B1, which is A1, because 5 > 4.
- Hence, (A1,B1) is the Nash equilibrium of the game.
- Page 539

Query #2 - Answer

	Player B	
Player A	B1	B2
A1	5,6	7,2
A2	4,5	9,1

- Alternatively, you can identify every player's best response:
- Let's start with the row player, player A:
  - If player B chooses B1, then player A responds with A1, since 5>4.
  - If player B chooses B2, then player A responds with A2, since 9>7.

Query #2 - Answer

		Player B	
Player A		B1	B2
	A1	5,6	7,2
	A2	4,5	9,1

- Let's continue with the column player, player B:
  - If player A chooses A1, then player B responds with B1, since 6>2.
  - If player A chooses A2, then player B responds with B1, since 5>1.

Query #2 - Answer

		Player B	
Player A		B1	B2
	A1	5,6	7,2
	A2	4,5	9,1

- Summarizing the best responses for each player:
  - We see that the only cell in which both players are selecting mutual best responses is (A1,B1).
    - Graphically, this cell has both payoffs underlined.
  - Hence, (A1,B1) is the unique Nash equilibrium of the game.

But do all games have a NE?

- So far, we have seen games:
  - with a single NE
    - e.g., Prisoner's Dilemma, Building a Plant, etc.,
  - with more than one NE.
    - Chicken game and applications: Satellite radio, etc. (Coordination Games).
    - Bank Runs (Anticoordination Games)
- But do all games have a NE?
  - They do, but in some games players select a particular strategy with 100% probability...
  - While in other games, they might be randomizing between two or more strategies.

Mixed Strategies

- Pure strategy: a specific choice of a strategy, with 100% probability.
- Mixed strategy: the choice among two or more strategies according to a specified probability.
  - The player randomizes between their actions.
  - *Example:* Choose A with probability 30% and B with the remaining probability, 70%.

1999 Final match of the Women's World Cup

- The game was between the USA and China. The game was tied 0-0 so the winner would be decided by penalty kicks:
  - Dive left or right for the goalie.
  - Kick left or right for the kicker.

US Kicker

		Aim Right	Aim Left
Chinese Goalie	Dive right	0, 0	-10, 10
	Dive Left	-10, 10	0, 0

1999 Final match of the Women's World Cup

- Does any player have a dominant strategy?
  - NO:
  - The goalie wants to get the ball, so she wants to dive towards the same direction as the kicker kicks the ball.
  - The kicker, in contrast, wants to kick to the opposite direction that the goalie dives.

US Kicker

		Aim Right	Aim Left
Chinese Goalie	Dive right	0, 0	-10, 10
	Dive Left	-10, 10	0, 0

1999 Final match of the Women's World Cup

- What is the Nash Equilibrium of the game?
  - None, if we restrict our attention to pure strategies.
  - Indeed, there is no cell where both players' payoffs were underlined because of being best responses.

US Kicker

		Aim Right	Aim Left
Chinese Goalie	Dive right	0, 0	-10, 10
	Dive Left	-10, 10	0, 0

1999 Final match of the Women's World Cup

- What if we allow players to randomize their strategy? Then we can always find a NE
- Let's define:
  - p is the probability that the Chinese goalie dives Right
  - q is the probability that the US kicker aims Right

Placing these probabilities in the payoff matrix:
- Probability p and 1-p in rows
- Probability q and 1-q in columns

US Kicker

Chinese Goalie

	q Aim Right	Aim Left 1-q
p Dive right	0, 0	-10, 10
1-p Dive Left	-10, 10	0, 0

Let's first consider the US kicker

1) If the US kicker is not selecting a particular action with 100% probability, it must be that she is indifferent between her two options: Kick aiming right and kick aiming left. That is, the expected utility of both options coincide.

$EU_{USA}(aim right) = EU_{USA}(aim left)$ where EU is Expected Utility

Rearranging, $0(p) + 10(1-p) = 10p + 0(1-p)$

$10 - 10p = 10p$

$10 = 20p$

Solve for p, $p = \frac{1}{2}$

Hence, the US kicker is indifferent between aiming right or aim left when the Chinese Goalie dives right with a 50% probability

Similarly, the Chinese goalie is not choosing a strategy with a 100% probability. Hence, it must be that she is indifferent between diving right, and diving left. That is,

$EU_{Chinese}(dive\ right) = EU_{Chinese}(dive\ left)$ where EU is Expected Utility

$0(q) + (-10)(1-q) = -10q + 0(1-q)$

$-10 - 10q = -10q$

$10 = 20q$

$q = \frac{1}{2}$

Hence, the Chinese Goalie is indifferent between diving right or left when the USA kicker aims right with a 50% probability

Summarizing, the mixed-strategy Nash Equilibrium of the game is :

$$msNE : (\underbrace{\frac{1}{2}R + \frac{1}{2}L}_{USKicker}, \underbrace{\frac{1}{2}R + \frac{1}{2}L}_{Chinese goalie})$$

Query #3

		Player B	
Player A		B1 (q)	B2 (1-q)
	A1 (p)	7,3	5,10
	A2 (1-p)	3,8	9,6

In the game above, in the Nash equilibrium in mixed strategies player A chooses A1 with a probability, *p*, of exactly:

a) 2/9 probability.
b) 3/9 probability.
c) 4/9 probability.
d) 5/9 probability.

Query #3 - Answer

- Answer A
- We are looking for the probability of A1, so we need to solve for p.
- Recall that, if players are playing a mixed strategy equilibrium (in which they both randomize among his two available strategies), it must be that Player B is indifferent between B1 and B2.
  - Otherwise, he would just select one of these strategies without the need to randomize (i.e., 100% of the time).
- Hence, if player B is indifferent between B1 and B2, we must have that
  - EU(B1) = EU(B2)

Query #3 - Answer

- Let's find each of these expected utilities separately:
  - $EU(B1) = 3p + 8(1-p)$
  - $EU(B2) = 10p + 6(1-p)$

$$EU(B1) = EU(B2)$$

- We can now set

$$3p + 8 - 8p = 10p + 6 - 6p$$
$$-5 + 8 = 4p + 6$$
$$-9 = -2$$
$$p = \frac{2}{9}$$

- Pages 548 - 549

- **Repeated games** are very usual in real life:
  1. Treasury bill auctions (some of them are organized monthly, but some are even weekly),
  2. Cournot competition is repeated over time by the same group of firms,
  3. OPEC cartel is also repeated over time.
- In addition, players' interaction in a repeated game can help us rationalize cooperation…
  - in settings where such cooperation could not be sustained if players interact once.

Let's see one example with the prisoner's dilemma →

Repeated Prisoner's Dilemma

Player 2

Player 1

	Cheat	Cooperate
Cheat	5, 5	14, 1
Cooperate	1, 14	10, 10

•Hence, N.E. of the unrepeated game is (cheat, cheat).
•However, if the game is repeated in the future, I might prefer to sustain cooperation (obtaining $10 in each period) than cheat.

But, how can we show this more formally?

- Consider the following "Grim-Trigger" strategy:
  - Every player starts cooperating in the first period of interaction.
  - He continues to cooperate as long as he observes that all players have cooperated in all past periods.
  - If, instead, he observes some past cheating, then he plays Cheat thereafter.
- Let us next show that such Grim-Trigger strategy induces both players to cooperate along all periods, if they care enough about the payoffs they can receive in the future.

But, how can we show this more formally?

- If cooperating along all periods, I obtain

$$10+10\delta+10\delta^2+10\delta^3\ldots=10\left[1+\delta+\delta^2+\delta^3\ldots\right]=10/(1-\delta)=10\cdot 1/(1-\delta)$$

- If, instead, I cheat today, I get $14 today, but the other player detects my defection and plays cheat thereafter (this is punishment prescribed in the grim-trigger strategy)

$$14+5\delta+5\delta^2+\ldots=14+5\left[\delta+\delta^2+\ldots\right]=14+5\delta\left[1+\delta+\ldots\right]$$

$$=14+5/(1-\delta)$$

- Hence, I prefer to cooperate if

$$10/(1-\delta)>14+5/(1-\delta)$$

Rearranging…

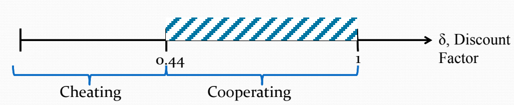
- Multiplying both sides by (1- δ), we obtain

$$10>14(1-\delta)+5\delta$$

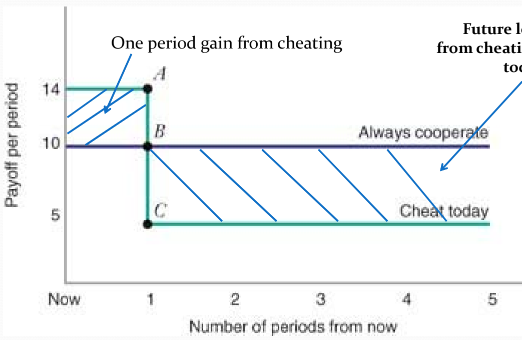
And rearranging… $10>14-14\delta+5\delta$

and simplifying 9δ>4, which implies δ>4/9=0.44

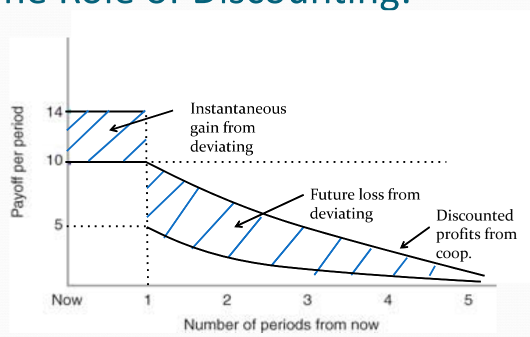
- Intuitively, this implies that I cooperate as long as I assign a sufficiently high value to future payoffs (high δ), i.e., as long as I am not "too impatient."
  - Recall that δ=0 implies that the player is super impatient: he assigns zero value to his future payoffs, i.e., he only cares about today.
  - If, in contrast, δ=1, then the player is super patient: he assigns exactly the same value to his current and future payoffs.



Using a "grim-trigger" strategy: Cooperate until a cheating is detected



The Role of Discounting:



A decrease in δ hence shrinks the (discounted) future loss from cheating, making it less important, and ultimately encouraging cheating today.

Cooperation in the repeated game is more likely if…

- *BENEFITS FROM CHEATING*:
  - Benefits from cheating are small (blue area is short). e.g, $11 rather than $14
  - Cheating is immediately detected (blue area is narrow). e.g. detected in only one period (rather than 3)
- *COSTS ASSOCIATED TO CHEATING:*
  - The punishment after one player cheats is sufficiently long, or even permanent (red area is sufficiently long).
  - Players care about future payoffs, i.e., players are not too impatient. δ is close to 1. (If they are, the red area shrinks).

Sequential-move games

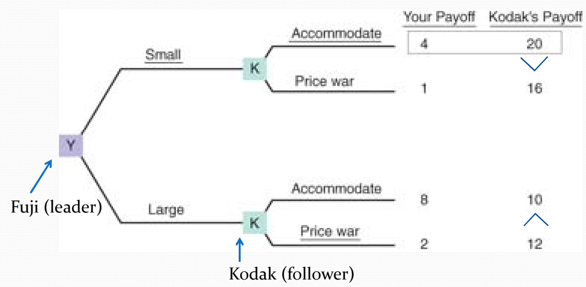
- In this type of games, one player takes an action before the other.
- We represent these games using "game trees":
  - A diagram that shows the different strategies of each player and when these strategies become available to him/her.
- How can we solve these games?
  - Using "backward induction" (also referred as rollback equilibrium)
  - Let us see a few examples…

Sequential-move games

- Backward induction (or Rollback equilibrium). General procedure:
  - First, we start analyzing the last mover in the game. We ask: which is his best strategy in all possible contingencies in which he/she is called on to move?
    - As a trick, mark with an arrow the branch corresponding to the last mover's optimal strategy in each of these cases.
  - Once you have examined the last player, move backwards (rolling back the game tree) towards the previous to the last mover, doing a similar analysis of his optimal strategies, and marking them with an arrow.
  - Repeating this process you eventually reach the first mover (at the "root" of the game tree), which implies that you are done!

Entry Game

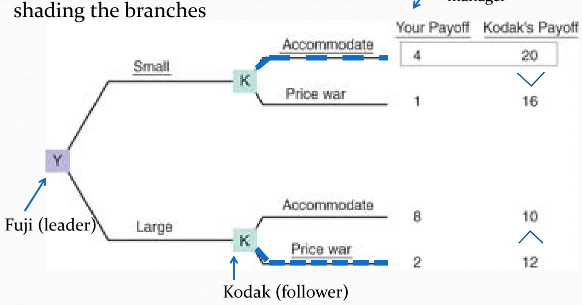
Start your analysis in the last mover: You are Kodak's manager



Entry Game

We mark Kodak's optimal actions by shading the branches

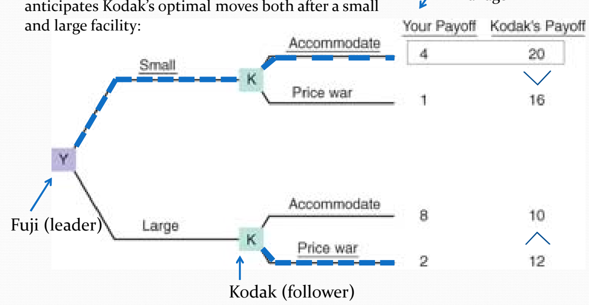
Start your analysis in the last mover: You are Kodak's manager



Entry Game

We can now analyze the previous mover, Fuji, who anticipates Kodak's optimal moves both after a small and large facility:

Start your analysis in the last mover: You are Kodak's manager

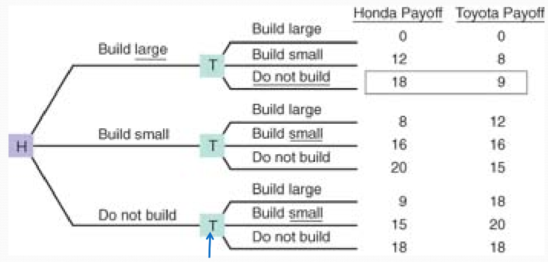


Hence, the equilibrium is : { Small, (Acc, Price War)}

Fuji Kodiak

- Notice that we specify what Kodak does in equilibrium (after observing that Fuji chose small) and off-the-equilibrium (after obseerving that Fuji chose large).
- Describing players' strategies off-the-equilibrium path is important: if Kodak had responded with Accommodate after a large facility, Fuji would have no incentive to choose Small (payoff of 4) instead of Large (payoff of 8)

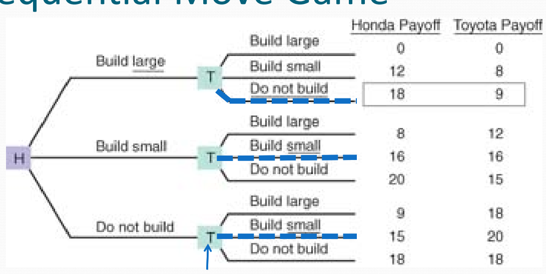
Sequential Move Game



Toyota (Follower)

Let's start with the last mover (Toyota).

Sequential Move Game

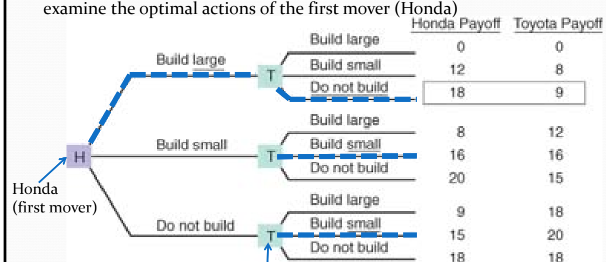


Toyota (Follower)

We have shaded the branches corresponding to optimal actions by Toyota after observing that Honda builds a large plant (at the top of the game tree), a small plant (in the middle of the game), or no plant (at the bottom of the game tree).

Sequential Move Game

Anticipating the optimal moves of the follower (Toyota), let us now examine the optimal actions of the first mover (Honda)



Toyota (Follower)

Hence the equilibrium of this sequential-move game is:
{Build large, (Not build, Small, Small)}
Honda Toyota

The strategic value of limiting your options:

- In the automaker example, Honda is better off pre-committing to a large capacity (limiting its options) than when it can choose any capacity at the same time as Toyota does.
  - Indeed, Honda made $16 in the equilibrium of the simultaneous-move game, but makes $18 as a leader in the sequential-move game.
- Limiting your options can then be profitable!
- We refer to such a strategy as a
  - **Strategic pre-commitment.**

More examples on irreversibility…

- When Airbus/Boeing were thinking about starting to build the "superjumbo", Airbus made irreversible investments in asset that couldn't be sold for much to other industries (too specific).

More examples on irreversibility…

- Hernan Cortes's conquest of the Aztec empire.
  - I am not proud of him as a fellow Spaniard, but he did something clever (from a strategic, not humanitarian, point of view)…
  - After traveling from Cuba to Mexico, he burned the Spanish ships, making retreat to Cuba impossible.
  - Cortes's men had no choice but to fight hard to win. It really worked out!
    - That's as irreversible as you can get!

More examples on irreversibility…

- When Orbitz and other firms use most favored customer clause (MFCC):
  - **Without MFCC**: You book a hotel room two months in advance for $150. Close to the date, Orbitz anticipates that half of the hotel will be empty during the day you will be there. They have the incentive to lower the price, something that might lead you to wait for "last-minute bargains".
  - **What other firm offers MFCC?**

More examples on irreversibility…

- When Orbitz and other firms use most favored customer clause (MFCC):
  - **With MFCC**: Orbitz's incentive to lower prices at the last minute is reduced when they must extend the discount to customers who previously booked their hotel at a higher price. This might lead you to book your next trip at current prices, increasing sales.

More examples on irreversibility…

- We don't negotiate with terrorists/pirates.
  - Most countries claim that. But when a hostage-taking situation arises, authorities have the incentive to accept pirates conditions (small cost for a developed country).
  - If the country does that, however, pirates will have strong incentives to take hostages in the future.
  - How to make the "no negotiation practice" irreversible? Some countries even include it in their constitutions, so deviating from that is almost impossible.

Limiting your options (cont.)

- For a strategic pre-commitment to be effective it must be:
  - Visible
    - Toyota must see the capacity decision that Honda made, and understand the consequences of such capacity in the posterior competition between the two firms.
  - Irreversible (or very hard to reverse).
    - Honda cannot say "Hey, Mr. Toyota, I will build a large plat. Believe me!" This is not irreversible, and Honda can step back and build a small plant afterwards.
    - If instead, Honda already makes an irreversible investment, Toyota can certainly believe it.

Announcement…

- If you enjoyed thinking about
  - ***Strategy and Game Theory***
  - Then you will really enjoy the EconS 424 course (Strategy and Game Theory).
  - We will talk about many applications to economics and business.
- It will be offered this upcoming Spring 2017 semester.
  - Undergrad level, easy math.
  - Most of the time it is easier than EconS 301!!
  - It will really help you if you are considering Masters, MBA, etc. in the future.