Monopoly and Monopsony

Chapter 11

Overview

- Unlike firms who face perfect competition and therefore have no choice over what quantity and price to set for their product, monopolies set the market price for their good.
- Perfect competition → one firm's production has no consequence on the market price (price taker)
- Monopolist → one firm's production has total impact on price (set the market price of his product)

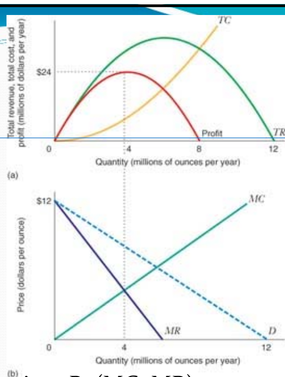
- But the monopolist cannot set an infinitely high price because even monopolists face demand curves.
  - That is, as a monopolists increases price, consumers will demand fewer units (as in any other market structure).

- So, how does a monopolists choose an optimal production level?
  - By equating the marginal revenue and marginal cost

$$MC = MR$$

- Let us show why this is the profit-maximizing condition for the monopolist

1) Graphically
2) Algebraically



P>MC →

Increases output until the point where MR=MC

Points with
MR>MC
MR<MC
cannot be optimal

- Note that since P>(MC=MR), a monopoly can earn a positive profit, unlike a firm in a perfectly competitive market.

Profit Maximization for theMonopolist

$$\max_q \pi(q) = TR(q) - TC(q) = p(q) \bullet q - TC(q)$$

↑ Inverse demand curve. For instance, $p(q) = a - bq$

Taking F.O.C.s with respect to q,

$$\frac{\partial \pi(q)}{\partial q} = \underbrace{p(q) + \frac{\partial p(q)}{\partial q} \bullet q}_{} - \frac{\partial TC(q)}{\partial q} = 0$$

↑ Derivative of the product $p(q) \cdot q$

↑ Reflects MR i.e., How total revenue increases if the monopolist raises output.

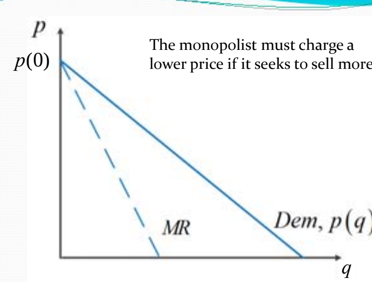
↑ This is the MC, i.e., How TC increases if the monopolist raises output.

Hence, MR-MC=0, or MR=MC

- Why is the *MR* curve below the market demand curve?

$$MR = \frac{\partial (pq)}{\partial q} = p(q) + \frac{\partial p(q)}{\partial q} q < p(q)$$

  - (1) Since the change in P is negative, i.e.,$\partial p / \partial q < 0$, the second term, $(\partial p / \partial q) q$, is also negative.
    - Therefore MR is less than $p(q)$ as depicted in the next figure.



$$MR = p(q) + \frac{\partial p(q)}{\partial q} \bullet q < p(q) = Dem \text{ since } \underbrace{\frac{\partial p(q)}{\partial q}}_{-} \bullet q < 0$$

That is, MR < Demand

- Note that at Q=0*(vertical intercept)*, MR *is* equal to P (both MR and Demand originate at the same height) because the second part of the MR equation becomes zero and we are left with

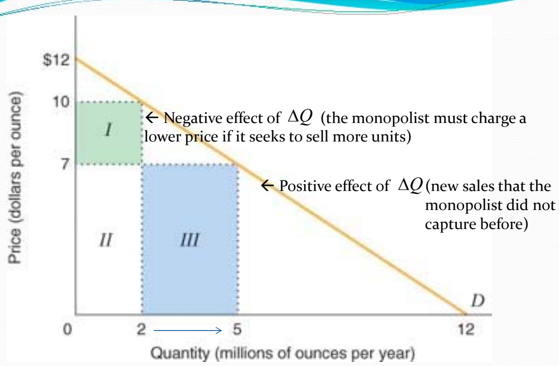
MR=P(0) + 0 = *P*(0)

$$MR(0) = p(q) + \frac{\partial p(q)}{\partial q}\underbrace{q}_{0} \rightarrow \underbrace{p(q)}_{MR=demand\ at\ q=0}$$

- More on Marginal Revenue:

Let's analyze how total revenue is affected by a marginal increase in output. In particular, the next figure depicts the:

- change in monopolist revenue = III – I,
  - where area I is the revenue lost from decreasing the price of some of the sold product and area III is the revenue gained from selling more units than before.
- Let's observe each area on the next figure...



(Below the marginal increase in Q)

- **<u>Average Revenue</u>**

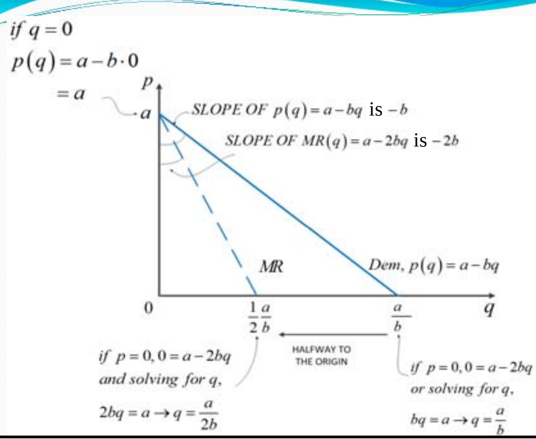
$$AR = \frac{TR}{q} = \frac{p(q)\cdot q}{q} = p(q)$$

- In monopoly theory, the Average Revenue (revenue per unit sold) is equal to *p(q)*. That is, the AR curve is identical to the market demand curve, p(q).

- **General Example of MR Curve:**
  - Consider an increase demand curve is given by $p = a - bQ$
  - We know $MR = p(q) + \frac{\partial p(q)}{\partial q} \bullet q$. Applying this expression to $p(q) = a - bq$

$$MR = \underbrace{(a - bq)}_{p(q)} + \underbrace{(-b)}_{\frac{\partial p(q)}{\partial q}} q = a - 2bq$$

- Hence, $MR = a - 2bq$
- Graphically, this implies that *MR* has the…
  - **Same vertical intercept as the market demand curve,** $p(0) = a$
  - **Twice the slope of the demand curve** (-2b) **rather than –b.**
- Let's depict the inverse demand p(q)=a-bq and compare it with its associated marginal revenue, MR(q)=a-2bq.



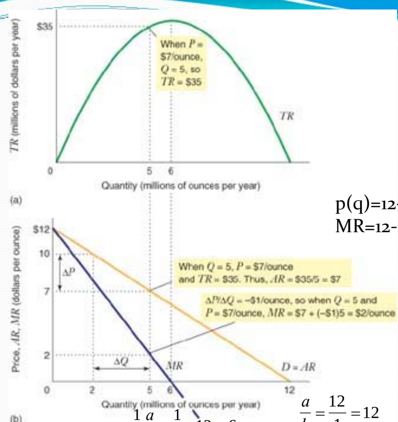
Let us next consider a numerical example of this inverse demand curve, where

$$p(q)=12-q,$$

That is, a=12 and b=1. Therefore, the associated marginal revenue is

$$MR(q)=12-2q.$$

Numerical Example



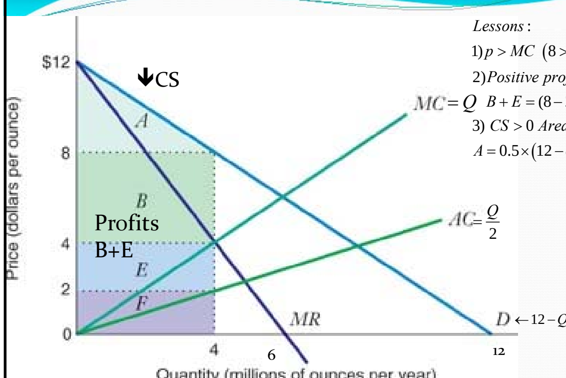
p(q)=12-q
MR=12-2q

$\frac{1}{2}\frac{a}{b} = \frac{1}{2} \bullet 12 = 6$ $\qquad$ $\frac{a}{b} = \frac{12}{1} = 12$

Monopoly Optimization

- Let's use our profit maximizing condition for the monopolist MR(q)=MC(q) in the above numerical example where.... $p = 12 - Q$

  $MC = Q$
- Step 1: $TR = pQ = (12-Q)Q = 12Q - Q^2 \rightarrow MR = 12 - 2Q$
- Step 2: $MR = MC$
  - $12 - 2Q = Q \rightarrow 12 = 3Q \rightarrow Q^m = 4$ units produced
  - Plugging this profit-maximizing output, $Q^m = 4$, into the demand function, we obtain a monopoly price of $p = 12 - 4 = \$8$



Lessons:

1) $p > MC$ $(8 > 4)$

2) *Positive profits of* $B + E = (8-2)\times 4$

3) $CS > 0$ *Area* $A = 0.5\times(12-8)\times 4 = \$8$

The monopolist doesn't have a Supply Curve

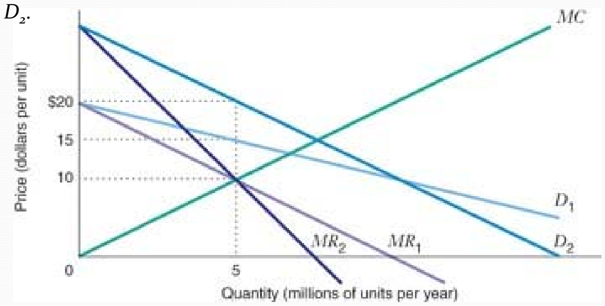
- **In perfect competition:** Every firm observes a market price (exogenous and fixed) and determines how much to produce, *q*. In particular, it increases q until p=MC(q).
- There is a specific relationship between p and q.

  → Supply curve: $q = f(p)$
- Example: if p=$10 and MC(q)=5+q, then 10=5+q entails q=10-5=5 units, and more generally its supply curve is q=p-5.

The monopolist doesn't have a Supply Curve

- **In Monopoly:** Firm determines simultaneously the market output and price without taking into account any other firm.
  - *There is no specific functional relationship between price and quantity.*
  - *Next figure for an example:*

Indeed, for different demand curves, a Monopolist can produce the same output (5 units) at different prices: \$15 for D_1 and \$20 for D_2.



Hence, there is no unique correspondence between p and q.
(No Supply Function)

Price Elasticity and Mark-up Prices

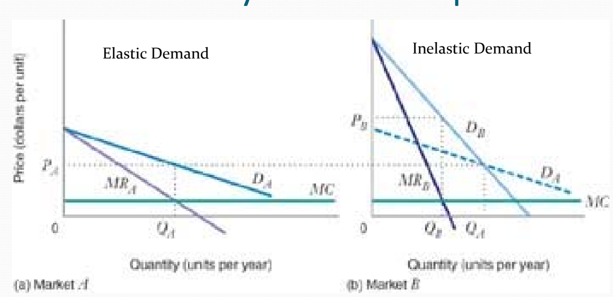
- Price elasticities play an important role in determining how much a monopolist can mark up prices.

$$(p - MC)$$

- When the elasticity of the demand curve is...
  - Elastic, the price mark up is small
  - Inelastic, the price mark up is large

- Let's see the relationship between markup, p-MC, and elasticity of demand in two extreme figures:

Price Elasticity and Mark-up Prices



Mark-up P_a-MC is small   Mark-up P_b-MC is large

Example: Exist/Don't exist close substitutes for the good in the market A and B, respectively

- There seems to exist a relationship between elasticities of demand and the monopolist's ability to set prices significantly above its marginal cost of production (high markups)

- Let's prove this relationship more formally.

- *What is the relationship between elasticity and marginal cost?*

We know the total revenue is $TR = p(q) \times q$, hence, marginal revenue is

$$MR = p + \frac{\partial p(q)}{\partial q} q$$

And rearranging...

$$MR = p\left[1 + \frac{\partial p(q)}{\partial q}\frac{q}{p}\right]$$

$$= p\left[1 + \frac{1}{\frac{p}{q} \bullet \frac{\partial q(p)}{\partial p}}\right]$$

- *What is the relationship between elasticity and marginal cost?*

From the formula of price-elasticity of demand, we know that

$$\varepsilon_{Q,P} = \frac{p}{q}\frac{\partial q(p)}{\partial p}$$

We can use the formula of elasticity in the denominator of MR in the previous slide. Hence, *MR* can be rewritten as

$$MR = p\left[1 + \frac{1}{\varepsilon_{Q,P}}\right]$$

- *What is the relation between elasticity and marginal cost?*
- Since *MR*=*MC*, we can use the above result $MR = p\left[1 + \frac{1}{\varepsilon_{Q,P}}\right]$ to obtain

$$p\left[1 + \frac{1}{\varepsilon_{Q,P}}\right] = MC$$

- Rearranging... $MC = p + p\frac{1}{\varepsilon_{Q,P}}$
- and further rearranging, we obtain

$$MC - p = p \bullet \frac{1}{\varepsilon_{Q,p}} \rightarrow \frac{MC - p}{p} = \frac{1}{\varepsilon_{Q,P}} \rightarrow$$

$$\frac{p - MC}{p} = -\frac{1}{\varepsilon_{Q,P}}$$

- This is the **Inverse Elasticity Pricing Rule (IEPR)**

Inverse Elasticity Pricing Rule (IEPR)

- The left side of the IEPR equation is the monopolist's optimal price markup, *p-MC*, expressed as a percentage of price.
- The monopolist's optimal mark-up of p above *MC*, as a % of price, is equal to minus the inverse of the price elasticity of demand.

$$\frac{p-MC}{p}=-\frac{1}{\varepsilon_{Q,p}}$$

- Note that as demand becomes more elastic

$$\uparrow\left|\varepsilon_{Q,p}\right| \Rightarrow \downarrow\left|\frac{1}{\varepsilon_{Q,p}}\right| \Rightarrow \downarrow\frac{p-MC}{p}$$

- Intuitively, as demand becomes more elastic, the mark-up that the monopolist can charge declines.
  - This might happen when consumers have close substitutes for the good sold by the monopolist.

Example

Consider a constant Marginal Cost, $MC = 50$

Demand, $Q = a\cdot p^{-b}$ ←Constant elasticity demand curve where the exponent $-b$ is the elasticity.

Indeed, let's confirm that the exponent –b is the price elasticity of demand

$$\varepsilon_{Q,P}=\frac{\partial q(p)}{\partial p}\frac{p}{q}=\left[-b\bullet ap^{-b-1}\right]\frac{p}{q}=$$

$$-b\bullet ap^{-b}\bullet p^{-1}\frac{p}{ap^{-b}}=p\bullet(-b)\bullet\frac{1}{p}=-b$$

Example

a) Find p^* if demand is $Q=100\cdot p^{-2}$ (then $\varepsilon_{Q,p}=-2$) by using IEPR (super fast!)

$$\frac{p-MC}{p}=-\frac{1}{-2}\rightarrow\frac{p-50}{p}=\frac{1}{2}$$

(MC ↙ pointing to 50)

And solving for price *p* yields a monopoly price of

$$2p-100=0\rightarrow p=\$100$$

Example (cont.)

b) Find p^* if demand changes to $Q = 100p^{-5}$ (then $\varepsilon_{Q,P} = -5$)

Using IEPR,

$$\frac{p - MC}{p} = -\frac{1}{\varepsilon_{Q,P}} \rightarrow \frac{p-50}{p} = -\frac{1}{-5}$$

solving for p yields a monopoly price of

$$5p - 250 = p \rightarrow 4p = 250 \rightarrow p = \$62.50$$

Example (cont.)

Summarizing…

When demand is more elastic (as in section B of the above exercise) we have that the monopolist's mark-up is smaller:

a) $p - MC = 62.50 - 50 = 12.50$ ↑ Mark up when $\varepsilon_{Q,P} = -5$ (section B)

b) $p - MC = 100 - 50 = 50$ ↑ Mark up when $\varepsilon_{Q,P} = -2$ (Section A)

Query #1

Suppose a monopolist faces a demand curve

$$Q = aP^{-b}$$

and that the monopolist has a constant marginal cost of MC(q)=*c*. The monopolist's profit-maximizing price is

a) $P = c\left(1 - \left(\frac{1}{b}\right)\right)$

b) $P = c\left(\frac{1}{b}\right)$

c) $P = c\left[\frac{1}{\left(1 - \left(\frac{1}{b}\right)\right)}\right]$

d) $P = c\left(-\frac{1}{b}\right)$

Query #1 - Answer

- Answer C
- The relationship between marginal revenue and the price elasticity of demand gives us another way to express the monopolist's profit-maximization.
- The Inverse Elasticity Pricing Rule (IEPR) states:

$$MC(Q^*) = P \times \left(1 + \frac{1}{\varepsilon_{Q,P}}\right)$$

- Recall that the equation $Q = aP^{-b}$ is the general formula for the Constant Elasticity Demand Curve from Chapter 3, where the price elasticity is equal to the $-b$ term.
- Using this information, we can substitute $-b$ in for $\varepsilon_{Q,P}$ and c for $MC(Q^*)$ in our IEPR.

$$c = p\left(1 + \frac{1}{-b}\right), \; solving \; for \; p, \; \frac{c}{1 - \frac{1}{b}} = p$$

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Application about mark-ups $\frac{p-MC}{P}$

- <u>Chewing Gum at the check-out counter</u>: Low elasticity, high mark-up 40%!!!

  ↑
  Impulse purchase items

- <u>Baby Food</u>: High elasticity, low mark-up (around 10%)

  ↑
  Considerable thought into their purchase decisions.
  Price comparison across stores.

What if the demand function is linear?

Consider again a figure with constant marginal cost

$MC = \$50$, but assume now the following linear inverse demand function $p(q) = 100 - 0.5q$. Solving for q, we can find the direct demand function 0.5q=100-p, or $q = 200 - 2p$

First, note that elasticity is

$$\varepsilon_{Q,P} = \frac{\partial q}{\partial p} \bullet \frac{p}{q} = -2 \bullet \frac{p}{200-2p} = \frac{-2p}{200-2p}$$

Using the IEPR,

$$\frac{p-50}{p} = -\frac{1}{\frac{-2p}{200-2p}} = \frac{200-2p}{2p}$$

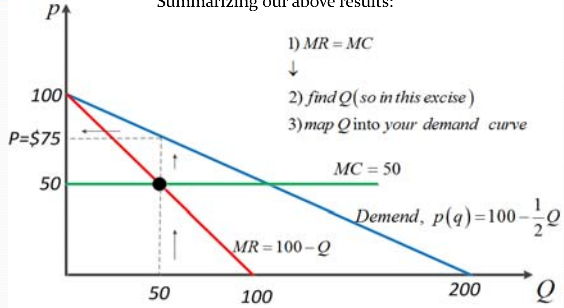
What if the demand function is linear?

Multiplying both sides by 2p, yields

$$2p \bullet \frac{p-50}{p} = 2p \bullet \frac{200-2p}{2p} \rightarrow 2p - 100 = 200 - 20$$

Solving for p, we obtain a monopoly price of $p = \$75$
which entails a monopoly output of q=200-2·75=50 units

Summarizing our above results:



$MR = MC$

$100 - Q = 50 \rightarrow p(50) = 100 - \frac{1}{2} \times 50 = \$75$

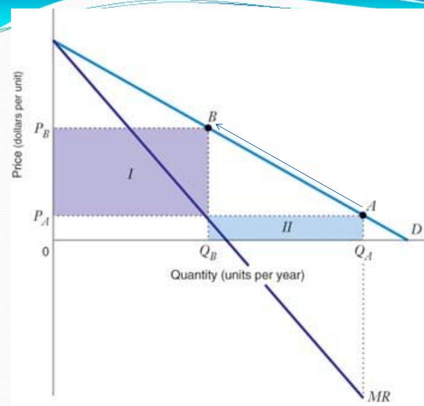
$Q = 50units$

Production in the Elastic Region

- The monopolist always produces in the **elastic** region of the Demand curve.

  Why?

- In the elastic region of the demand curve, the monopolist could decrease his production (raise his price) and increase profits.
- In the next figure, region I is larger (which is the profit gained if quantity decreases from A to B) than region II (profit lost from decreasing output from A to B).



Moving from A to B the monopolist:
-Losses II
-But gains I

Example

- In the previous exercise $p(q) = 100 - 0.5 \cdot q$, and $\varepsilon_{Q,P} = \frac{-2p}{200-2p}$
- We found that the profit maximizing price for the monopolist was …*p=$75*
- Plugging this price in $\varepsilon_{Q,P}$, we obtain

$$\varepsilon_{Q,P} = \frac{-2 \bullet 75}{200-2(75)} = \frac{-150}{50} = -3$$

- We just confirmed that the monopolist produces in the elastic region of the demand curve, since

$$\varepsilon_{Q,P} = -3 < -1$$

- That is, in the absolute value of elasticity is larger than 1.

- This result of course applies to any linear inverse demand function $p(q)=a-bq$, and constant marginal cost $MC(q)=c$.
- In particular, the monopolist maximizes profits by setting $MR(q)=MC(q)$, or

$$a-2bq=c$$

which solving for q yields a monopoly output of

$$q=\frac{a-c}{2b}$$

and a monopoly price of

$$p = a - b\left(\frac{a-c}{2b}\right) = \frac{2ab-ab-cb}{2b} = \frac{a+c}{2}$$

Hence, evaluating the price-elasticity, ε, at the profit-maximizing output and price for the monopolist, we obtain

$$\varepsilon(p^m) = \frac{\partial q(p)}{\partial p} \cdot \frac{p^m}{q^m} = -\frac{1}{b} \cdot \frac{\frac{a+c}{2}}{\frac{a-c}{2}} = -\frac{a+c}{a-c} < 0$$

Therefore, elasticity is lower than -1, i.e., the monopolist produces in the *elastic* region of the demand curve if

$$-1 > -\frac{a+c}{a-c}$$

$$1 < \frac{a+c}{a-c} \longleftrightarrow a-c < a+c$$ which holds for all positive marginal costs, c>0

Lerner Index of Market Power

- Lerner Index is a measure of the Market Power

$$\frac{p-MC}{p} = Lerner\ Index$$

$\frac{p-MC}{p}$
→ 0 for perfect competition, where $p=MC$
→ 1 (100%) $\frac{p-MC}{p} = 1 \rightarrow p - MC = p \rightarrow MC = 0$

- The Lerner index is:
  - high for inelastic demands (no close substitutes, a lot of market power), and
  - low for elastic demands (many close substitutes, not much market power).

Application

- Breakfast cereals, why are their prices so high?
- Are firms acting as a monopolist (colluding)?

- We can test that by using the Lerner Index and $\varepsilon_{Q,P}$.
  1. If they colluded $\frac{p-MC}{p} = \frac{-1}{\varepsilon_{Q,P}}$ should be above 65%
  2. What is their true mark-up? Around 41%

- So, probably firms are <u>not</u> colluding:
  - Firms are obtaining large profits, but that is probably because of product differentiation and some loyal customers.
  - Not because they coordinate their output decisions, producing as a single monopolist (cartel).

Query #2

The Lerner Index for a firm operating in a perfectly competitive industry would be

a) less than zero.
b) zero.
c) between zero and one.
d) one.

Query #2-Answer

- Answer B
- We know that the Lerner index is a measure of monopoly power that is the percent markup of price over marginal cost:
  - $(p - MC)/p$
  - Notice that this is the left-hand side of the IEPR
- Because perfectly competitive firms produce at a point where *P*=*MC*, we can see that the term in the numerator would be reduced to zero, also causing the Lerner index to equal zero.
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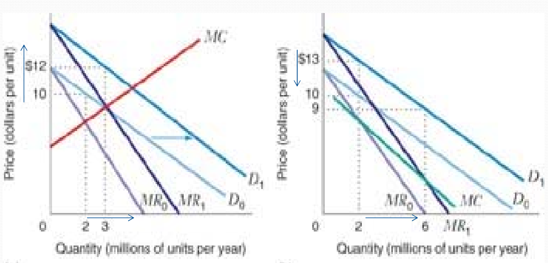
Comparative Statics

Two types:

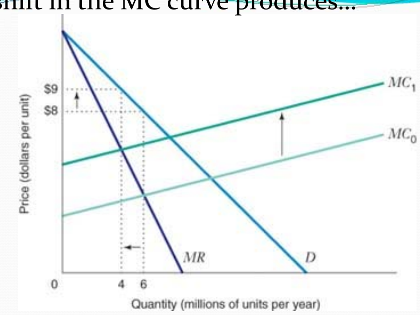
- How does a shift in the demand curve affect the monopolist profit-maximization decisions?
- Or, how does a shift in *MC* affect profit maximization?

Comparative Statics

- An **increase in demand** leads to:
  - An unambiguous increase in quantity,
  - Prices might increase (left figure) or decrease (right figure), depending on the slope of the MC curve



2) A shift in the MC curve produces…



- An upward shift in MC produces ↑*p* and↓*Q*
  - The direction of these effects is unambiguous.
  - Their relative size, however, depends on the elasticity of demand

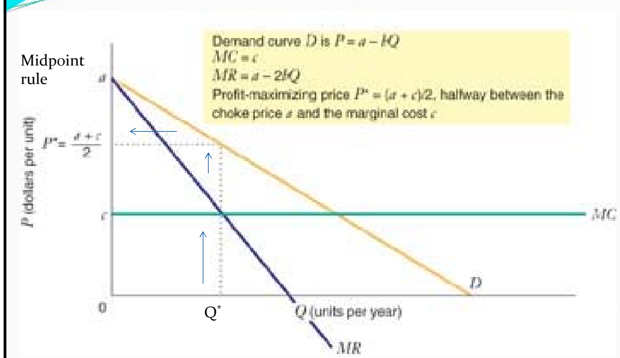
Monopoly Midpoint Rule

$p(q) = a - bq$ ↗ $MC = c$ ↗

- For monopolies with linear demand and a constant MC, they can find their optimal price through the **Monopoly Midpoint Rule.**
- It determines that a profit-maximizing monopoly sets a price that lies at the midpoint between the vertical intercept of the demand curve (choke price, or a) and the MC.

$$\text{that is,} \quad p^* = \frac{a+c}{2}$$

Figure ➔



Example

$$p(q) = a - bq$$
$$MC = c$$
$$MR = a - 2bq$$

$MR = MC$ Profit max output for Monopoly

➔ $a - 2bq = c$
➔ $a - c = 2bq$ ➔ $\boxed{\frac{a-c}{2b} = q}$

$$q = \frac{a-c}{2b} \rightarrow p = a - b\left(\frac{a-c}{2b}\right) = a - \frac{a-c}{2} = \frac{2a-a+c}{2} = \boxed{\frac{a+c}{2}}$$

Example

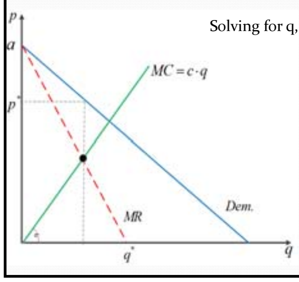
$$p(q) = 100 - 5q$$
$$MC = \$3$$
$$p^* = \frac{a+c}{2} = \frac{100+3}{2} = \frac{103}{2}$$

$p^* = \frac{a+c}{2}$ "*midpoint rule*"

Midpoint between the choke price, a, and MC, c.
(See Figure).

Midpoint rule is only valid if $MC = c$, i.e., if MC is constant in q (Flat Horizontal Line)

If instead MC=c q, (marginal cost is increasing in q, as in the figure) then MR=MC yields an output level



Solving for q,

$$MR \rightarrow a - 2bq = c \cdot q \leftarrow MC$$

$$a = (c + 2b) \cdot q \rightarrow q^* = \frac{a}{c + 2b}$$

Monopoly price is hence

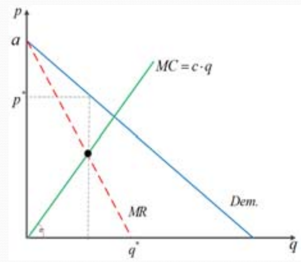
$$p(q) = a - b\left(\frac{a}{c + 2b}\right) = \frac{ac + 2ab - ab}{c + 2b}$$

$$= \frac{a(c + b)}{c + 2b} \neq \boxed{\frac{a + c}{2}}$$

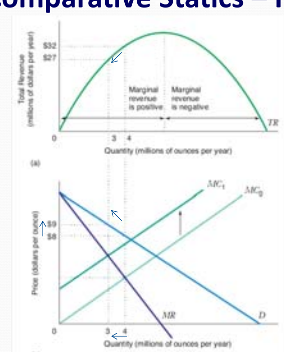
Incorrect Price

Correct Price

Midpoint rule is not valid when $MC(q) \neq c$

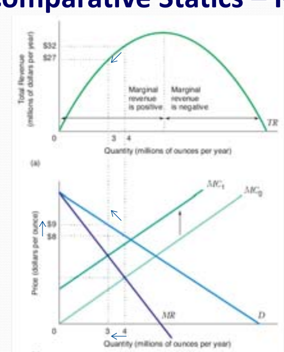


Comparative Statics – Revenue and MC shifts



- Upward shift of *MC* decreases the total revenue of a profit-maximizing monopolist.
- *Application*: if the imposition of an excise tax on beer produces an increase in TR for beer producers, this implies that the industry is *not* acting collusively.

Comparative Statics – Revenue and MC shifts



- If, however, the imposition of an excise tax on beer produces a decrease in TR for beer produces, then we would have some evidence (although still incomplete) that the industry is acting collusively as a single monopolist

Comparative Statics – Revenue and MC shifts

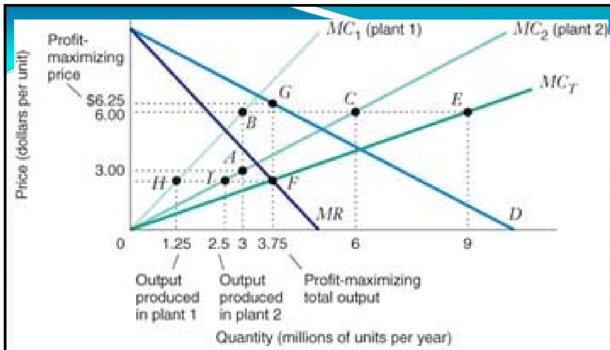
- **Application 11.5: No “smoking gun” for tobacco producers.**
- Tobacco industry is highly concentrated, with the four largest firms accounting for 92% of the market.
- But, are they acting collusively, i.e., as a single profit-maximizing monopolist?
  - Evidence suggests that is *not* the case, since increases in *MC* do not entail a decrease in TR, which is what we should observe if firms were acting as a single monopolist.
  - Why then tobacco firms make so much profits?

  Other forms of capturing surplus from consumers (product differentiation, advertising, etc.) that we will describe in Chapter 12.

Multiplant Monopolies

- Some monopolies operate multiple plants. How does this kind of monopoly maximize profits?
  - We must find both what the firm will produce overall (total product between both plants) and how the monopoly will divide its production between its two plants.

- ***To maximize profit***, a multiplant monopolist must produce at a level where the *MC*’s of all plants are equal.
  - Otherwise, it would have incentives to shift production from the plant with the highest MC (of producing its last unit) to the plant with the lowest MC.



Procedure:

1) Determine MC_T (tricky)
2) Set $MR=MC_T$ (this gives you total output, Q_T) ← point F
3) Plug MC_T into MC_1 and MC_2 to obtain Q1 and Q2 ← points H and I.

Multiplant Monopoly-Example

- Process for profit maximization in Multiplant Monopolies…
  - (1) Find MC_T by horizontally summing individual marginal costs. You should be left with MC_T = some function of Q
  - (2) Invert the expression you found in the previous step (solving for MC_T) so
  - (3) Equate MC_T and MR, and solve for Q_T to find the total quantity produced across all plants
  - (4) Evaluate MC_T at Q_T
  - (3) Plug this value into MC_1 and $MC2$ (inverted from step1) to obtain the production in each plant, Q_1 and Q_2 (how the firm allocates total production across its two plants)

Example of a Multiplant Monopoly

Consider a multiplant monopolist facing linear inverse demand function $p(q) = 120 - 3Q$ with marginal revenue given by $MR = 120 - 6Q$

The monopolist produces in two plants 1 and 2, with marginal cost functions

$MC_1(Q_1) = 10 + 20Q_1$ and $MC_2(Q_2) = 60 + 5Q_2$

Example of a Multiplant Monopoly

1) Find MC_T:
   - MC_T is the horizontal sum of MC_1 and MC_2.
   - In order to sum them, we first need to invert them,

$$\frac{MC_1 - 10}{20} = Q_1 \quad and \quad \frac{MC_2 - 60}{5} = Q_2$$

   - Adding them up, yields

$$Q_T = Q_1 + Q_2 = \frac{MC_T - 10}{20} + \frac{MC_T - 60}{5} = 0.25MC_T - 12.5$$

2) Invert the above expression, so we have MC_T, as a function of Q.

$Q = 0.25MC_T - 12.5$ solving for MC_T, we obtain

$MC_T = 50 + 4Q$

3) Set $MR = MC_T$ (as in any monopoly profit-maximization problem),

$120 - 6Q = 50 + 4Q$

$70 = 10Q$, which yields a total output of $Q_T = 7 units$

Therefore, evaluating the demand function at q=7 units, we obtain a monopoly price of

$p = 120 - 3 \times 7 = \$99$

4) Evaluate MC_T at Q_T=7 units

$MC_T(Q = 7) = 50 + 4 \times 7 = \$78$ ← This is the height of the MC curve at Q_T=7

5) Plug this value, $MC_T(Q = 7) = \$78$, into MC_1 and MC_1(inverted from step 1) in order to find Q_1. Similarly for MC_2 and Q_2.

$$Q_1 = \frac{78 - 10}{20} = 3.4 units \quad Q_2 = \frac{78 - 60}{5} = 3.6 units$$

Of course, we can check that the sum of the production in all plants, $Q_1 + Q_2$, coincides with Q_T=7 units since

$Q_1 + Q_2 = Q_T$ since

$3.4 + 3.6 = 7$ units

Output Choice with Two Markets (single plant)

When analyzing a multiplant monopolist, we consider that he operates two plants but faced a single demand (e.g., world demand):

Single market demand

Output from plant 1, Q_1 → Single market demand ← Output from plant 2, Q_2

Output Choice with Two Markets (single plant)

However, we could consider the opposite:

A monopolist that, operating a single plant, sells its goods to two different markets (each with a different demand function):

Demand 1 ← Single plant → Demand 2

Output Choice with Two Markets (single plant)

- We assume the monopoly charges the same price in both markets…(in subsequent chapters we will allow for Price Discrimination)
- **Procedure to follow:**

  (1) First, we want to aggregate the individual market demands so that we can determine the aggregate marginal revenue for this firm.

  (2) We then set aggregate MR equal to the MC to find optimal price.

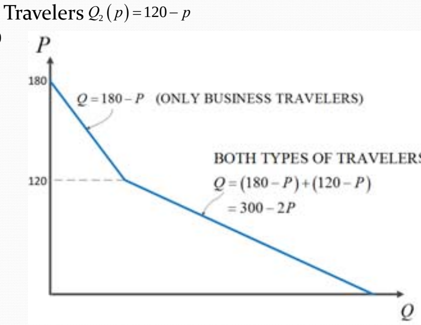
  (3)We then find Q by using the aggregate demand curve.

Example – Output Choice with two markets

Business Travelers $Q_1(p) = 180 - p$

Vacation Travelers $Q_2(p) = 120 - p$

$MC(Q) = 30$



When pε[120,180] only business travelers demand positive units, and :

- Inverse demand is $p(Q)=180-Q$
- With associated MR given by $MR(Q)=180-2Q$

- When p<120, both types of customers demand positive amounts, demand is $Q=300-2p$, so inverse demand is $p(Q)=150-0.5Q$, with associated $MR(Q)=150-Q$

How to determine a unique Monopoly price if we have 2 demand functions (with a MR each)?

1) Can the price *p* be above $120? → $p(q)=180-q$

Let's assume that is the case $MR=180-2q$

MR=MC

180-2Q=30

$180-2Q=30 \quad Q=75units$ entailing a price of $p=180-75=105$

But then *p* is not greater than 120!!! → contradicts our initial assumption of *p*>$120.

How to determine a unique Monopoly price if we have 2 demand functions (with a MR each)?

2) Can the price *p* be below $120? → $p(q)=150-\frac{1}{2}q$

$MR=150-q$

MR=MC

$150-Q=30 \quad Q=120units \quad p=150-0.5\times120=\$90$

which is indeed below $120

Hence, $Q_1(p)=180-90=90$ units (business)
$Q_2(p)=120-90=30$ units (vacation)

120 units overall as we determined above

↑
P=$90

Cartel

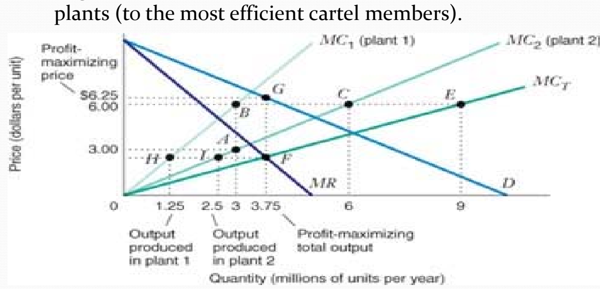
- Cooperation by separate firms in order to maximize joint profits.
  - OPEC is a popular example of a cartel
  - Auction houses Sotheby's and Christie's is another example of a cartel (collusion in the commissions they charge for selling art pieces).
  - Lysine price-fixing, as portrayed in the funny movie *The Informant*, with Matt Damon.
  - Crazy (but real!) example of a cartel: 3 biggest chicken farms in Chile reduced production and increased prices. Estimated damage to consumers: US$ 1 billion!

Cartel

- In a cartel, ***we can determine the profit-maximizing output level and price by the same process we used for multiplant monopolist.***
  - Practically speaking, a cartel is no different than a multiplant monopolist
- Application in your textbook: OPEC doing a bad job at maximizing joints profits! That is, their joint profit could be even higher if they truly behaved as a single monopolist (more pain at the pump)!

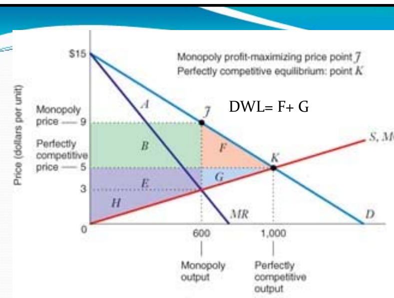
Cartel

- *Usual misconception*: the cartel does not need to set the same output level to each of its members.
- Similarly to a multiplant monopolist, the cartel allocates a larger share of its total production to the most efficient plants (to the most efficient cartel members).



Consumer Welfare under Monopoly

- How is the consumer benefitted or harmed in the monopolistic market versus a perfectly competitive market?
  - Under monopolists, there is a deadweight loss to consumer welfare
- *Let's look at a graph to illustrate this change in consumer welfare…*



	Perfect Competition	Monopoly	Impact of Monopoly
Consumer surplus	$A + B + F$	A	$-B-F$
Producer surplus	$E + G + H$	$B + E + H$	$B-G$
Net economic benefit	$A + B + E + F + G + H$	$A + B + E + H$	$-F-G$

Deadweight loss of monopoly

- Note, however, that the DWL of a monopoly might be understated because of the existence of rent-seeking activities (e.g., lobbying)
- That is, the lower bound of the welfare loss of monopoly is area F+G
- While the upper bound of DWL of monopoly is
  (F + G) + (B + E + H)
  where B+E+H represents the monopoly profits, which this firm can use (at least in part) to lobby so that entry is not allowed in the industry. (Entry of several firms would erode all these profits, as the market structure approaches perfect competition).

Monopoly profit (B+E+H)= maximum amount that a firm is willing to spend on rent-seeking activities to protect or preserve its monopoly power.

Example: Cable companies spent millions in the 1990's lobbying Congress to preserve regulations that limit the ability of satellite broadcasters to compete with traditional cable service.

While they succeeded at delaying the "green light" on satellite broadcast, they ultimately failed at stopping them from entering into the traditional cable service.

Query #3

Suppose that the perfectly competitive soybean industry in the United States is monopolized.
- Under perfect competition, the equilibrium price was \$2 and quantity was 100,000.
- The monopolist raises price to \$5 and restricts quantity to 70,000.

- Assume that the monopolist is maximizing profits and that it faces a linear, upward-sloping marginal cost curve that begins at the origin. Also assume that this marginal cost curve is the industry supply curve under perfect competition.
- What is the loss in consumer surplus that the monopolist captures in the form of profit?

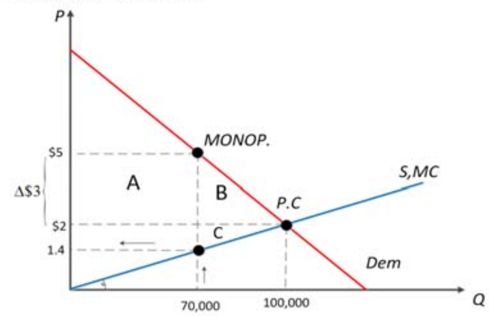
a) \$500,000
b) \$350,000
c) \$300,000
d) \$210,000

Query #3 - Answer

- Answer D
- The loss in consumer surplus that the monopoly captures as profit is equal to the change in price relative to perfect competition multiplied by $Q_{monopoly}$
  - (See rectangle A in the figure of the next slide)
- **Monopoly**: Q* = 70,000 and P* = \$5
- **Perfect Competition**: Q* = 100,000 and P* = \$2
- ΔPrice = \$3 and $\text{Quantity}_{\text{Monopoly}}$ = 70,000
  - Hence, the loss in Consumer Surplus due to monopoly is

$$\$3 \times 70,000 = \$210,000$$

QUERY # 3-ANSWER



RECTANGLE $A=(5-2)\times 70,000=\$210,000$ ← BUT ONLY THIS PART IS CAPTURED BY THE MONOP. IN FORM OF PROFITS.

TRIANGLE $B=\frac{1}{2}\times(5-2)\times 30,000=$ 45,000

255,000 *LOSS IN CS.*

Additional Query #3*b*

- What is the deadweight loss (DWL) due to monopoly in the example analyzed in Query #3?
  - The triangle from 70,000 to 100,000 units, above the MC curve and below the Demand curve (triangle B+C).
  - But, what is its height? For that, we need to know the expression of the MC curve.
    - Easy to find! First, note that the MC curve originates at zero, and reaches a price of $2 when Q=100,000.
      - Then, it satisfies $2=100,000*a* where, solving for the slope *a*, we obtain *a*=1/50,000.
      - Hence, the MC curve is $MC=\left(\frac{1}{50,000}\right)\cdot Q$
    - Evaluating such MC curve at Q=70,000, we obtain a price of p=1/50,000*70,000=$1.4.
    - Hence, the triangle of the DWL (areas B+C) is given by

$$\frac{1}{2}\times(5-1.4)\times(100,000-70,000)=54,000$$

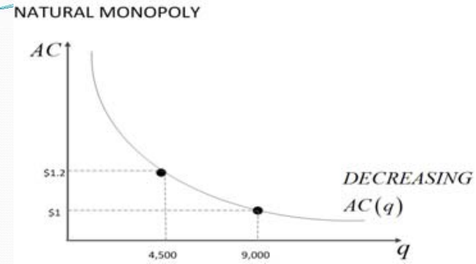
Why do Monopolies Exist?

- ***Natural Monopolies:*** A market in which, for any relevant level of industry output, the total cost incurred by a single firm producing that output is less than the combined total cost that two or more firms would incur if they divided that output among themselves.

(Note that demand must be low).

- ***Example:*** Water distribution companies, phone companies in the 1950s, Natural gas distribution companies
- Figure on next page

NATURAL MONOPOLY



$$TC_{one\ firm}=\$1\bullet 9,000\,units=\$9000$$

$$TC_{two\ firms}=2\bullet[\$1.20\bullet 4,500\,units]=\$10,800$$

One firm is better! ➔ *Example*: Satellite TV in the U.K. (small country with only one firm)

Barriers to Entry (other reasons why monopolies exist)

- ***Structural:*** incumbent firms have cost or demand advantages that would make it unattractive for a new firm to enter the industry (e.g., positive network externalities). Think of eBay example from book.
  - Critical Mass: As a seller, you prefer to go to eBay (rather than some new and unknown competitor to eBay) because many potential buyers visit the site. (Similarly if you're a buyer)
- ***Legal:*** incumbent firm is legally protected against competition. Patents are an example of this.
  - Optimal duration of a patent, from a social welfare point of view (still some controversy, think about patents for new drugs).
- ***Strategic:*** incumbent firm takes explicit steps to deter entry (this might be where an incumbent firm has a reputation of fighting off newcomers through price wars)…
  - Alternative: Overproduction under incomplete information.

More on Strategic barriers to entry:

- In 1994 United Airlines and Frontier Airlines fought a "price war" in the Billings-Denver Route (sometimes with half prices)
- However, after a year, Frontier withdrew from the route.
- What happened after?
  - You are right…UA increased prices from $100 to $250 (or more!)
  - Frontier logged a complaint with the DOJ arguing United Airlines used predatory pricing (that is, to force Frontier out of the market and afterwards behave as a monopolist), and Frontier won.

Tax Incidence on Consumers

- Reading from Perloff's textbook (posted on Angel)
- Can the imposition of a $1 tax to a monopolist lead to an increase in the price consumers pay of more than $1?

Let's look at an example…

- Consider the following Constant Elasticity Inverse Demand Function

$$p(Q)=Q^{\frac{1}{\varepsilon}}$$

where the exponent $\frac{1}{\varepsilon}$ is the price elasticity.

Tax Incidence on Consumers

Hence, total revenue is $TR(Q)=pQ=Q^{1/\varepsilon}\cdot Q=Q^{1+\frac{1}{\varepsilon}}$,

thus implying that marginal revenue is

$$MR(Q)=\frac{\partial TR}{\partial Q}=\left(1+\frac{1}{\varepsilon}\right)\cdot Q^{\frac{1}{\varepsilon}}$$

- Assume that the monopolist's marginal costs are MC(Q)=c, but after the tax they become c+τ, where τ denotes the per-unit tax.
- We can now use the monopolist's profit-maximizing condition MR=MC, we obtain

$$\left[1+\frac{1}{\varepsilon}\right]\cdot Q^{\frac{1}{\varepsilon}} = c+\tau$$

Solving for *Q*, yields a monopoly output of

$$Q^{\frac{1}{\varepsilon}} = \frac{c+\tau}{1+\frac{1}{\varepsilon}} \rightarrow Q = \left(\frac{c+\tau}{1+\frac{1}{\varepsilon}}\right)^{\varepsilon}$$

- Plugging this monopoly output into the inverse demand , $p(Q)=Q^{\frac{1}{\varepsilon}}$, yields

$$p(Q) = Q^{\frac{1}{\varepsilon}} = \left[\frac{c+\tau^{\varepsilon}}{1+\frac{1}{\varepsilon}}\right]^{\varepsilon\bullet\frac{1}{\varepsilon}}$$

- How has the monopoly price changed after introducing (or increasing) a sales tax?
  - That is, how does *p* change in *τ*? Let's differentiate with respect to *τ* to figure it out

$$\frac{\partial p}{\partial \tau} = \frac{1}{1+\frac{1}{\varepsilon}}$$

But we know that ε<-1 for the monopolist (recall that the monopolist produces in the elastic segment of the demand curve), so the imposition of the tax increases p by more than the tax

Example:

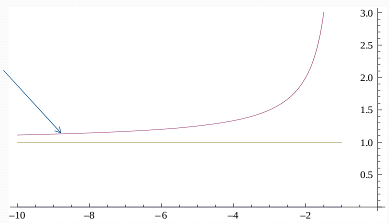
If ε=3, then the previous derivative becomes

$$\frac{\partial p}{\partial \tau} = \frac{1}{1+\frac{1}{-3}} = \frac{-3}{-3+1} = 1.5 > 1$$

indicating that a $1 tax on the monopolist produces an increase in prices of more than $1, i.e., of $1.5.

Tax incidence in a monopoly

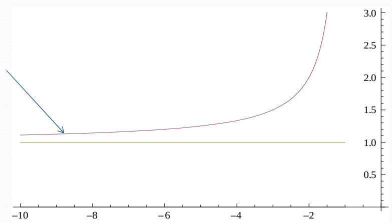
$$\frac{\partial p}{\partial \tau} = \frac{1}{1+\frac{1}{\varepsilon}}$$



- The horizontal axes measures price-elasticity, ε, while the vertical axes measures the derivative of *p* with respect to taxes.
- A given increase in taxes produces a *more-than-proportional* increase in monopoly prices (above the 100%). This is graphically depicted by the fact that $\frac{\partial p}{\partial \tau}$ lies above the flat line at 1 for all price elasticities.

Tax incidence in a monopoly

$$\frac{\partial p}{\partial \tau} = \frac{1}{1+\frac{1}{\varepsilon}}$$



- In addition, the more inelastic demand is (elasticity smaller than -1, and thus close to the origin), the larger the price increase that the monopolist can pass onto consumers.

Tax incidence under monopoly

- The previous example, where the monopolist faces a constant-elasticity inverse demand function, $p(Q) = Q^{1/\varepsilon}$, suggests that…
  - a given 1% increase in taxes leads to an increase in monopoly prices of *more than* 1%.
- This is, however, not always the case.
- The responsiveness of monopoly prices to a given increase in taxes depends on the particular demand function the monopolist faces.
- Let's look at one example…

Example

- Let's look at an example where a tax is imposed on a monopolist.
- What happens to price and quantity after tax?

$$p = 24 - Q$$
$$MR = 24 - 2Q$$
$$MC = 2Q, \text{where } \tau = 8$$

- (1) **No Tax**…MR=MC
  - $24 - 2Q = 2Q \rightarrow Q = 6$
  - $p = 24 - 6 = \$18$
- (2) **Tax**… MR=MC
  - $24 - 2Q = 2Q + 8(\text{this is the tax}) \rightarrow Q = 4$
  - $P = 24 - 4 = \$20$

Example Conclusion:

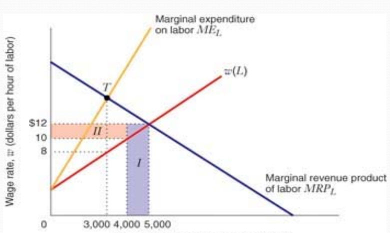
- So, Δp=$2.
- ***Tax incidences*** out of $8 total tax…
  - $2 are passed on the consumer
  - $6 are absorbed by the monopolist

Monopsony

- A market of a single buyer and many firms (sellers).
  - *Example*: coal mine in small town in West Virginia, or Walmart in a really small town.
  - There are almost no other employers (buyers of labor services) but there are many potential employees (sellers of labor services).

Monopsony

- ***To profit maximize,*** this firm will want to find a level of production where $\mathbf{MRP_L = ME_L}$ (Marginal revenue product of labor = marginal expenditure of labor).



Why does profits maximization entail $\mathbf{MRP_L = ME_L}$?

- Consider a production function Q=f(L) where, as usual, labor is productive, f'(L)>0, but at a decreasing rate, f''(L)<0
- The coal mine is competing in a perfectly competitive market for coal, selling every ton of coal at a given market price *p*

$$TR = p*Q = p*f(L)$$

Why does profits maximization entail $MRP_L = ME_L$?

And the effect of hiring one more worker on total revenue $TR = p \cdot f(L)$ is

$$MRP_L = \frac{\partial TR}{\partial L} = p * f'(L)$$

Intuitively, the additional worker produces f'(L) units of output, and these units are sold at a price of *p* at the international market for coal.

However, hiring workers also entails costs...

TC= w(L)•L

Hence, the marginal cost of hiring an additional worker is

$$\frac{\partial TC}{\partial L} = \underbrace{w(L) \bullet 1}_{\text{Area I in the figure}} + \underbrace{\frac{\partial w(L)}{\partial L} \bullet L}_{\text{Area II in the figure}}$$

It represents the extra cost of hiring one more worker (salary).

It represents the extra cost from raising the wage to all existing workers.

Therefore, the marginal expenditure on labor, ME_L, is exactly this marginal cost of hiring one more worker, that is,

$$ME_L = \frac{\partial TC}{\partial L} = w(L) + \frac{\partial w(L)}{\partial L} \bullet L$$

and since the derivative $\frac{\partial w(L)}{\partial L}$ is greater than 0, then the second term is positive, implying that $ME_L > w(L)$

Graphically, ME_L lies *above* the supply curve, w(L)

- More generally, since profits are

  Profits=TR –TC

  If we take first order conditions with respect to L, we obtain,

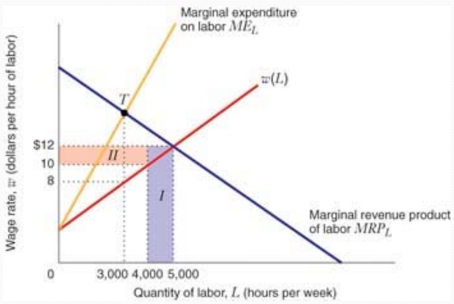
$$\frac{\partial TR}{\partial L} - \frac{\partial TC}{\partial L} = 0, \text{ or } \frac{\partial TR}{\partial L} = \frac{\partial TC}{\partial L}$$

And relabeling these terms using the definitions from previous slides, we obtain

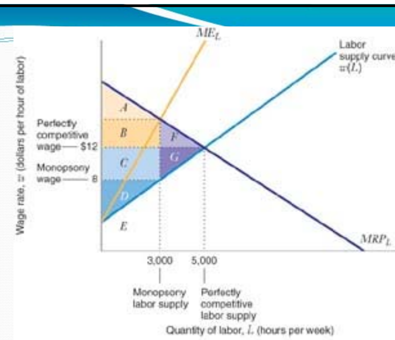
$$MRP_L = ME_L$$

Hence, in order to maximize its profits, the monopsonist increases L until the point in which MRP_L crosses ME_L

In order to maximize its profits, the monopsonist increases L until the point in which MRP_L crosses ME_L



Deadweight Loss of Monopsony: Areas *F*+*G*



	Perfect Competition	Monopsony	Impact of Monopsony
Consumer surplus	$A+B+F$	$A+B+C$	$C-F$
Producer surplus	$C+D+G$	D	$-C-G$
Net economic benefit	$A+B+C+D+F+G$	$A+B+C+D$	$-F-G$

CS + PS

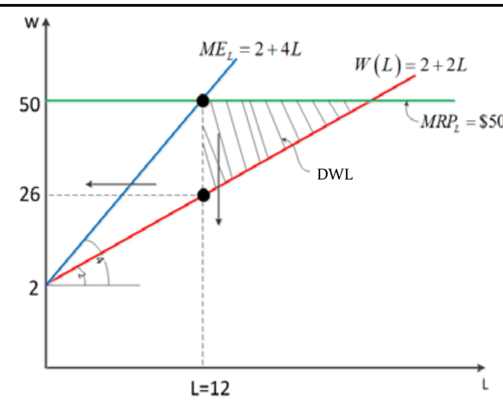
Example-Monopsony

- Consider a Production Function Q = 5L and assume that the monopsony sells each unit in an international perfectly competitive market at a price of *p*=$10.
  - In addition, assume that the Labor Supply Curve is
    - w(L)= 2 + 2L
  - Hence

$$ME_L = w(L) + L\frac{\partial w(L)}{\partial L} \rightarrow (2+2L) + L\cdot 2 = 2+4L$$

$$MRP_L = p\cdot MP_L = \$10\cdot\frac{\partial Q}{\partial L} = \$10\cdot 5 = 50$$

Let's depict them →



Equilibrium occurs at $MRP_L = ME_L$ 50=2+4L → L=12 workers
Plugging L=12 into W(L)=2+2*12=$26 ⟵ salary

- In the above example:
  - Under **monopsony**, wage is w=\$26 and 12 workers are hired
  - Under **perfect competition**, wage would be determined by $MRP_L=W(L)$ (rather than $MRP_L=ME_L$), entailing a higher wage of w=\$50 and 50=2+2L, i.e., L=24 workers being hired.
  - Hence, the presence of a monopsony gives rise to a salary wedge relative to perfectly competitive labor markets.
    - In this example, such salary wedge is 50-26=\$24.

Wedge between perfectly competitive salaries and monopsonistic salaries

- Empirical evidence of the salary wedge for Walmart (it is the most important employer is many small localities):
  - Insignificant in large metropolitan areas.
    - Hence, w_{PC} is similar to $w_{Monopsony}$.
  - Significant in small localities (especially in the South).
    - In particular, the difference $w_{PC} - w_{Monopsony}$ is positive and larger than 5% for some localities.