Cheap Talk in a Lobbying Game

Reading references:

- *– Grossman and Helpman, Chapter 4*
- *– Tadelis, section 18.2 - 18.3*
- *– Osborne, section 10.8*

Consider a lobby trying to influence the decisions of a politician. In particular:

1) Lobbying (interest group) knows the state of the world θ.
Examples: Costs of health care in a hospital, or the environmental degradation of a region.

2) Politician doesn't know the realization of θ, but receives a message from the interest group in order to inform him before selecting a certain policy, p.

Utility function of the Politician:
$G(p,\theta) = -(p-\theta)^2$: Minimize the distance between the policy implemented by the politician, p, and the true state of the world, θ. Therefore, a policy $p = \theta$ maximizes the politician's utility.

Utility function of the Special Interest Group (SIG):
$u(p,\theta) = -(p-\theta-\delta)^2$: Minimize the distance between the policy implemented by the politician, p, and the ideal policy for the SIG, which is $\theta+\delta$. Therefore, policy $p = \theta+\delta$ maximizes the SIG's utility. Furthermore, parameter δ can be understood as the bias between the politician and the SIG preferences. When δ=0 their utility functions coincide, and so does their ideal policy $p = \theta$.

In standard signalling models, sending a higher message was more costly for the sender.
Example: Acquiring a higher education in the labor market signaling game, or producing a larger output in the limit pricing signaling game.

In cheap-talk sending different messages entail the same cost for the sender, and such a cost is zero, i.e., messages are always free. Formally, we say that messages are payoff inconsequential for the sender, as they do not affect his payoff function. In addition, note that messages are also payoff inconsequential for the receiver (the politician) who only cares about the true state of the world θ and how far his implemented policy lies from such state of the world.

For simplicity, we first consider the setting in which there are only two different states of the world, then we extend our analysis to a setting with three states of the world, and finally generalize to a continuum of states of the world.

Two States of the World

$$\theta = \begin{cases} \theta_H, & \text{with probability } \alpha \\ \theta_L, & \text{with probability } 1-\alpha \end{cases}$$

Example: high/low envrionmental degradation (only known by SIG)

Responder. We next follow the standard steps to check for a separating PBE in which the lobbyist truthfully reports the true state of the world to the politician. First, we examine the receiver's optimal responses after each of the messages he could receive:

a) The politician implements a policy $p = \theta_H$ if he is told $\theta = \theta_H$:

$$G(p,\theta) = -(p-\theta_H)^2$$
$$\Rightarrow -2(p-\theta_H) = 0$$
$$\Rightarrow p = \theta_H$$

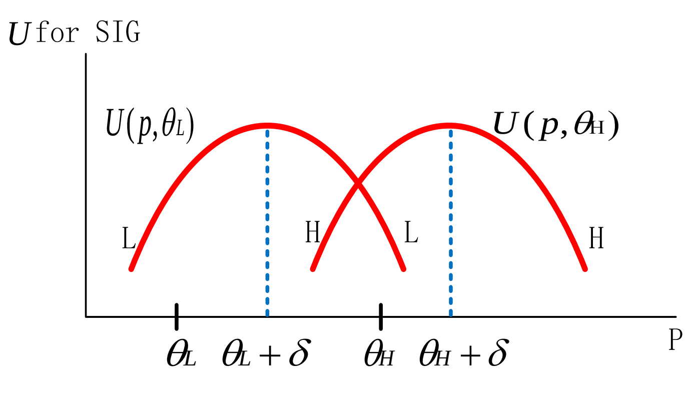
b) The politician implements a policy $p = \theta_L$ if he is told $\theta = \theta_L$:

$$G(p,\theta) = -(p-\theta_L)^2$$
$$\Rightarrow p = \theta_L$$

c) The politician implements a policy $p = \alpha\theta_H + (1-\alpha)\theta_L$ if he is not told anything. Thus his expected utility is

$$G(p,\theta) = -\alpha(p-\theta_H)^2 - (1-\alpha)(p-\theta_L)^2$$
$$\Rightarrow -2\alpha(p-\theta_H) - 2(1-\alpha)(p-\theta_L) = 0$$
$$\Rightarrow p = \alpha\theta_H + (1-\alpha)\theta_L$$

We want to find conditions for informative lobbying: policy maker believes lobbyist's messages



Sender. We now analyze the incentives of the SIG to truthfully report the state of the world:

a) If θ_H is the true state of the world, and sending $\theta = \theta_H$ generates a policy response of $p = \theta_H$, then SIG would never deviate towards θ_L.

b) If θ_L is the true state of the world, and sending a message $\theta = \theta_L$ entails a policy response of $p = \theta_L$. But why not send $\theta = \theta_H$ instead? Since the SIG is upward biased, they might want to report the high state of the world if θ_H is closer to its ideal, $\theta_L + \delta$, than θ_L is.

- By truthtelling the SIG obtains a utility:

$$u(p, \theta_L) = -(\theta_L - (\theta_L + \delta))^2$$

- By misrepresenting the true state of the world, the utility of the SIG becomes:

$$u(p, \theta_L) = -(\theta_H - (\theta_L + \delta))^2$$

- Therefore, the SIG truthfully reports the state of the world if and only if

$$-(\theta_L - (\theta_L + \delta))^2 \geq -(\theta_H - (\theta_L + \delta))^2$$

or solving for the SIG's bias, δ, we obtain

$$\Rightarrow \delta \leq \frac{\theta_H - \theta_L}{2}$$

That is, the SIG has incentives to convey the true state of the world to the politician if and only if its bias is sufficiently small. As a special case, when the bias is zero truthful reporting can be sustained. In addition, when the two states of the world become more distant to one another, the range of biases sustained truthful reporting expands.

Separating PBE Summary. We can next summarize under which conditions a separating PBE can be sustained in this cheap talk game. This type of PBE is often referred to as a "fully informative PBE," since all observed information by the SIG is conveyed to the politician. In particular:

1) Beliefs are $\mu(\theta_H \mid \theta_H) = 1$ and $\mu(\theta_L \mid \theta_L) = 1$

2) The politician's optimal responses are:

If θ_H is received, then θ_H is believed, and $G(p, \theta) = -(p - \theta_H)^2 \Rightarrow p = \theta_H$

If θ_L is received, then θ_L is believed, and $G(p, \theta) = -(p - \theta_L)^2 \Rightarrow p = \theta_L$

3) The SIG's optimal messages are:

If θ_H is the true state of the world, a message $\theta = \theta_H$ is sent;

If θ_L is the true state of the world, a message $\theta = \theta_L$ is sent if and only if

$$\delta \leq \frac{\theta_H - \theta_L}{2}$$

Note that if, instead, the bias satisfies $\delta > \frac{\theta_H - \theta_L}{2}$, then both senders have incentives to send θ_H. In this context, when the politician receives a message of θ_H he doesn't know whether such a message originates from a high or low state of the world. In this case, a pooling PBE emerges, as we describe below:

Pooling PBE in which the SIG sends message θ_H in both states of the world. Summary:

1) Politician's beliefs are
$\mu(\theta_H \mid \theta_H) = \mu(\theta_L \mid \theta_H) = 1/2$
$\mu(\theta_H \mid \theta_L)$ and $\mu(\theta_L \mid \theta_L)$ are arbitrary

3) Politician's optimal responses:
If a message θ_H is observed, then the true state of the world can be anything, and he implements a policy $p = \alpha\theta_H + (1-\alpha)\theta_L = \theta_H/2 + \theta_L/2$
If a message θ_L is observed, then θ_L is believed and he implements a policy $p = \theta_L$.

4) SIG messages:
If θ_H is the true state of the world, then the SIG sends $\theta = \theta_H$.
If θ_L is the true state of the world, then the SIG also sends $\theta = \theta_H$ if his bias is sufficiently high, $\delta > \frac{\theta_H - \theta_L}{2}$ (Recall that this condition was found from comparing the SIG's payoffs, $-(\theta_L - (\theta_L + \delta))^2 \leq -(\theta_H - (\theta_L + \delta))^2$)

Intuition: If the divergence between the preferences of the policy maker and the SIG is sufficiently large, then the PBE is uninformative.

Pooling PBE in which the SIG sends message θ_L in both states of the world.
Let's examine if this strategy profile can be sustained as a PBE.

1) Politician's beliefs are:
$\mu(\theta_H \mid \theta_L) = \mu(\theta_L \mid \theta_L) = 1/2$
$\mu(\theta_H \mid \theta_H)$ and $\mu(\theta_L \mid \theta_H)$ are arbitrary

3) Politician's optimal responses are:
If a message θ_L is observed (in equilibrium), then he implements a policy $p = \alpha\theta_H + (1-\alpha)\theta_L = \theta_H/2 + \theta_L/2$
If a message θ_H is observed (off-the-equilibrium), then θ_H is believed (for instance), and he implements a policy $p = \theta_H$

4) SIG's messages:
If θ_L is the true state of the world, he sends $\theta = \theta_L$ if

$$-(\theta_L - (\theta_L + \delta))^2 \geq -(\theta_H - (\theta_L + \delta))^2 \Rightarrow \delta < \frac{\theta_H - \theta_L}{2}$$

If θ_H is the true state of the world, he sends $\theta = \theta_L$ if

$$-(\theta_L-(\theta_H+\delta))^2 \geq -(\theta_H-(\theta_H+\delta))^2 \Rightarrow \delta < \frac{\theta_L-\theta_H}{2}$$

which is impossible since $\delta \geq 0$ by definition.

Hence, if the SIG observing θ_H never wants to send a message of $\theta = \theta_L$. Therefore, this pooling strategy profile where all types of SIG send θ_L cannot be sustained as a PBE.

Three States of the World

Let us first explore under which conditions a separating (informative) PBE can be sustained in this context.

1) Politician's beliefs:

$$\left.\begin{matrix} \mu(\theta_H \mid \theta_H)=1 & 0 & 0 \\ 0 & \mu(\theta_M \mid \theta_M)=1 & 0 \\ 0 & 0 & \mu(\theta_L \mid \theta_L)=1 \end{matrix}\right\} \textit{Trusted Messages}$$

2) Politician's optimal responses:

If message θ_H is received, then θ_H is believed and he implements a policy $p=\theta_H$.

If message θ_M is received, then θ_M is believed and he implements a policy $p=\theta_M$.

If message θ_L is received, then θ_L is believed and he implements a policy $p=\theta_L$.

3) SIG's messages:

• a) If θ_H, then the SIG sends $\theta = \theta_H$.

• b) If θ_L, then the SIG could send $\theta = \theta_H$ or $\theta = \theta_M$ which we need to check:

- If we find a condition such that θ_M is not sent, then θ_H is not sent either (as it is even further away from the SIG's ideal policy, $\theta_L+\delta$)
- By truthtelling, the SIG's utility is $u(p,\theta_L) = -(\theta_L-(\theta_L+\delta))^2$
- By misrepresenting, the SIG's utility is $u(p,\theta_L) = -(\theta_M-(\theta_L+\delta))^2$
- Therefore, truthtelling can be sustained when θ_L is the true state of the world if and only if

$$-(\theta_L-(\theta_L+\delta))^2 \geq -(\theta_M-(\theta_L+\delta))^2$$
$$\Rightarrow \delta \leq \frac{\theta_M-\theta_L}{2}$$

- That is, if the bias satisfies $\delta \leq (\theta_M-\theta_L)/2$, the SIG will truthly reveal θ_L, rather than sending θ_M or θ_H

• c) If θ_M, then $\theta = \theta_L$ will never be sent. In addition, sending an untruthfull message $\theta = \theta_H$ must be unprofitable. Following a similar argument as above, that happens when the bias satisfies $\delta \le (\theta_H - \theta_M)/2$.

***** By adding a third state of the world, we have added more conditions on the preference divergence (bias) between the politician and the SIG that ensures truthful reporting.

***** These two conditions are necessary and sufficient for a fully informative (separating) PBE with 3 types.

What if one of the above conditions is violated?

If $\delta \le (\theta_M - \theta_L)/2$ holds, then the SIG with type θ_L truthfully reports θ_L.

If, instead, $\delta > (\theta_H - \theta_M)/2$ holds, then the SIG with type θ_M deviates and reports θ_H. We can then sustain a "partially informative equilibrium," as we show next.

Partially informative equilibrium:

1) Politician's beliefs

$\mu(\theta_L \mid \theta_L) = 1$

$\mu(\theta_M \mid \theta_H) = \mu(\theta_H \mid \theta_H) = 1/2$

$\mu(\cdot \mid \theta_M)$ arbitrary since it is never observed

3) Politician's optimal responses:

If θ_L is received, $\Rightarrow p = \theta_L$

If θ_H is received, $\Rightarrow$ θ_M and θ_H are equally likely, and $p = (\theta_M + \theta_H)/2$

If θ_M is received, $\Rightarrow$ anything is believed, any policy is implemented

4) SIG's messages:

• a) If θ_H is the true state $\rightarrow$ SIG sends "not low" and induces a policy $p = (\theta_M + \theta_H)/2$, which is better than saying "low" and getting $p = \theta_L$

• b) If θ_M is the true state $\rightarrow$ SIG prefers to send "not low" than θ_L if and only if

$$-\left(\frac{\theta_H + \theta_M}{2} - (\theta_M + \delta)\right)^2 \ge -\left(\theta_L - (\theta_M + \delta)\right)^2$$

$$\Leftrightarrow \delta \ge \frac{\theta_H - \theta_M}{4} - \frac{\theta_M - \theta_L}{2}$$

• c) If θ_L is the true state $\rightarrow$ SIG prefers "low" to "not low" if and only if

$$\delta \le (\theta_H - \theta_M)/4 + (\theta_M - \theta_L)/2$$

Numerical Example: Consider parameter values $\theta_L = 0, \theta_M = 4, \theta_H = 6$. A fully informative PBE requires:

$$\delta \leq \frac{4-0}{2} = 2,\ and\ \delta \leq \frac{6-4}{2} = 1$$

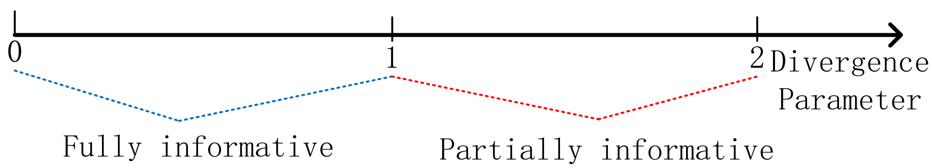
that is

$$\delta \leq 1$$

Instead, a partially informative PBE requires:

$$\delta \geq \frac{6-4}{4} - \frac{4-0}{2} = -\frac{3}{2},\ and\ \delta \leq \frac{1}{2} + 2 = \frac{5}{2}$$

The following figure depicts the bias between the politician and the SIG. When it is small, a fully informative PBE can be sustained while when the bias is relatively large a partially informative equilibrium can be supported.



Pooling PBE. Let us next analyze under which condition a pooling equilibrium can be sustained, in which no information is transmitted from the SIG to the politician. In particular, the SIG sends a single message (for instance, θ_H) regardless of the true state of the world.

1) Politician's beliefs:

Any state of the world can be (no update of priors) $\begin{cases} \mu(\theta_H \mid \theta_H) = \mu(\theta_L \mid \theta_H) = 1/3 \\ \mu(\cdot \mid \theta_M) = 1/3 \\ \mu(\cdot \mid \theta_L) = 1/3 \end{cases}$

2) Politician's optimal responses:

If a message θ_H is received, then

$$\max_p G(p,\theta) = -\frac{1}{3}(p-\theta_H)^2 - \frac{1}{3}(p-\theta_M)^2 - \frac{1}{3}(p-\theta_L)^2$$

that is, the politician implements a policy $p = (\theta_H + \theta_M + \theta_L)/3$, which is just the average of all three possible states of the world (as they were all equally likely according to the politician's priors). A similar argument applies when the politician receives any other message, as we describe next.

If a message θ_M is received, then the politician implements $p = (\theta_H + \theta_M + \theta_L)/3$

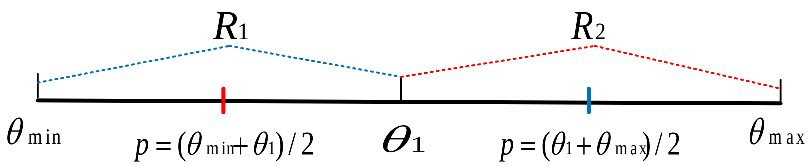
If a message θ_L is received, then the politician implements $p = (\theta_H + \theta_M + \theta_L)/3$

4) SIG's optimal messages:

• If θ_H is the true state: nothing is believed, any message can be sent
• If θ_M is the true state: nothing is believed, any message can be sent
• If θ_L is the true state: nothing is believed, any message can be sent

Cheap talk game with a continuum of types

Consider now that the true state of the world is uniformly distributed between θ_{min} and θ_{max}, where $\theta_{max} > \theta_{min}$. Let us start analyzing whether a "two-step equilibrium" can be sustained, whereby the SIG sends a message when the true state of the world lies between θ_{min} and θ_1, and a different message when the true state of the world lies between θ_1 and θ_{max}, as depicted in the next figure.



The SIG that observes a true state of the world of exactly θ_1 must then be indifferent between sending a message R1, which induces a policy of $p = (\theta_{min} + \theta_1)/2$, and sending a message R2, which induces a policy $p = (\theta_1 + \theta_{max})/2$. That is,

$$-\left(\frac{\theta_{min} + \theta_1}{2} - (\theta_1 + \delta)\right)^2 = -\left(\frac{\theta_1 + \theta_{max}}{2} - (\theta_1 + \delta)\right)^2$$

$$\Leftrightarrow (\theta_1 + \delta) - \frac{\theta_{min} + \theta_1}{2} = \frac{\theta_1 + \theta_{max}}{2} - (\theta_1 + \delta)$$

$$\Leftrightarrow 2(\theta_1 + \delta) = \frac{\theta_{min} + \theta_1}{2} + \frac{\theta_1 + \theta_{max}}{2}$$

$$\Leftrightarrow \theta_1 + \delta = \frac{1}{2}\left[\frac{\theta_{min} + \theta_1}{2} + \frac{\theta_1 + \theta_{max}}{2}\right]$$

$$\Leftrightarrow \theta_1 + \delta = \frac{1}{4}\left(\theta_{min} + \theta_{max} + 2\theta_1\right)$$

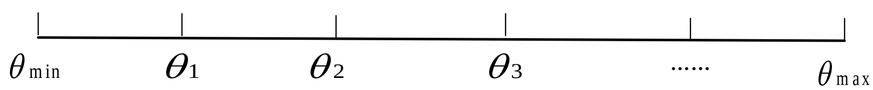
$$\Leftrightarrow \theta_1 = \frac{1}{2}\left(\theta_{min} + \theta_{max}\right) - 2\delta_1$$

Numerical example: If $\theta_{min} = 0$ and $\theta_{max} = 1$, then the cutoff becomes

$$\theta_1 = \frac{1}{2} \cdot 1 - 2 \cdot \delta = \frac{1}{2} - 2\delta ,$$

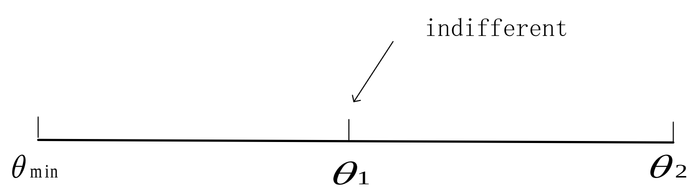
which implies that θ_1 is positive if and only if $\delta < 1/4$. This is the condition on the bias parameter that we were looking for in order to sustain a "two-step equilibrium." Intuitively, a two-step equilibrium can be sustained, and then some information is transmitted from the SIG to the politician, if the SIG's bias is not very large. Note that this is a common result we found in previous cheap talk games with a discrete number of types, whereby communication can emerge between the privately informed SIG and the uninformed politician if their preference bias is sufficiently small.
If, instead, the bias is relatively large, i.e., $\delta > 1/4$, a pooling (or babbling equilibrium) is the unique PBE that we can sustain.

From our previous analysis, we found that the type space can be partitioned into two segments when the preference bias is small. But, can it be partitioned into more segments (steps)? Since a larger number of partitions entail a more accurate communication between the SIG and the politician, we could even ask a more general question: How many partitions can we sustain in cheap talk games with a continuum of types?



1) Politician's beliefs:
$\mu(R_1 \mid R_1) = 1$;
$\mu(R_2 \mid R_2) = 1$;
$\vdots$

2) Politician's optimal responses:
If R_1 is received, then the politician implements $p = (\theta_{min} + \theta_1)/2$;
If R_2 is received, then the politician implements $p = (\theta_1 + \theta_2)/2$;
$\vdots$



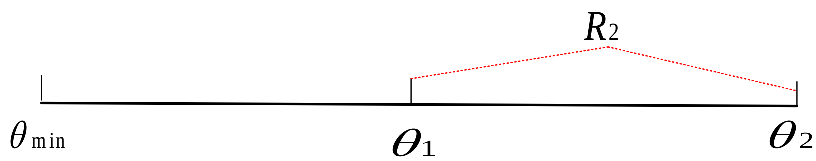
3) SIG's optimal messages:

a) If $\theta \in [\theta_{min}, \theta_1]$, then we just need to check whether θ is in R_2. All messages above R_2 entail larger policies (overshooting), so there is no need to check them as potential deviations.

$$-\left(\left(\frac{\theta_{min}+\theta_1}{2}\right)-(\theta_1+\delta)\right)^2 \geq -\left(\left(\frac{\theta_1+\theta_2}{2}\right)-(\theta_1+\delta)\right)^2$$
$$\Leftrightarrow (\theta_1+\delta)-\frac{\theta_{min}+\theta_1}{2} \leq \frac{\theta_1+\theta_2}{2}-(\theta_1+\delta)$$
$$\Leftrightarrow \theta_2 \geq 2\theta_1+4\delta-\theta_{min}$$

b) If $\theta \in R_2$, we have to check that the SIG has no incentives to

$$\begin{bmatrix} underreport & \theta \in R_1 \\ overreport & \theta \in R_3 \end{bmatrix}$$



• (b_1) No underreporting:

If $\theta = \theta_1 \in R_2$

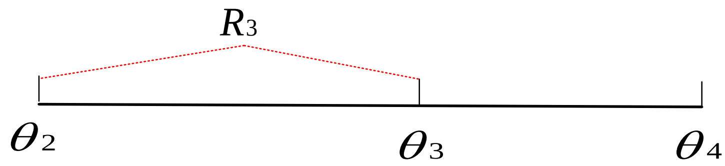
$$-\left(\frac{\theta_1+\theta_2}{2}-(\theta_1+\delta)\right)^2 \geq -\left(\frac{\theta_{min}+\theta_1}{2}-(\theta_1+\delta)\right)^2$$
$$\Leftrightarrow (\theta_1+\delta)-\frac{\theta_1+\theta_2}{2} \leq \frac{\theta_{min}+\theta_1}{2}-(\theta_1+\delta)$$
$$\Leftrightarrow \theta_2 \leq 2\theta_1+4\delta-\theta_{min}$$
$$\Leftrightarrow \theta_2 = 2\theta_1+4\delta-\theta_{min}$$

• (b_2) No overreporting:

If $\theta = \theta_2 \in R_2$

$$-\left(\frac{\theta_1+\theta_2}{2}-(\theta_2+\delta)\right)^2 \geq -\left(\frac{\theta_2+\theta_3}{2}-(\theta_1+\delta)\right)^2$$
$$\Leftrightarrow (\theta_2+\delta)-\frac{\theta_1+\theta_2}{2} \geq \frac{\theta_2+\theta_3}{2}-(\theta_1+\delta)$$
$$\Leftrightarrow \theta_3 \geq 2\theta_2+4\delta-\theta_1$$

c) If $\theta \in R_3$, we must check $\begin{bmatrix} no\ underreporting & \theta \in R_2 \\ no\ overreporting & \theta \in R_3 \end{bmatrix}$



- (c_1) No underreporting:

If $\theta = \theta_2 \in R_3$

$$-\left(\frac{\theta_2+\theta_3}{2}-(\theta_2+\delta)\right)^2 \geq -\left(\frac{\theta_1+\theta_2}{2}-(\theta_2+\delta)\right)^2$$

$$\Leftrightarrow (\theta_2+\delta)-\frac{\theta_2+\theta_3}{2} \leq \frac{\theta_1+\theta_2}{2}-(\theta_2+\delta)$$

$$\Leftrightarrow \theta_3 \leq 2\theta_2+4\delta-\theta_1$$

$$\Leftrightarrow \theta_3 = 2\theta_2+4\delta-\theta_1$$

- (c_2) No overreporting:

If $\theta = \theta_2 \in R_2$

$$-\left(\frac{\theta_1+\theta_2}{2}-(\theta_2+\delta)\right)^2 \geq -\left(\frac{\theta_2+\theta_3}{2}-(\theta_1+\delta)\right)^2$$

$$\Leftrightarrow (\theta_2+\delta)-\frac{\theta_1+\theta_2}{2} \geq \frac{\theta_2+\theta_3}{2}-(\theta_1+\delta)$$

$$\Leftrightarrow \theta_3 \geq 2\theta_2+4\delta-\theta_1$$

$$\vdots$$

$$\theta_j = 2\theta_{j-1}+4\delta-\theta_{j-2}, \textit{ where } \theta_0 = \theta_{min} \textit{ and } \theta_n = \theta_{max}$$

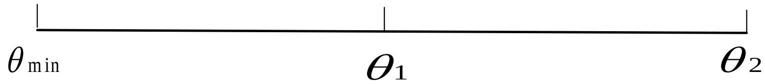
This describes the partially informative PBE.

Numerical Example: Consider that $\delta > 6$, $\theta_{min} = 0$, $\theta_{max} = 24$

• A two-step equilibrium, i.e., n=2, entails that

$$\theta_2 = \theta_{max} = 24, \ \theta_1 = 2 \cdot \theta_{min} + 4\delta > 24$$

which is not possible.



• A one-step equilibrium, i.e., n=1, entails that

$$\theta_1 = \theta_{max} = 24$$

- In this one-step equilibrium, the lobbyist reports "The true value of θ is between θ_{min} and θ_{max}" and the policy maker responds setting

$p = (\theta_{min} + \theta_{max})/2 = 12$.

If δ is a small number, you can find separating PBEs with more than 2 partitions.

Generally, equilibrium outcomes in the continuous type space scenario must satisfy

$$\theta_2 = 2\theta_1 + 4\delta - \theta_{min} \quad (1)$$

$$\theta_j = 2\theta_{j-1} + 4\delta - \theta_{j-2} \quad (2)$$

$$\theta_n = \theta_{max} \quad (3)$$

Note that (2) is actually a second-order differential equation in θ_j with

initial condition $\theta_0 = \theta_{min}$

final condition $\theta_n = \theta_{max}$

whose solution is

$$\theta_j = \frac{j}{n}\theta_{max} + \frac{n-j}{n}\theta_{min} - 2j(n-j)\delta$$

In this context, to have at least one partition, we need $\theta_1 > \theta_{min}$ (in many applications, θ_{min} is assumed to be zero), which entails

$$\frac{1}{n}\theta_{max} + \frac{n-1}{n}\theta_{min} - 2(n-1)\delta > \theta_{min}$$

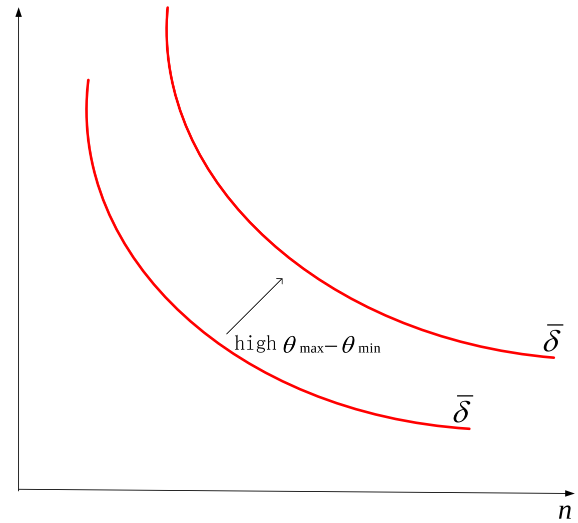
and rearranging,

$$2n(n-1)\delta < \theta_{max} - \theta_{min}$$

and solving for δ, we obtain

$$\delta < \frac{\theta_{max} - \theta_{min}}{2n(n-1)} \equiv \overline{\delta}$$

That is, the bias needs to be lower than cutoff $\overline{\delta}$ for a *n*-partition equilibrium to be sustained.



In the above figure, as the number of partitions (n) increases, the equilibrium is more difficult to sustain. Intuitively, more accurate information between the SIG and the politican (larger number of partitions) can only be sustained when the bias becomes extremelly small. In addition, as $\theta_{max} - \theta_{min}$ increases (in the numerator of the above cutoff $\bar{\delta}$), equilibria with more partitions become sustainable under a wider range of biases.