Inputs and Production Functions

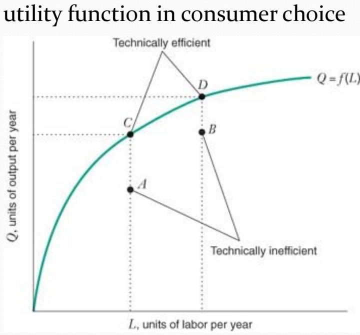
Chapter 6 Lecture Slides

Inputs and the Production Function

- ***Inputs (factors of production)*** are resources, such as labor, capital equipment, and raw materials, that are combined to produce finished goods.
- The ***production function*** is a mathematical representation that shows the maximum quantity of output a firm can produce given the quantities of inputs that it might employ.

$$Q = f\left(L, K\right)$$

- The *production function* tells us the maximum amount of production, Q, for a given amount of inputs L and K. (analogous to utility function in consumer choice theory).
- Example…



- This graph shows the maximum amount of output this firm will produce from any given quantity of labor.
- Real World Example: Technical Inefficiency among U.S Manufacturers, 1990, where inefficiency is measured as $\frac{\text{Observed output}}{\text{Potential output, } f(L,K)}$
  - (63% of efficiency or alternatively, 39% of inefficiency).
  - But efficiency is much higher (closer to 1, i.e., closer to the frontier) where the firm:
    - Faces competition
    - The firm is not a major player in its industry

Marginal and Average Products

- Looking at the production function we can derive two distinct types of productivity for a given input: the average productivity and the marginal productivity of a certain input.

- The first is the ***Average Product of Labor***, which tells us the average output per worker, which we write as the AP_L.

$$AP_L = \frac{total\ product}{quantity\ of\ labor} = \frac{Q}{L}$$

- The second is the ***Marginal Product of Labor***, which tells us the rate at which total output rises as the firm increases its quantity of labor. We write this as MP_L.

$$MP_L = \frac{change\ in\ total\ product}{change\ in\ quantity\ of\ labor} = \frac{\Delta Q}{\Delta L} \quad \frac{\partial Q}{\partial L}$$

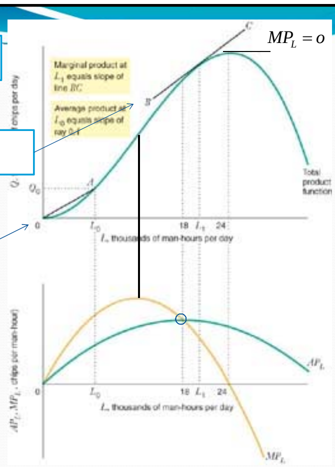
The top graph is the Total Product Curve.

MP_L= slope of segment BC for L_1 quantity of labor

AP_L=slope segment between point A and the origin.

↓

$$AP_L = \frac{Q_o}{L_o}$$



- **Relationship between AP and MP**
  - Consider the example of adding one test score to a number of scores…
- average grade increases ➔ the marginal effect of the last test was positive
- average grade decreases ➔ the marginal effect of the last test was negative

Similarly,

- AP is increasing ➔ $MP_L > AP_L$
- AP is decreasing ➔ $MP_L < AP_L$
- AP is flat ➔ $MP_L = AP_L$

$\text{Labor Productivity} = AP_L$

Growth in Labor Productivity in the U.S., 1947-2009

Year	Annual Growth Rate in Labor Productivity	
1947-1955	3.21 %	
1955-1965	2.61 %	
1965-1975	2.18 %	
1975-1985	1.38 %	Oil shocks
1985-1995	1.51 %	
1995-2005	2.94 %	Capital "deepening": Increase in K and in its quality.
2005-2009	1.90 %	

- Example

$$q = f(L,K) = 0.1LK + 3L^2K - 0.1L^3K$$

Assume $\bar{k} = 10$, *so that* $f(L,10) = 0.1L \bullet 10 + 3L^2 \bullet 10 - 0.1L^3 \bullet 10$

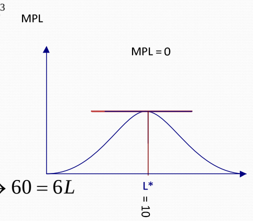
$$= L + 30L^2 - L^3$$

1) $MP_L = \dfrac{dq}{dL} = 1 + 60L - 3L^2$

Where does MPL reach a max?

$$\frac{d(1+60L-3L^2)}{dL} = 60 - 6L = 0 \rightarrow 60 = 6L$$

and solving for L, we obtain L*=10

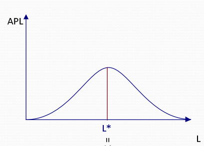


$$AP_L = \frac{f(L,10)}{L} = \frac{L + 30L^2 - L^3}{L} = 1 + 30L - L^2$$

2) Where does it reach a max?

$$\frac{\partial\left(1+30L-L^2\right)}{\partial L} = 30 - 2L = 0$$

which, solving for L, yields L=15.



3) Is this point L=15 the crossing point between AP_L and MP_L?

$$MP_L = AP_L$$
$$1 + 60L - 3L^2 = 1 + 30L - L^2$$
$$30L = 2L^2$$
$$30 = 2L \rightarrow L = 15$$

4) Generally, why does AP_L and MP_L cross each other at the max of AP_L?

If $AP_L = \dfrac{f(L)}{L}$, then AP_L reaches it max at the point where its derivative becomes zero:

$$\frac{dAP_L}{dL} = \frac{f'(L) \bullet L - 1 \bullet f(L)}{L^2} = \frac{f'(L)}{L} - \frac{f(L)}{L^2} = 0$$

$$\leftrightarrow \left(f'(L) - \frac{f(L)}{L}\right) \bullet L = \underbrace{(MP_L - AP_L)}_{MP_L = AP_L} \bullet L = 0$$

Law of Diminishing Marginal Returns

- As we continually increase the quantity of one input while holding other inputs constant, the marginal product for that increasing input will eventually decrease.
- This is so pervasive that economists refer to it as the "Law of Diminishing Marginal Returns."

Two Input Production Function

- So far we analyzed one input production function Q=f(L)
- It is more realistic to consider production functions in which the firm is allowed to use more than one input, e.g., labor and capital

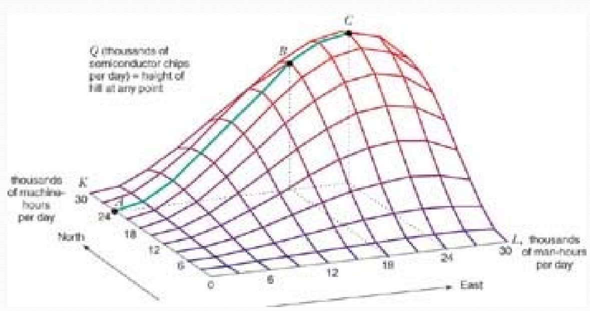
$$Q = f(L,K)$$

Two Input Production Functions

- A two-input production function gives us a three dimensional graph called the **Total Product Hill**.
- On the horizontal axis's are the two inputs, and on the vertical axis is the quantity produced.

Figure……

- Note: height of the hill=Q



- How much does output increase…

…as a result of an increase in labor?

$$MP_L = \frac{\Delta Q}{\Delta L}\Big|_{K\ constnt} = \frac{\partial f(L,K)}{\partial L}$$

(This is the slope of the mountain as we move east. Note that we maintain K fixed and increase only L)

…as a result of an increase in capital?

$$MP_K = \frac{\Delta Q}{\Delta K}\Big|_{L\ constnt} = \frac{\partial f(L,K)}{\partial K}$$

(This is the slope of the mountain as we move north. Note that we now keep L fixed and increase only K)

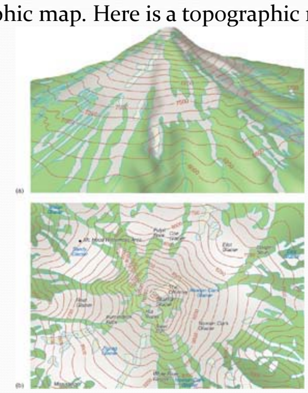
Example

- K=24 and ΔL (eastward movement up hill), we move from A to B to C (peak).
  - Given the red line is our *TP* when K is fixed at K = 24.
  - MP_L is the slope of the red line, when we hold *K* fixed at K =24.

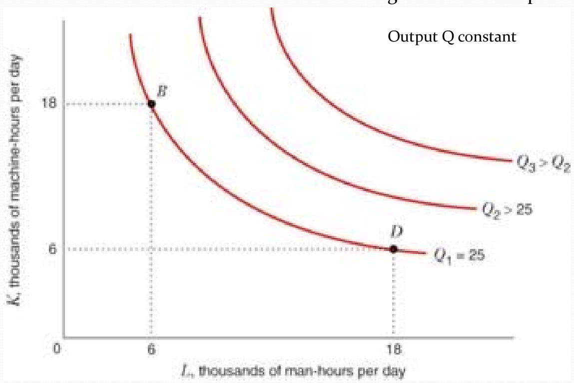
Isoquants

- An **isoquant** is a curve that shows all the combinations of labor and capital that can produce a given level of output.
- Graphically, they are the level curves representing all the points of the mountain (combinations of L and K) yielding the same height (same output Q).

- You can think of an isoquant as one of the lines on a topographic map. Here is a topographic map of Mt. Hood…



- The isoquants are downward sloping because a firm can substitute between L and K to maintain a given level of output.



- **Example:** We take a certain production function and fix the level of Q to mathematically illustrate an isoquant…
- Q=20, what is the isoquant?

$$Q = \sqrt{KL} \rightarrow 20 = \sqrt{KL} \rightarrow 400 = KL \rightarrow K = \frac{400}{L}$$ → Isoquant for an output level of Q = 20

- And more generally, for any output level Q, we have that

$$Q = \sqrt{KL} \Rightarrow Q^2 = KL \rightarrow K = \frac{Q^2}{L}$$ → Isoquant for any output level Q

  - In order to obtain the isoquant, we only need to solve for the variable in the vertical axis, K
  - For instance, for Q = 10, the isoquant is given by

$$K = \frac{10^2}{L} = \frac{100}{L}$$

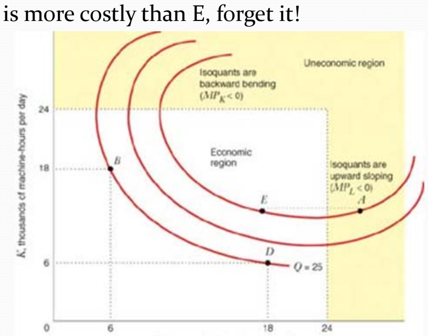
- But why don't we include the full circle of the isoquant (think of topographic map) on the actual plot of isoquants?

- **<u>Distinction between Economic and Uneconomic Regions:</u>**
- In the upward and backward sloping regions of the isoquant, that firm would be producing a certain output level at an unnecessarily high cost (negative marginal product).
- In short, they could produce the same level of output by using far fewer inputs.

Figure…..

- The firm can produce the same output at point A and E, but….

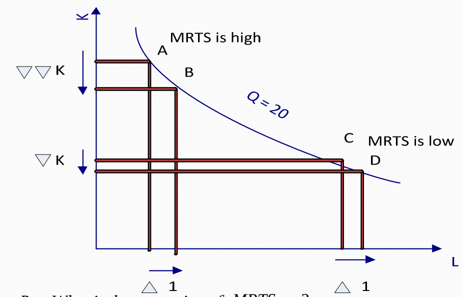
  …A is more costly than E, forget it!



Marginal Rate of Technical Substitution

- Intuitively, it represents how many machines the firm can substitute for one worker (or vice versa), keeping output level unaffected.
- ***MRTS*** is the rate at which the quantity of capital can be reduced for every one unit of increase in the quantity of labor, holding the quantity of output constant.
  - Put simply, it is the slope of the isoquant.
  - Analogous to the concept of MRS in Consumer Choice

- MRTS is diminishing:



But, What is the expression of $MRTS_{L,K}$? →

- Intuitively, the MRTS is diminishing because:
  - When capital is abundant (point A in the previous figure), the firm can give up a **large amount of capital** in order to hire one more worker, keeping output unaffected.
  - However, when capital is scarce (point C), the firm can only give up a **small amount of capital** when hiring one more worker, and still keep its output unaffected.

Finding the MRTS (slope of the isoquant) for any given Production Function

$$Q = f(K,L)$$

We first totally differentiate:

$$dQ = \overbrace{\frac{\partial f(K,L)}{\partial K}}^{MP_K} dK + \overbrace{\frac{\partial f(K,L)}{\partial L}}^{MP_L} dL$$

Since we move along the same isoquant, Q doesn't change, i.e., dQ = 0,

$$0 = MP_k \cdot dK + MP_L \cdot dL$$

Rearranging,

$$-MP_L \cdot dL = MP_K \cdot dK$$

And solving for $\frac{dK}{dL}$ we obtain,

$$-\frac{MP_L}{MP_K} = \frac{dK}{dL} \rightarrow \frac{MP_L}{MP_K} = -\frac{dK}{dL}$$

↘ Slope of isoquant, $-\frac{dK}{dL}$, is equal to the ratio of marginal productivities

Example

$Q = KL$

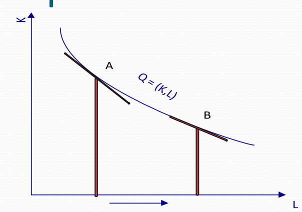
The marginal products are:

$MP_L = K$

$MP_K = L$

Hence,

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K}{L}$$



- $MRTS_{L,K}$ diminishes as L increases and K falls, i.e., as we move along the isoquant.
- So MRTS of Labor for Capital is hence diminishing. (Flatter Isoquant as we move rightward)

- **Real World Example:**
  - The MRTS between low- and high-skilled workers in the United States ≈ 6.
  - In words, firms would be able (and willing!) to substitute 6 low-skilled workers for one high-skilled worker and still maintain their output unaffected.
  - Let's continue studying…

Query #1

Consider the production function $Q = 5K + 10L$. The $MRTS_{L,K}$ is

a) 2.00

b) 1.50

c) 1.00

d) 0.50

Answer Query #1

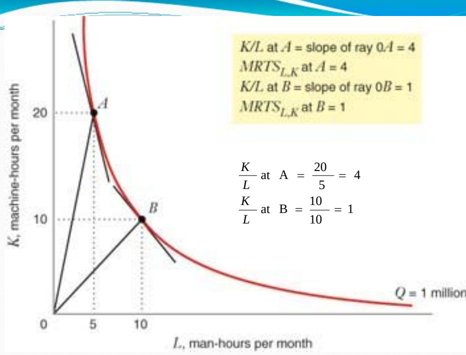
- Answer A
- The Marginal Rate of Technical Substitution of Labor for Capital, $MRTS_{L,K}$, is the rate at which the quantity of capital can be reduced for every one unit increase in the quantity of labor, holding the quantity of output constant.
- The $MRTS_{L,K}$ is equal to MP_L / MP_K
- Since MP_L= 10 and MP_K = 5, then $MRTS_{L,K}$=10/5 = 2.00
- Pages 201-203

Elasticity of Substitution

- But how easy is it for a firm to substitute between L and K?
  - The ***Elasticity of Substitution*** is a measure of how easy it is for a firm to substitute L for K. It is equal to the percentage change in the K-L ratio for every one percent change in the $MRTS_{L,K}$ as we move along an isoquant.

$$\sigma = \frac{\left(\%\Delta \left(K/L\right)\right)}{\left(\%\Delta MRTS_{L,K}\right)}$$

$$\Delta 1\% \text{ in } MRTS_{L,K} \Rightarrow \Delta\sigma\% \text{ in } \frac{K}{L}$$



- Remember, K/L is the slope of the rays from origin to a given point on the isoquant.
- MRTS is the slope of the isoquant at a given point, e.g., A or B

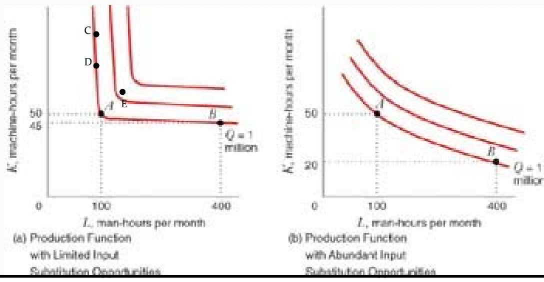
Example (From previous Figure)

$$\sigma = \frac{\%\Delta \frac{K}{L}}{\%\Delta MRTS_{L,K}} = \frac{\dfrac{\left(\frac{K}{L}\right)_B - \left(\frac{K}{L}\right)_A}{\left(\frac{K}{L}\right)_A}}{\dfrac{MRTS^B_{L,K} - MRTS^A_{L,K}}{MRTS^A_{L,K}}} =$$

$$= \frac{\frac{1-4}{4}}{\frac{1-4}{4}} = \frac{-0.75}{-0.75} = 1$$

Elasticity of Substitution- Extreme Cases

- If σ is close to zero, then MRTS changes drastically (large denominator in σ), as in Figure 6.11 (a)
- If σ is large, then MRTS is almost constant (small denominator in σ), as in Figure 6.11 (b)



Special Production Functions

- ***(1) Linear Production Function***:
  - General Form: Q=aL + bK, where a and b are positive constants

    Example: Q=20K+10L

    If we want to depict the isoquant of Q=200

    200=20K+10L

    Solving for K, $K = 10 - \frac{1}{2}L$ ← Equation of Isoquant

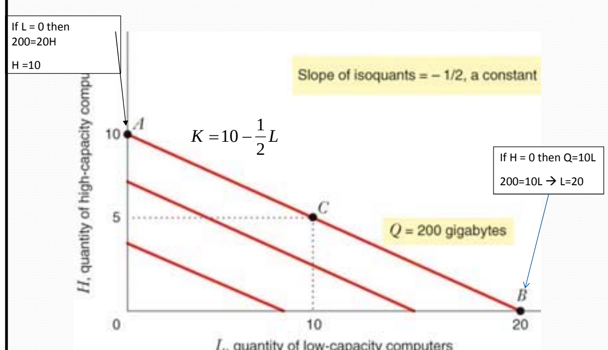
  Slope of Isoquant ($MRTS_{K,L}$) is -0.5, which does not depend on L.

  Hence, $MRTS_{K,L}$ is constant in L.

Special Production Functions

- MRTS is constant, e.g, 0.5 in our example (isoquants are therefore straight lines).
- For instance, a firm that is flexible enough to use either oil and gas has…
  - a linear production function because these inputs are perfect substitutes for the firm.

- Q=20H + 10L (always additive)



- What about its elasticity of substitution, σ?
- The linear production functions have an elasticity of substitution of ∞ because the denominator of the equation (%ΔMRTS) equals zero:

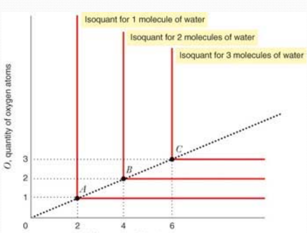
$$\sigma = \frac{\%\Delta \frac{K}{L}}{\%\Delta MRTS_{K,L}} = \frac{\%\Delta \frac{K}{L}}{0} = +\infty$$

Since the slope of the isoquant ($MRTS_{L,K}$) doesn't change along all points of the isoquant, the denominator equals 0.

- (2) ***Fixed Proportions Production Functions***
  - The K/L ratio does not change because the two inputs must be used in a constant proportion if we seek to increase production.
    - That is, an increase in one input without a proportional increase in the other input will not result in any added production.
  - ***General form***: Q=min{aL,bK}, where a>0 and b>0.
    - Remember that 'min' means that you take the minimum of the two numbers in parentheses.

Fixed Proportions Production Function - *Example*

- Q= min {L,2K}, i.e., 2 additional units of labor must be accompanied by one unit of K in order to raise output in one more unit.



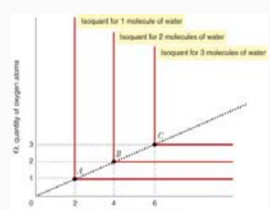
$$Q = \min\{\frac{L}{2}, K\}$$

$$3 = \min\{\frac{6}{2}, 3\}$$

$$3 = \min\{\frac{10}{2}, 3\}$$

Fixed Proportions Production Function - *Example*

- Usual trick in order to find the kink of the L-shaped isoquants:
  - Set the two arguments of the min equal to each other. That is , L=2K, and then solve for the input on the vertical axis (usually K), which yields K=L/2.
  - Graphically, this implies that the kink is crossed by a straight line originating at (0,0) and with a slope of ½.



- With Fixed Proportions production functions, the elasticity of substitution is σ=0. Why?

$$\sigma = \frac{\%\Delta \frac{K}{L}}{\%\Delta MRTS_{K,L}}$$ where $MRTS_{L,K}$ (slope of Isoquant) goes from ∞ to 0. Hence %ΔMRTS=∞

$$\sigma = \frac{\%\Delta \frac{K}{L}}{\%\Delta MRTS_{K,L}} = \frac{\%\Delta \frac{K}{L}}{\infty} = 0$$

  - This implies that it is perfectly difficult to substitute between the two inputs because the substitution must occur in constant proportions.
  - *Example*: Chemical industry, where every unit of output must contain a constant proportion of inputs.

- ***(3) Cobb-Douglas***
  - This production function is the intermediate of the previous two. That is, the K/L and MRTS ratios change as we move along the isoquant.
  - ***General Form***: $Q = AL^{\alpha}K^{\beta}$, where A, α and β are positive constants.

$$Hence,\ MRTS_{L,K} = \frac{-MP_L}{MP_K} = -\frac{\alpha AL^{\alpha-1}K^{\beta}}{\beta AL^{\alpha}K^{\beta-1}} = -\frac{\alpha K}{\beta L}$$

  - Interestingly, the elasticity of substitution for Cobb-Douglas functions is always 1, i.e., $\sigma = 1$.
    - Let's see why...

- Rearranging the MRTS, we obtain

$$-MRTS_{L,K}\frac{\beta}{\alpha} = \frac{K}{L} \qquad (1)$$

Hence, $\Delta \frac{K}{L} = \frac{-\beta}{\alpha} \Delta MRTS_{L,K}$

$$\frac{\Delta \frac{K}{L}}{\Delta MRTS_{L,K}} = -\frac{\beta}{\alpha} \qquad (2)$$

We also know from (1) that:

$$-MRTS_{L,K} = \frac{\alpha}{\beta}\frac{K}{L} \leftrightarrow \frac{MRTS_{L,K}}{\frac{K}{L}} = \frac{-\alpha}{\beta} \qquad (3)$$

We can then use our results (2) and (3) to obtain the elasticity of substitution, σ.

- Hence, the elasticity of substitution is...

$$\sigma = \frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta MRTS_{L,K}} = \frac{\frac{\Delta\frac{K}{L}}{\frac{K}{L}}}{\frac{\Delta MRTS}{MRTS}} = \frac{\Delta\frac{K}{L}}{\Delta MRTS} \bullet \frac{MRTS}{\frac{K}{L}} = \underbrace{\left(\frac{-\beta}{\alpha}\right)}_{\text{From (2)}}\underbrace{\left(\frac{-\alpha}{\beta}\right)}_{\text{From (3)}} = 1$$

- Hence, the Cobb-Douglas production function has an elasticity of substitution, σ=1, for <u>any</u> values of parameters α, β and A.

Cobb-Douglas production function - *Example*

$$Q = AL^{\alpha}K^{\beta}$$

$$MRTS_{L,K} = -\frac{-MP_L}{MP_K} = -\frac{-\alpha AL^{\alpha-1}K^{\beta}}{\beta AL^{\alpha}K^{\beta-1}} = \frac{-\alpha K}{\beta L}$$

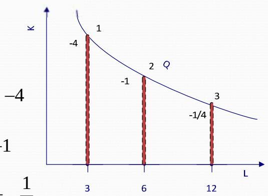
Example: α=β=0.5

$MRTS_{L,K}$ becomes...

$$1)(K,L) = (12,3) \rightarrow -\frac{0.5}{0.5} \bullet \frac{12}{3} = -4$$

$$2)(K,L) = (6,6) \rightarrow -\frac{0.5}{0.5} \bullet \frac{6}{6} = -1$$

$$3)(K,L) = (3,12) \rightarrow -\frac{0.5}{0.5} \bullet \frac{3}{12} = -\frac{1}{4}$$



But, what is the elasticity of substitution, σ, in the Cobb-Douglas production function? σ=1.

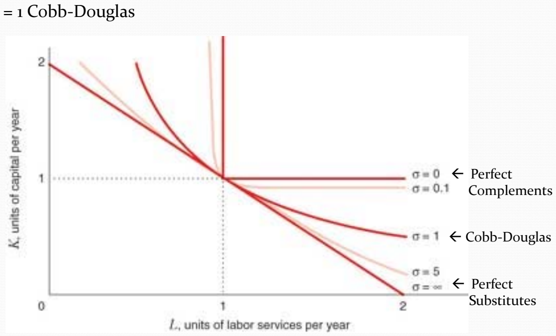
- ***(4) CES – Constant Elasticity of Substitution Production Function***
  - The three other production functions are special cases of this production function.
  - That is, the other 3 functions can be seen in this function:

$$Q = [aL^{\frac{\sigma-1}{\sigma}} + bK^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

where σ is the elasticity of substitution.

In particular...

- σ = ∞ (linear) substitutes (inputs are infinitely easy to substitute)
- σ = 0 (fixed proportions) Complements (Inputs cannot be substituted without affecting total output).
- σ = 1 Cobb-Douglas



- **Real World Application**
  - Let's see some elasticities of substitution in the real world (German Industries):

Industry	*Elasticity of substitution, σ*
Chemicals	0.37
Iron	0.50
Motor vehicles	0.10
Food	0.66

Low elasticity of substitution between L and K (Hard to substitute, almost right-angled isoquants).

High elasticity of substitution between L and K (easy to substitute, smooth curved isoquants, not straight yet).

Query #2

Suppose every molecule of salt requires exactly one sodium atom, Na, and one chlorine atom, Cl. The production function that describes this is

a) Q = Na + Cl
b) Q = Na x Cl
c) Q = min(Na, Cl)
d) Q = max(Na, Cl)

Answer Query #2

- Answer C
- The isoquants for for a fixed-proportions production function are L-shaped, because each additional atom of Chlorine is useless without an additional unit of Sodium, and vice versa.
- These inputs are referred to as *perfect complements*
  - The notation "*min*" means take the minimum value of the two numbers in parentheses.
- Page 209 in your textbook.

Returns to Scale

- We will now see how output changes if we simultaneously increase all inputs in the same proportion.
  - Remember that if inputs have positive marginal products, an increase in inputs must result in an increase in output…
    - BUT, <u>by how much</u> will output increase?
  - Returns to scale give us the percentage increase in output for a given increase in all inputs.

- Returns to Scale are mathematically expressed as…

$$\text{Returns to Scale} = \frac{\%\Delta(\textit{quantity of outputs})}{\%\Delta(\textit{quantity of all inputs})}$$

OR…

For a production function

Q=f(L,K)
Hence, ϕQ= f(λL, λK) Where λ>1

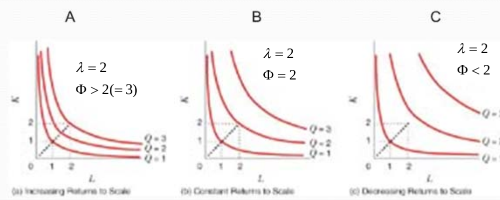
$$\text{Returns to scale} = \frac{\%\Delta Q}{\%\Delta \text{ in all inputs}} \begin{matrix} \leftarrow \phi \\ \leftarrow \lambda \end{matrix}$$

Therefore,
if $\Phi > \lambda$ we have *increasing* returns to scale
if $\Phi < \lambda$ we have *decreasing* returns to scale
if $\Phi = \lambda$ we have *constant* returns to scale

Returns to Scale (in words)

- A proportional increase in all inputs by λ, produces a…
  - A) <u>more than proportional</u> increase in output (increasing returns to scale)
  - B) <u>proportional</u> increase in output (constant returns to scale)
  - C) <u>less than proportional</u> increase in output (decreasing returns to scale)

- Figure A below represents <u>increasing</u> returns to scale
  - $\Phi > \lambda$
- Figure B below represents <u>constant</u> returns to scale
  - $\Phi = \lambda$
- Figure C below represents <u>decreasing</u> returns to scale
  - $\Phi < \lambda$



Exercise

- Consider a Cobb-Douglas production function, where L=5 and K=3.

$$Q = L^{\alpha} K^{\beta} \quad \to \text{e.g. } L=5,\ K=3 \quad Q = 5^{\alpha} \cdot 3^{\beta}$$

Let us now analyze by how much output increases if all inputs experience a common increase of λ

For instance, all inputs double, implying that λ =2.

If all inputs double, the above Cobb-Douglas production function becomes

$$(2 \times 5)^{\alpha} \cdot (2 \times 3)^{\beta} = 2^{\alpha} 5^{\alpha} 2^{\beta} 3^{\beta} = 2^{\alpha+\beta}\left(5^{\alpha} 3^{\beta}\right) = 2^{\alpha+\beta} Q$$

Exercise

This is generally true for any Cobb-Douglas production function. In particular,

$$A(\lambda L)^{\alpha}(\lambda K)^{\beta} = \lambda^{\alpha+\beta} \underbrace{A L^{\alpha} K^{\beta}}_{Q} = \lambda^{\alpha+\beta} Q$$

- If $\lambda^{\alpha+\beta} > \lambda$, which occurs when($\alpha+\beta > 1$) then the production function has <u>increasing</u> returns to scale
- if $\lambda^{\alpha+\beta} = \lambda$, which occurs when ($\alpha+\beta = 1$) then the production function has <u>constant</u> returns to scale
- if $\lambda^{\alpha+\beta} < \lambda$, which occurs when ($\alpha+\beta < 1$) then the production function has <u>decreasing</u> returns to scale

Empirical estimates of Returns to Scale

	$\alpha+\beta$
Decreasing Returns to Scale	
Tobacco	0.51
Food	0.91
Transportation equipment	0.92
Constant Returns to Scale	
Apparel and Textile	1.01
Furniture	1.02
Electronics	1.02
Increasing Returns to Scale	
Paper Products	1.09
Petroleum and Coal	1.18
Primary Metal	1.24

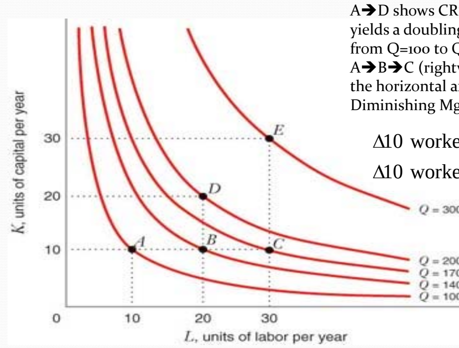
- **Why are Returns to Scale important?**
  - If a firm exhibits increasing Returns to Scale, there are cost advantages to large scale production.
  - That is, the firm will be able to produce at a lower cost per unit than the aggregate cost that two firms would incur when each produces half of the single firm output.
    - One firm is better than two! An argument for promoting industry concentration.

    Empirical evidence:
    - Electrical power generation
      - (Observed in 1950-60, but less today; see application 6.7 in your textbook).
    - Oil pipeline transportation

Tricky Question

- Can a firm exhibit constant returns to scale, yet experience a diminishing marginal product for all inputs?
  - Yes!
  - Let's see why.

Difference between Returns to Scale and Diminishing Mg. Returns



A→D shows CRS (doubling inputs yields a doubling effect in output from Q=100 to Q=200)
A→B→C (rightward movements in the horizontal axis) shows Diminishing Mg returns to labor

Δ10 workers$\Rightarrow$ Δ40Q(A$\rightarrow$B)

Δ10 workers$\Rightarrow$ Δ30Q(B$\rightarrow$C)

Query #3

Returns to scale refers to:

a) the increase in output that accompanies an increase in one input, all other inputs held constant.

b) a change in a production process that enables a firm to achieve more output from a given combination of inputs.

c) the number of units of increase in output that can be obtained from an increase in one unit of input.

d) the percentage by which output will increase when all inputs are increased by a given percentage.

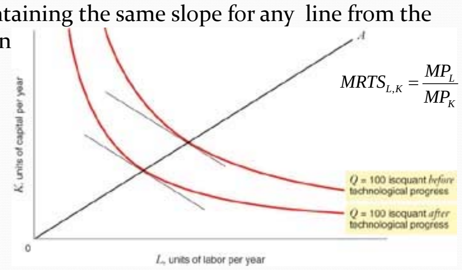
Answer Query #3

- Answer D
- By definition,
  - Returns to Scale = % Δ(quantity of output) / % Δ(quantity of all inputs)
- Page 212

Technological Progress

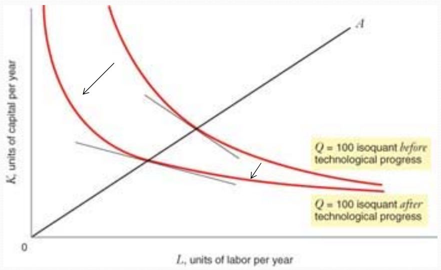
- A firm's production function shifts over time because of increased know-how and new investment and research.
- We refer to this phenomena as ***technological progress***:
  - a change in a production process that enables a firm to achieve more output from a given combination of inputs;
  - or, equivalently, the same amount of output from fewer inputs.

- ***Neutral Technological Progress*** decreases the amount of inputs needed to achieve a certain level of production without affecting the firm's marginal rate of technical substitution.
- Graphically, the isoquants will shift inward while maintaining the same slope for any line from the origin

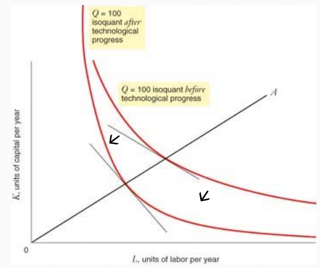


- ***Labor Saving Technological Progress:*** it causes the marginal product of capital to increase in relation to marginal product of labor.
  - Flatter isoquant: if $MRTS_{L,K}=\frac{MP_L}{MP_K}$ goes down, it must be that MP_K grows more rapidly than MP_L. "Fire workers!"

Estimated as true for counties such as the UK.



- ***Capital Saving Technological Progress:*** *it* causes the marginal product of labor to increase in relation to the marginal product of capital.
  - Steeper isoquant
  - If $MRTS_{L,K}$ grows it must be that MP_L grows more rapidly than MP_K.
  - "Get rid of those machines."



$$\uparrow \text{slope Isoq} \Rightarrow MRTS_{L,K} \Rightarrow \uparrow \frac{MP_L}{MP_K} \Rightarrow \uparrow\uparrow MP_L > \uparrow MP_k$$

Example - Cobb-Douglas

- Here the firm's production function changes…

$$Q_1=\sqrt{KL} \quad \textit{where over time this changes to } Q_2 = L\sqrt{K}$$

$$MP_K = 0.5\left(\sqrt{\frac{L}{K}}\right) \qquad MP_K = 0.5L\left(\sqrt{\frac{1}{K}}\right)$$

$$MP_L = 0.5\left(\sqrt{\frac{K}{L}}\right) \qquad MP_L = \sqrt{K}$$

- A) Is there any tech. progress?
  - Yes, because $Q_1<Q_2$ for any level of K>0 and L >0.
  - Indeed, $\sqrt{KL}=\sqrt{K}\sqrt{L}<L\sqrt{K}$, which simplifies to $\sqrt{L}<L$. This holds for any number of workers, L.

- B) The progress is Capital Saving since…

$$\left(MRTS_{L,K}=\frac{0.5\left(K/L\right)^{0.5}}{0.5\left(L/K\right)^{0.5}}=\frac{K}{L}\right)_{Q_1} < \left(MRTS_{L,K}=\frac{(K)^{0.5}}{0.5L\left(1/K\right)^{0.5}}=\frac{2K}{L}\right)_{Q_2}$$

That is, the isoquant becomes steeper (increase in MRTS) after the technological progress, as in the previous figure.

APPLICATION 6.8

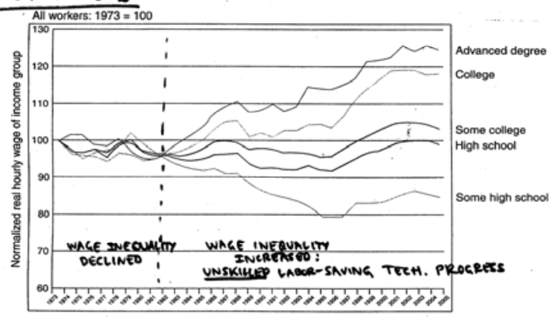


FIGURE 6.23 **Real Wages by Educational Attainment in the United States, 1973–2005**
The figure shows the trend between 1973 and 2005 in real (i.e., inflation-adjusted) hourly wages of U.S. workers, according to the worker's level of educational attainment. The wages are normalized so that 1973 = 100.
Source: Economic Policy Institute, http://www.epi.org/content.cfm/datazone_dznational (accessed November 10, 2008).