

EconS 503 – Advanced Microeconomics II¹

Adverse Selection

Handout on Two-part tariffs (Second-degree price discrimination)

1. Introduction

Consider a setting where an uninformed firm is attempting to sell an item to a privately informed customer. The firm's profit function is $F - cq$, where $c > 0$ represents the firm's marginal costs, and F is the fee paid from the customer to the firm in exchange for q units of the good (price for the package of units, rather than a unit price). The customer's utility function is $u(q, T, \theta) = \theta \cdot v(q) - F$, where $u' > 0$ and $u'' < 0$. Parameter θ is privately observed by the consumer, and takes on either θ_L with probability β or θ_H with probability $1 - \beta$, where $\theta_H > \theta_L$.

2. Complete Information

[2nd Stage] For a given pair of fee T_i and quantity q_i , (T_i, q_i) , consumers with valuation θ_i purchase the good if and only if $\theta_i v(q_i) - T_i \geq 0$

[1st Stage] Observing θ_i (as we are in the complete-information version) and anticipating the buyers decision rule in the second stage, $\theta_i v(q_i) - T_i \geq 0$, the firm solves the PMP

$$\max_{T_i, q_i} T_i - cq_i$$

$$\text{subject to } \theta_i v(q_i) - T_i \geq 0 \leftarrow P.C.$$

The participation constraint (P.C.) must bind. Otherwise T_i can be further increased, thus increasing profits. Hence, $\theta_i v(q_i) = T_i$, which simplifies the above problem to the following unconstrained maximization problem

$$\max_{q_i} \theta_i v(q_i) - cq_i$$

Taking F.O.C with respect to q_i ,

$$\underbrace{\theta_i v'(q_i) - c}_{MV = MC} \leq 0 \quad (= 0 \text{ in interior solutions})$$

Hence, under complete information, q_i is increased until the point in which the consumer's marginal utility of additional units coincides with the firm's marginal cost. As we next show, when the firm is uninformed about the customer's type, this result doesn't necessarily arise.

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3. Incomplete Information

The firm cannot observe the realization of θ . The firm could offer contracts of the form $(T(q), q)$, with function $T(q)$ being as general as you can imagine.

For simplicity, let's consider three types of contracts:

- Linear pricing: $T(q) = p \cdot q$ customers pay p for every unit they buy.
- Nonlinear pricing (single two-part tariff for all types of customers)
- Nonlinear pricing (two two-part tariffs one for each type of customer)

3.1. Linear pricing, $T(q) = p \cdot q$

[2nd Stage] Every customer with type θ_i pays a price p per unit of q purchased, thus obtaining a utility

$$\theta_i u(q) - pq \quad \text{for all } i = \{H, L\}$$

In order to maximize his utility (for every given p), he increases q until

$$\theta_i u^1(q) = p$$

Solving for q , we find θ_i –Walrasian demand

$$q_i = D_i(p)$$

Hence, θ_i –customer's utility is

$$\underbrace{\theta_i u(D_i(p))}_{q_i} - \underbrace{p \cdot D_i(p)}_{q_i}$$

[1st Stage] By backward induction, the monopolist anticipates the demand function $D_i(p)$ for θ_i –type buyer. Hence, the firm maximizes expected profits:

$$\max_p (p - c) \cdot [\beta \cdot D_L(p) + (1 - \beta) \cdot D_H(p)]$$

Let $D(p) \equiv \beta \cdot D_L(p) + (1 - \beta) \cdot D_H(p)$ denote the expected demand, which helps us simplify the above program to

$$\max_p (p - c) \cdot D(p)$$

Taking FOC with respect to p yields

$$D(p) + p D'(p) - c = 0$$

Solving for p , we obtain a linear price of

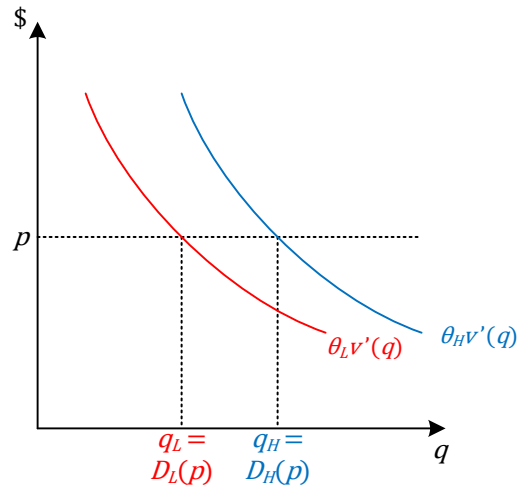
$$p^{LP} = c - \frac{D(p^{LP})}{D'(p^{LP})}$$

where $p^{LP} > c$ if $D'(p^{LP}) < 0$. Depending on the parameter values, it might be profitable for the seller to only serve θ_H buyers.

3.2. Single two-part tariff

The firm sets a single two-part tariff (F, p) to both types of customers, and each type of buyer decides to take it or leave it.

Fee. From the UMP of each type of consumer, we obtained FOC of $\theta_i v'(q) = p$. Plotting them on the same figure, we find:



where functions $\theta_i v'(q)$ are decreasing in q by the concavity of $v(\cdot)$, i.e., $v'' < 0$ for all q . Hence, $D_H(p) > D_L(p)$, thus implying that net surpluses, $S_i(p)$, satisfy

$$S_H(p) = \theta_H v[D_H(p)] - p \cdot D_H(p) > \theta_L v[D_L(p)] - p \cdot D_L(p) = S_L(p)$$

That is, $S_H(p) > S_L(p)$.

If the firm seeks the participation of both types of customers, we need the fee to satisfy

$$F \leq S_L(p) < S_H(p)$$

More explicitly:

- In the second stage, every customer with type θ_i purchases the good if and only if $F \leq S_i(p)$.
- In the first stage, the firm anticipates the customers' decision rule of $F \leq S_i(p)$, and chooses the single two part tariff that maximizes profits.

Mathematically,

$$\begin{aligned} \max_{(F,p)} & \beta[F + (p - c) \cdot D_L(p)] + (1 - \beta)[F + (p - c) \cdot D_H(p)] \\ & = F + (p - c) \underbrace{[\beta \cdot D_L(p) + (1 - \beta) \cdot D_H(p)]}_{D(p), \text{ i.e., expected demand}} \end{aligned}$$

$$\text{subject to } F \leq S_i(p) \text{ for all } i = \{H, L\}$$

However, the seller can increase F until $F = S_L(p)$. Raising it any further would lead the low-type customers to reject the purchase. Plugging $F = S_L(p)$ into the above problem helps us obtain an unconstrained PMP (with only one choice variable, p), as follows

$$\max_p S_L(p) + (p - c) \cdot D(p)$$

Taking first-order conditions with respect to p yields

$$S'_L(p) + D(p) + (p - c)D'(p) = 0$$

Solving for p and rearranging, we obtain a price of the single two part tariff, p^{STPT} , of

$$p^{STPT} = \underbrace{c - \frac{D(p)}{D'(p)}}_{p^{LP}, \text{ price under linear pricing}} + \underbrace{\frac{S'_L(p)}{D'(p)}}_{+}$$

Where the last term is positive since $S'_L(p) < 0$ and $D'(p) < 0$.

Remark: $S'_i(p)$ can be found by applying the Envelope Theorem on

$$S_i(p) = \theta_i \cdot v[D_i(p)] - p \cdot D_i(p)$$

In particular, second-order effects are absent, so that $D_i(p)$ is unaffected by a price change. As a consequence

$$S'_i(p) = 0 - D_i(p) = -D_i(p) < 0$$

Hence, prices in each setting are ranked as follows:

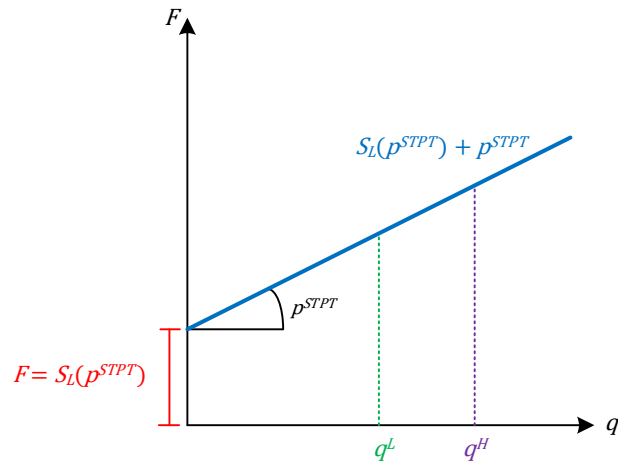
$$p^{STPT} > p^{LP} > c \text{ (price under perfect competition)}$$

The firm then sets a single two-part tariff

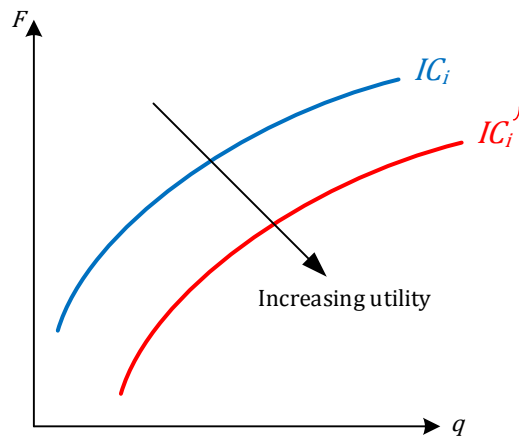
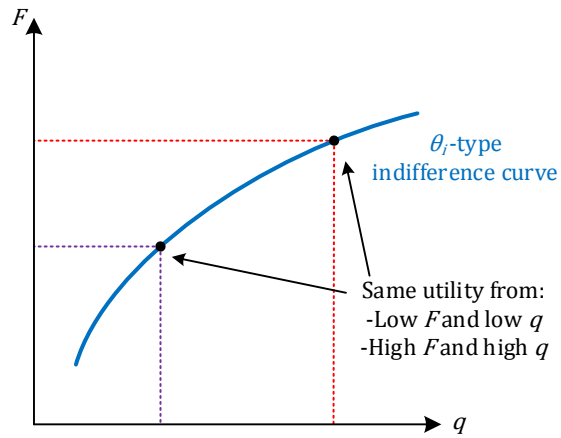
$$(F, p) = (S_L(p^{STPT}), p^{STPT})$$

Practice: Considering a demand function $D_i(p) = \theta_i - p$, where $\theta_i = \{1, 2\}$ and $\beta = \frac{1}{2}$, find the profit-maximizing two-part tariff.

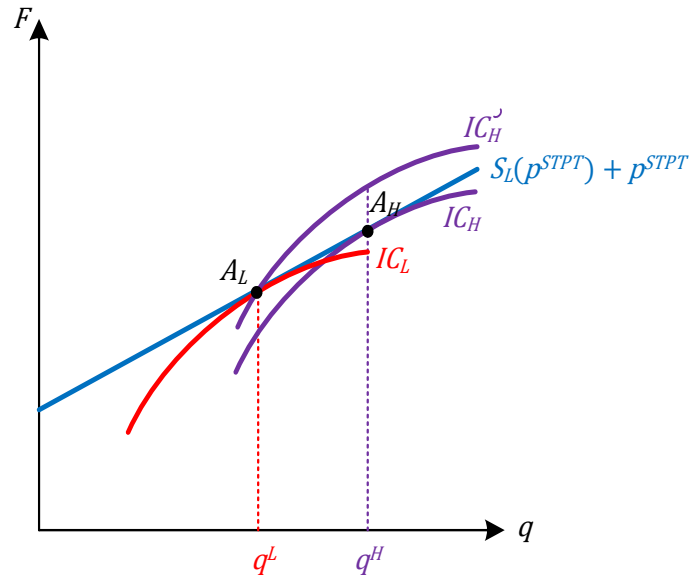
In addition, $q_H = D_H(p^{STPT}) > D_L(p^{STPT}) = q_L$. We can depict this two-part tariff in the (F, p) -quadrant, as follows.



Graphical representation of the indifference curves using the same (F, p) -quadrant:



We can now superimpose IC on top of the two-part tariff, obtaining:



Some points about equilibrium behavior in the case of a single two-part tariff that we just analyzed:

- Customer θ_H is better off at A_H than at A_L
- Customer θ_L is better off at A_L than at A_H

Motivation to move to other contracts (in particular, toward two two-part tariffs, which we analyze in the following section):

- The seller could do better if he sets a contract that yields point A_H to θ_H -buyer (since this buyer is indifferent about accepting the contract meant for him or that of the θ_L -customer).

3.3. Several (or menu) two-part tariffs

Consider a setting where the monopolist cannot observe the type of each consumer. Rather than offering a uniform price for all types of customers, or a single two-part tariff to all types of customers, the monopolist can design a menu of two-part tariffs, (F_L, q_L) and (F_H, q_H) , with the property that the customer with type $i = \{L, H\}$ has the incentives to self-select the two-part tariff (F_i, q_i) meant for him.

In this setting, the monopolist must guarantee that:

- Both types of customers are willing to participate (i.e., the two-part tariff meant for each type of customer provides him with a weakly positive utility level), and
- Both types of customers do not have incentives to choose the two-part tariff meant for the other type of customer, that is, type i customer prefers (F_i, q_i) over (F_j, q_j) where $j \neq i$.

Figure 1 PC condition binds (upper panel) and IC condition binds (lower panel)

High-demand customer. Let us first focus on the high-demand consumer and show that IC_H is binding, (the lower panel of figure 1 arises for this type of customer).

Proof. An indirect way to show that IC_H binds, i.e., $F_H = \theta_H[u(q_H) - u(q_L)] + F_L$, is to demonstrate that $F_H < \theta_H u(q_H)$ (i.e., as depicted be in the lower panel of Figure 1). By contradiction, consider that $F_H = \theta_H u(q_H)$. If this condition holds, then IC_H can be rewritten as

$$F_H - \theta_H u(q_L) + F_L \geq F_H, \text{ which simplifies to } F_L \geq \theta_H u(q_L)$$

In addition, we can combine this result with the property that $\theta_H > \theta_L$ to obtain

$$F_L \geq \theta_H u(q_L) > \theta_L u(q_L)$$

That is, $F_L > \theta_L u(q_L)$. This finding, however, violates the participation constraint of the low-demand customer, PC_L , indicating that we have reached a contradiction and, therefore, $F_H < \theta_H u(q_H)$ (i.e., PC_H is not binding). Thus, IC_H is binding but PC_H is not, confirming that for the high-demand customer the lower panel of Figure 1 applies (i.e., $F_H = \theta_H[u(q_H) - u(q_L)] + F_L$). *Q.E.D.*

Low-demand customer. Let us now use a similar approach to show that the top panel of Figure 1 arises for the low-demand customer (i.e., PC_L binds since $F_L = \theta_L u(q_L)$).

Proof. Similarly as for high-demand customers, we can prove this result by instead showing that $F_L < \theta_L[u(q_L) - u(q_H)] + F_H$ holds. Proving this result by contradiction, assume that $F_L = \theta_L[u(q_L) - u(q_H)] + F_H$. Plugging this expression into IC_H (which binds, as shown in our discussion of the high-demand customer), we obtain

$$\theta_H[u(q_H) - u(q_L)] + \theta_L[u(q_L) - u(q_H)] + F_H = F_H,$$

This expression simplifies to

$$\theta_H[u(q_H) - u(q_L)] = \theta_L[u(q_L) - u(q_H)]$$

and ultimately reduces to $\theta_H = \theta_L$, violating the initial assumption $\theta_H > \theta_L$. Therefore, $F_L = \theta_L[u(q_L) - u(q_H)] + F_H$ cannot hold, but instead $F_L < \theta_L[u(q_L) - u(q_H)] + F_H$ must be true. As a consequence, the top panel of Figure 1 applies for the low-demand customer, ultimately implying that PC_L binds while IC_L does not. *Q.E.D.*

Summarizing, from the high-demand customer we have that $\theta_H[u(q_H) - u(q_L)] + F_L = F_H$ whereas from the low-demand customer we obtained that $\theta_L u(q_L) = F_L$. We can now plug this information about F_H and F_L into the monopolist's expected PMP, which now becomes an unconstrained maximization problem, as follows:

$$\max_{q_L, q_H \geq 0} p[F_H - cq_H] + (1 - p)[F_L - cq_L]$$

$$\begin{aligned}
&= p \left[\frac{\theta_H [u(q_H) - u(q_L)] + F_L - cq_H}{F_H} \right] + (1-p) \left[\frac{\theta_L u(q_L) - cq_L}{F_L} \right] \\
&= p \left[\theta_H [u(q_H) - u(q_L)] + \frac{\theta_L u(q_L)}{F_L} - cq_H \right] + (1-p) [\theta_L u(q_L) - cq_L]
\end{aligned}$$

which ultimately simplifies to

$$= p[\theta_H u(q_H) - (\theta_H - \theta_L)u(q_L) - cq_H] + (1-p)[\theta_L u(q_L) - cq_L]$$

Importantly, constraint PC_L binding implies that IC_L also holds (recall the lower panel of Figure 1), and constraint IC_H binding entails that PC_H is also satisfied. In other words, all four constraints hold. Taking first-order conditions with respect to q_H yields

$$p[\theta_H u'(q_H) - c] = 0, \text{ or } \theta_H u'(q_H) = c,$$

Therefore, the amount offered to high-demand customers, q_H , is socially efficient (their demand coincides with marginal cost). As we discuss next, such efficient outcome does not arise for low-demand customers. In particular, taking first-order conditions with respect to q_L we obtain

$$p[-(\theta_H - \theta_L)u'(q_L)] + (1-p)[\theta_L u'(q_L) - c] = 0,$$

which can be rewritten as

$$u'(q_L)[(1-p)\theta_L - p(\theta_H - \theta_L)] = (1-p)c$$

and further simplified to

$$u'(q_L)[\theta_L - \theta_H p] = (1-p)c$$

Dividing both sides by $(1-p)$, we obtain

$$u'(q_L) \left[\frac{\theta_L - \theta_H p}{1-p} \right] = c$$

Note that this expression can alternatively be written as²

$$u'(q_L) \left[\theta_L - \frac{p}{1-p} (\theta_H - \theta_L) \right] = c$$

Figure 2 separately depicts the left- and right-hand side of the last expression. For comparison purposes, it also plots $u'(q_L) * \theta_L$, which helps identify the socially optimal output q_L^{so} (i.e., that arising under complete information).

² In order to find an expression in which θ_L stands alone inside the parenthesis, we set up the equation $\frac{\theta_L - \theta_H p}{1-p} = \theta_L - x$. Solving for the unknown x , yields $\frac{p}{1-p} (\theta_H - \theta_L)$.

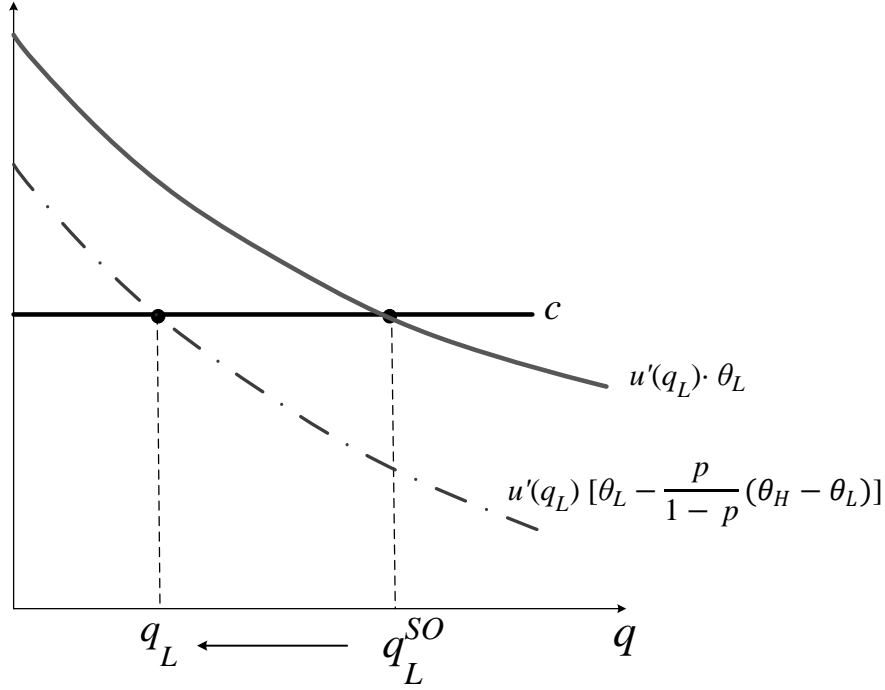


Figure 2 Output for the low-demand customer

Summarizing, the amount offered to high-demand customers is socially efficient (recall that $\theta_H u'(q_H) = c$). In other words, $q_H = q_H^{SO}$, and there is no output distortion for high-demand customers relative to complete information allocations. That is a common finding in principal-agent models where the principal (in this case the monopolist) cannot observe the private type of the agent (in this case the consumer). In contrast, the output offered to low-demand customers entails a distortion relative to complete information, $q_L < q_L^{SO}$, as depicted in Figure 2. Furthermore, this output distortion $q_L^{SO} - q_L$ is increasing in term $\frac{p}{1-p}(\theta_H - \theta_L)$. Specifically, it increases in the frequency of high-type buyers, p , and on the difference between high- and low-type buyers, $(\theta_H - \theta_L)$.

In addition, the fact that constraint PC_L binds while PC_H does not, entails that only the high-demand customer retains a positive utility level, i.e., $\theta_H u(q_H) - F_H > 0$. In other words, the firm's lack of information provides the high-demand customer with an "information rent." Intuitively, this information rent emerges from the seller's attempt to reduce the incentives of the high-type customer to select the contract meant for the low type. In particular, while the low-demand buyer pays a lower fee, the output that he receives is sufficiently low to make it unattractive for the high-demand buyer, $q_L < q_L^{SO}$. In other words, the output distortion $q_L^{SO} - q_L$ that we described above stems from the seller's purpose to reduce the information rent of the high-type buyer.

Example. Consider a monopolist selling a textbook to two types of graduate students, low- and high-demand, with utility function

$$U_i(q_i, F_i) = \theta_i \left(q_i - \frac{q_i^2}{2\theta_i} \right) - F_i,$$

where $i = \{L, H\}$ and $\theta_H > \theta_L$. In this context, we obtain the direct demand function $q_i = \theta_i - p$. In addition, assume that the proportion of high-demand (low-demand) students is γ ($1 - \gamma$, respectively). The monopolist's constant marginal cost is $c > 0$, which satisfies $\theta_i > c$ for all $i = \{L, H\}$. Consider for simplicity that $\theta_L > \frac{\theta_H + c}{2}$, which implies that each type of student would buy the textbook, both when the firm practices uniform pricing and when it sets two-part tariffs (as we next show).

Uniform pricing. Consider first that the monopolist does not practice price discrimination (i.e., it sets a uniform price that induces both types of customers to purchase positive units). In this setting, the monopolist sets a unique price p that solves the expected PMP

$$\max_p \gamma[(p - c)(\theta_L - p)] + (1 - \gamma)[(p - c)(\theta_H - p)],$$

where $q_i = \theta_i - p$ for every type- i customer. Taking first-order conditions with respect to p yields

$$\gamma(\theta_L - p) - \gamma(p - c) + (1 - \gamma)(\theta_H - p) - (1 - \gamma)(p - c) = 0$$

And solving for p we obtain the uniform price

$$p^{Uniform} = \frac{\gamma\theta_L + (1 - \gamma)\theta_H + c}{2},$$

which yields monopoly profits of

$$\pi^{Uniform} = \frac{[\gamma\theta_L + (1 - \gamma)\theta_H - c]^2}{4}$$

Note that the monopolist could use a uniform price to only serve high-demand students. The price that would maximize its profits in this case solves

$$\max_p (1 - \gamma)(p - c)(\theta_H - p)$$

thus ignoring the segment of low-demand students. Taking first-order conditions with respect to p and solving for p yields $p_H = \frac{\theta_H + c}{2}$. In this context, monopoly profits become

$$\pi^{Uniform-H} = (1 - \gamma) \frac{(\theta_H - c)^2}{4},$$

which are larger than those when serving both types of students (i.e., $\pi^{Uniform-H} > \pi^{Uniform}$) if the proportion of low-demand customers, γ , is sufficiently small, that is,

$$\gamma < \frac{(\theta_H - c)(\theta_H - 2\theta_L + c)}{(\theta_H - \theta_L)^2}.$$

Intuitively, the frequency of high-value customers is large, thus inducing the seller ignore low-value customers to focus on high-value customers alone. For instance, parameter values $\theta_H = 5$, $\theta_L = 2$, $c = 1$ and $\gamma = \frac{3}{4}$ satisfy this condition since

$$\gamma < \frac{(\theta_H - c)(\theta_H - 2\theta_L + c)}{(\theta_H - \theta_L)^2} = \frac{(5 - 1)(5 - 4 + 1)}{(5 - 2)^2} = \frac{8}{9}.$$

Otherwise, if $\gamma > \frac{8}{9} \cong 0.88$, the proportion of low-value customers is large enough to induce the seller to not ignore this type of buyers, and thus serve both types.

Two-part tariffs. Let us now consider that the monopolist offers a menu of two-part tariffs to each type of student (i.e., (F_L, q_L) and (F_H, q_H)). From the previous discussion, we know that IC_H and PC_L bind. Therefore,

$$F_L = \theta_L \left(q_L - \frac{q_L^2}{2\theta_L} \right) \text{ and } F_H = \theta_H \left[\left(q_H - \frac{q_H^2}{2\theta_H} \right) - \left(q_L - \frac{q_L^2}{2\theta_L} \right) \right] + \theta_L \left(q_L - \frac{q_L^2}{2\theta_L} \right).$$

Hence, the monopolist's PMP becomes

$$\begin{aligned} & \max_{q_L, q_H \geq 0} \gamma [F_H - cq_H] + (1 - \gamma) [F_L - cq_L] \\ &= \gamma \left[\underbrace{\theta_H \left[\left(q_H - \frac{q_H^2}{2\theta_H} \right) - \left(q_L - \frac{q_L^2}{2\theta_L} \right) \right] + \theta_L \left(q_L - \frac{q_L^2}{2\theta_L} \right) - cq_H}_{F_H} \right] + (1 - \gamma) \left[\underbrace{\theta_L \left(q_L - \frac{q_L^2}{2\theta_L} \right) - cq_L}_{F_L} \right] \\ &= \theta_H \gamma \left(q_H - \frac{q_H^2}{2\theta_H} \right) + (\theta_L - \theta_H \gamma) \left(q_L - \frac{q_L^2}{2\theta_L} \right) - c\gamma q_H - c(1 - \gamma)q_L \end{aligned}$$

Taking first-order conditions with respect to q_H and q_L yields

$$\begin{aligned} \gamma \theta_H \left(1 - \frac{q_H}{\theta_H} \right) &= c\gamma \rightarrow q_H = \theta_H - c \\ (\theta_L - \theta_H \gamma) \left(1 - \frac{q_L}{\theta_L} \right) &= c(1 - \gamma) \rightarrow q_L = \theta_L - c \frac{\theta_L(1 - \gamma)}{(\theta_L - \theta_H \gamma)} = \theta_L - c \frac{1 - \gamma}{1 - \frac{\theta_H}{\theta_L} \gamma}, \end{aligned}$$

which are both positive since $\theta_L > c \frac{1 - \gamma}{1 - \frac{\theta_H}{\theta_L} \gamma}$, given that $\theta_H > c$ by definition. On the other hand, socially optimal outputs can be solved by setting marginal utility equals to marginal cost (i.e., $u'_i(q_i) = \theta_i \left(1 - \frac{q_i}{\theta_i} \right) = c$), thus yielding $q_H^{SO} = \theta_H - c$ and $q_L^{SO} = \theta_L - c$.

We can then compare q_i against q_i^{SO} for every type- i customer obtaining that $q_H = q_H^{SO}$, and that $q_L < q_L^{SO}$, since

$$\theta_L - c \frac{1 - \gamma}{1 - \frac{\theta_H}{\theta_L} \gamma} < \theta_L - c$$

This reduces to $1 - \gamma > 1 - \frac{\theta_H}{\theta_L} \gamma$, which is true given that $\theta_H > \theta_L$ by definition. The monopolist can obtain larger profits by practicing second-degree price discrimination (two-part tariffs) than by setting a uniform price (either to attract both or only one type of customer). Using the same parameter values as under uniform pricing, $\theta_H = 5, \theta_L = 2, c = 1$ and $\gamma = \frac{3}{4}$, we obtain we obtain output levels $q_H = 4,$

$q_L = \frac{16}{7}$, and fees of $F_H = \frac{444}{49} \cong 9.06$ and $F_L = \frac{96}{49} \cong 1.95$. As a consequence, expected profits from two-part tariffs are

$$\pi^{TPT} = \gamma[F_H - cq_H] + (1 - \gamma)[F_L - cq_L] = \frac{26}{7} \cong 3.71$$

In contrast, those under uniform pricing become

$$\pi^{Uniform} = \frac{[\gamma\theta_L + (1-\gamma)\theta_H - c]^2}{4} = \frac{49}{64} \cong 0.76, \text{ and}$$

$$\pi^{Uniform-H} = (1 - \gamma) \frac{(\theta_H - c)^2}{4} = 1$$

Hence, practicing two-part tariffs is profit-enhancing for the monopolist since

$$\pi^{TPT} > \pi^{Uniform-H} > \pi^{Uniform}.$$